Design and Control of a Flapping Flight Micro Aerial Vehicle

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CMU-RI-TR-14-19

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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August 2014

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Abstract

Miniature flapping flight systems hold great promise in matching the agility of their natural counterparts, bees, flies, and hummingbirds. Characterized by reciprocating wing motion, unsteady aerodynamics, and the ability to hover, insect-like flapping flight presents an interesting locomotion strategy capable of functioning at small size scales and is still a current focus of research. A vehicle with the capabilities of a fly would have potential use as miniature nodes in sensor networks, near invisible surveillance platforms, and mobile vehicles in search and rescue. Designing and constructing such systems, however, is difficult. Beyond the limits of battery capacity and the difficulties of miniature sensor design, simply producing enough lift for liftoff is a challenge. A balance must be maintained between mechanical complexity, controllability, and weight. While more actuators generally lead to more controllable degrees of freedom, they also contribute significantly to system mass.

In light of these constraints, we choose to utilize passive behavior and mechanical resonance when possible. We develop platforms utilizing passive wing rotation, where the wing leading edge is driven and the trailing edge is allowed to rotate based on elastic energy storage, wing aerodynamics, and inertial effects. Wing flapping motion is allowed to resonate through the choice of cantilever actuator or added elastic element. Systems are constructed at two different size scales, using piezoelectric actuators and motors to drive the wing leading edge.

In this work the design of several controllable flapping flight micro aerial vehicles is discussed and platform underactuation, control development, and active and passive stability is examined. At under one gram, both a 700 mg and 160 mg system are constructed with a single piezoelectric actuator driving each wing. Design considerations including structure rigidity, controllability and mass centralization are considered, with body finite element analysis and wing coupling tests performed. The constructed 160 mg prototype is shown to achieve a lift-to-weight ratio of $\sim 3/8$. With an actuator driving each wing, the system is capable of producing altitude controlling forces as well as pitch and roll torques with a change in wing flapping amplitude. An alternative means of generating wing asymmetry for lift control is proposed and implemented with a shape memory based flexural hinge. Lift control is demonstrated on a modified flapping platform with an application of heat. At a larger 3 and 7 grams, a two and four wing motor-driven flapping platform is designed and constructed. With the use of an elastic element in parallel with the flapping motion, the motor driven design is able to resonate, resulting in a novel and simple lift-off capable system. Motor, flapping frequency, and wing size are chosen based on impedance matching criteria, and further experimentally optimized. Control of both piezo and motor-driven platforms is demonstrated in both simulation and in limited control experiments with a developed robust and linear controller respectively.
Acknowledgments

This thesis wouldn’t have been possible without the help and support of my advisor Dr. Metin Sitti as well as all the members of the NanoRobotics Lab here at Carnegie Mellon University. I would like to specifically thank Dr. Slava Arabagi, David Colmenares, and Dr. Domenico Campolo who have been my coworkers and collaborators on various iterations of the flapping-wing systems over the years. From sanity checks to inspiring discussions and general hard work, your assistance was very much appreciated.

To my family, thank you for your patience and for your support. This chapter of my life is over, and I can’t wait to see what’s next.
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Chapter 1

Introduction

1.1 Motivation

While engineers have created flying platforms that greatly outmatch the natural world in speed, carrying capacity, and flight time, nothing yet rivals the common housefly in sheer acrobatic ability and compact system size. Not only are they capable of hovering, they can also perform very fast maneuvers; one such example is the saccade wherein which a $90^\circ$ direction change can be accomplished in less than 50 ms [1]. Flapping flight in general scales favorably to smaller sizes [2], leading to flying insects almost invisible to the unaided human eye. *Dicopomorpha echmepterygis* (fairyflies) have body lengths as short as 0.11 mm and take advantage of their small size by attaching themselves to other flying insects for long trips [3]. Artificial systems with similar capabilities would be perfectly suited for applications were it is necessary to maneuver through very small spaces and navigate cluttered environments. They could be used as cheap, miniature nodes in sensor networks, near invisible spies in military applications, and in swarms for exploring rubble in search and rescue missions.

Designing a fully autonomous flapping wing micro aerial vehicle (FMAV) at the scale of a fly, however, is a great challenge. Beyond a strictly limited battery capacity and the requirement of new, of miniaturized sensors, simply producing enough lift for lift-off is still a focus of
ongoing research. Currently manufactured actuators, while generally powerful, tend to be comparatively large and heavy. As clearly demonstrated in the natural world, there is an effective upper limit of the size and weight of systems capable of sustained hovering. The largest animals that can hover are the *Patagona gigas* (giant hummingbird) and the *Glossophagine phyllostomid* (flower bat), weighing up to 22 grams and 32 grams, respectively [4], [5]. This is several orders of magnitude less than the largest flying animal, the Kori bustard, which weighs up to 16 kg [6].

In isometrically scaled systems, aerodynamic forces on flapping wings are typically proportional to wing area and scale in proportion to \( m^{0.67} \), where \( m \) is mass. As the system increases in size and correspondingly in weight, the flapping wings will eventually not be able to support the increased weight [7]. Actual size scale limits on flight performance, however, are thought to be a combination of both energetic and aerodynamic constraints, which in birds and insects, stem from a variety of bio-mechanical and physiological effects [7], [6]. Technological limitations present a similar bound on artificial systems, making matching insect performance a careful balance of platform mechanical complexity, weight, and controllability and inspire the development and optimization of new actuators and power saving approaches.

### 1.2 Previous Works

Over the years, researchers have worked to understand natural fliers to better apply and improve upon any advantageous physical mechanisms. Listing just a few, animals ranging from hawkmoths [8], hummingbirds [9], flies [10], and dragonflies [11] have been studied to gain a better understanding of both organism lift production and control. In flapping-wing systems, lift generation comes entirely from the repetitive wing motion; the system must be able meet the necessary power requirements to support its own weight, overcome skin friction drag, and accelerate the wings which have non-negligible inertia [12]. Characteristics such as flapping resonance and passive wing rotation have each been observed in natural systems and serve to reduce the power requirements for wing flapping [13], [11]. Flight performance is also significantly affected by the
decrease in Reynolds number in small scale systems. Reynolds number, \( Re \), is the ratio of inertial to viscous fluid forces, and as size scale decreases, viscous effects tend to dominate. Unsteady high-lift mechanisms such as rotational circulation, clap-and-fling, wake-capture, and dynamic stall have each been shown to boost lift in natural flapping-wing hovering systems, and are in part possible due to the laminar flow behavior characteristic of these relatively low Reynolds number regimes [6], [2], [14], [15].

Artificial flapping systems inspired by natural fliers have been constructed, but while bird-inspired ornithopter designs have been demonstrated with improved lift coefficients over fixed wings [16] and have proven successful in applications ranging from pest control to children’s toys [17], [18], systems capable of hover are more rare. Utilization of resonance when possible, underactuation, and designing for passive stability are approaches frequently used, though to date, only two systems have demonstrated controlled hovering flapping flight [19], [20].

1.2.1 Rotary Wing vs. Flapping Wing Systems

Helicopters are the only other platforms, besides systems with directed jet propulsion, that are capable of hovering. Lift is produced by rotating wings or blades which are typically driven with either engines or electric motors depending upon platform size. In large systems, the efficiency and magnitude of lift production using rotating airfoils is difficult to improve upon. As size scale decreases however, flapping wing flight has a number of advantages related to actuation, agility, and aerodynamic power expenditure.

Currently one of the smallest rotary wing systems is the magnetically actuated device by Miki et al. Without a battery, the system weighs only 3.5 mg and is composed of a structure with two soft, rotational wings each 900 \( \mu \text{m} \) long. Each wing was electroplated with a nickel-iron alloy to allow the blades to be rotated with a magnetic field. Under guide wires, it was capable of flight up to 8 seconds in length but was restricted to operation within a magnetic coil system [21], [22]. Glauvitz’s PolyMUMPs fabricated rotary wing design was similar in scale, with a
rotor diameter of only 2 mm. A scratch drive was used to rotate six blades, but the total weight of the system prohibited sufficient blade speed to achieve liftoff [23]. At 3 grams and a width of 4 cm, the larger Mesicopter relies on the traditional motors for blade actuation. The platform is composed of polymeric materials with four rotors and has demonstrated liftoff with a tether [24]. The smallest untethered helicopter to date is the Prox Dynamics Pico-flyer at 3.3 grams [25]. The larger Prox Dynamics PD-100 Black Hornet incorporates an onboard GPS at 16 grams [26].

While there have been several sub 5 gram rotary wing systems, in general, fast rotary motion is difficult to achieve as system size continues to decrease. Electromagnetic forces scale down poorly at $L^4$, where $L$ is the wire length in a magnetic field. The torque that can be produced by a motor drops significantly, making them an unfeasible choice in many platforms and leaving few alternative actuators that are capable of achieving full revolutions at sufficient speed.

Compared to a fully actuated hummingbird wing, helicopters or multi-rotor systems also suffer from reduced acrobatic agility due to limitations on the producible body forces and moments. The typical helicopter is controlled in part through the use of collective and cyclic control inputs allowed by a swash plate. Translational motion is achieved by cyclic change of blade angle of attack and the resulting body tilt in the desired direction of travel. A fully controlled oscillating airfoil, however, has the potential of creating an almost arbitrary mean body forces and wrench over a wing beat cycle, within limits, allowing fly and hummingbird more acrobatic maneuvers.

Though rotary wing systems can also take advantage of the lift improvements seen with delayed stall at low Reynolds numbers [27], rotational circulation, wake capture and clap-and-fling seem to be unique to flapping wing motion. Each can significantly increase overall lift production over a wing stroke from 10 to over 50% [28], [2]. Below a Reynolds number of 100, recent work by Zheng et al. has also shown that wing flapping is more efficient than wing rotary motion in a rigid planform. Using an insect inspired wing shape, the power required to produce equivalent lift was estimated for a spinning and flapping wing using computational fluid dynamic
As Reynolds number decreased, viscous drag dominated in the rotary wing, significantly increasing power expenditure [29].

### 1.2.2 Motor-Driven Flapping Platforms

The reciprocating wing motion in a flapping system can be generated through a range of actuators and transmissions, though typically one of two actuators is used depending on the desired system size scale. In Fig. 1.1, data taken and modified from [30], the power density of several actuators is plotted as mass varies. At less than 1 gram, the power density of piezoelectric actuators begins to rival and then exceed that offered by conventional motors. With a high blocking force and response time, the reciprocating motion of the piezoelectric cantilever actuators is well suited to flapping systems. As mass increases, motors are typically a better choice and are most frequently partnered with a crank and rocker mechanism to achieve the appropriate wing motion.

Developed at the Technical University of Delft, the family of platforms Delfly I, II, and Micro are among the smallest flapping flight systems capable of autonomy. With four wings, the design takes advantage of the clap-and-fling flapping mechanism to increase maximum system lift; as the wings are brought together and pulled apart, fluid flow at the opening gap creates an advantageous air circulation about the wing. The wings have a high amount of flexibility, aiding clap-and-fling and effectively creating passive camber deformation in their airfoil. While not capable of true hovering, the systems are capable of very slow forward flight. To allow control, magnetic actuators are used to change the rudder and elevator at the tail. The Delfy micro has a wingspan of 10 cm and a low total weight of 3 grams, including battery, camera, and wireless transmitter [31].

The flapping flight platform developed at Cornell has, depending upon design iteration, four wings or eight wings [32], [33]. Due to their symmetric placement about the body and added fins at the top and bottom of the platform, the system is passively stable about hover. In the smaller 3.9 g version of the prototype, wings and supporting body structure are 3D printed,
Figure 1.1: Power density versus mass of selected actuators. Figure is recreated from [30] and augmented with the brushed DC motor data tabulated in Tables B.1, B.2, B.3. In this figure, SMA refers to actuators formed of shape memory alloy. Actuators that create a force or displacement using thermal expansion are marked as ‘thermal’.

allowing fast design variation and testing [33]. As there is only a single actuator driving the wings, only total lift force can be changed, preventing the system from maneuvering without additional modifications.

Using a self-constructed electromagnetic actuator, Vanneste et al. created a 22 mg flapping system with a resonant thorax and polymer body. Flapping at 30 Hz, the system was capable of producing 75% of lift necessary for liftoff with passive wing rotation using a compliant link [34], [35]. Similar wing coupling with a single actuator, however, would prevent control even if liftoff was achieved.

In 2011, AeroVironment unveiled their hummingbird inspired platform capable of controlled flight and hovering. At 19 g with a wingspan of 16.5 cm, the platform is currently the only motor-driven system capable of hovering with two reciprocating wings [20]. Control is accomplished with asymmetric wing tensioning between left and right wings, allowing roll, pitch, and yaw torques to be produced. Outdoor flight in light wind conditions as well as maneuvers such as
the 360° lateral flip demonstrate system robustness and sheer acrobatic ability. However, the complexity of the mechanism would complicate mass manufacturing and could limit their use in a swarm application. In addition, any reduction of platform scale for miniaturization is similarly complicated.

1.2.3 Piezo-Driven Flapping Platforms

Piezoelectric (PZT) bending cantilever actuators present a suitable alternative to motors at the small size scale with high energy densities, high speed, and low weight, and have been investigated and optimized for use in flapping wing micro aerial vehicles [36], [37]. In each case, the platform is designed to resonate at its flapping frequency, significantly reducing the necessary power to flap its wings. Two main approaches have been taken to generating the appropriate wing trajectory.

In the first, the wing is allowed to passively rotate. The Harvard MicroFly was the first sub-gram platform with a lift-to-weight ratio of greater than one [38]. The platform has a total weight of 60 mg, a wing span of 3 cm, and a wing beat frequency of 110 Hz. In this first prototype, a single PZT actuator drives the leading edge of both wings while the trailing edge is allowed to rotate about a flexural hinge. In the work of Finio et al., additional small bending and twisting actuators are shown capable of creating wing flapping asymmetry without much additional weight, allowing the generation of controlling torques [39]. Later iterations with an actuator driving each wing have demonstrated controlled hovering [19]. Similarly in the work of Greenyer, passive wing rotation is induced by the oscillation of a resonant piezoelectric actuator and structure. The single actuator is used to flap two wings, which are able to achieve similar wing trajectories through wing deformation [40].

In contrast, the Micromechanical Flying Insect (MFI) developed at UC Berkeley has a total of four PZT bending actuators capable of controlling each wing’s leading and trailing edges independently [41]. The platform has a wingspan of 25 mm and operates at 275 Hz. With
control over wing trajectory and therefore body torques, the platform was shown to be stable about hover in simulation [42]. However, due the actuators’ additional weight, the lift produced by the platform prototype was not sufficient for liftoff.

1.2.4 Generating Controlling Forces and Torques

Of the mechanical flapping systems constructed, the MFI is the most similar to acrobatic insects in its approach to lift and controlling torque generation. It is known that insects are able to control their wing trailing edge, which has been shown to be useful during maneuvering in the more acrobatic species [43]. Actuators, however, typically contribute the majority of overall system mass and full wing actuation in this case came at the cost of lift-off.

Utilizing passive wing rotation has become a recent trend in the effort to reduce system mass. In fact, recent studies have shown that at least a portion of insect wing rotation is passively achieved through inertial and aerodynamic forces [11]. Passive wing rotation, from either wing deformation [33], [31], [40] or through the use of a hinge [38], [44], [45], [46] has been applied in a number of platforms. Of course, with the resulting underactuated wings, an alternative mechanism or mechanisms must be implemented in order to achieve system control.

In both the Harvard MicroFly and the Nano Hummingbird, the two systems capable of controlled hovering, differentiation is made between the actuators dedicated to providing the majority of system power and actuators dedicated to control. In Finio et al.’s design iteration of the MicroFly and in the final Nano Hummingbird, a single large actuator is used to drive the flapping motion of both wings [39], [20]. Smaller, light weight actuators are adding to create bilateral asymmetries between the individual wings. Originally inspired by the more complex insect orders including Diptera which have two distinct muscle groups controlling wing motion [47], this approach serves to reduce the impact additional actuators have on the overall weight of the system while allowing generation of controlling torques.
1.2.5 Controller Design

Flapping flight systems are naturally highly dynamic and time-variant which, when also considering inbuilt additional platform limitations, can make control a challenge. One approach, implemented in Cornell’s platform, is to design passive stability into the system, which ensures operation about hover [32],[33]. Practically, however, one cannot make the system too stable as this tends to eliminate all system maneuverability. In general, flapping wings act to damp the system’s motion, providing a stabilizing effect about certain body axes. Both Hedrick et al. and Cheng et al. have investigated body damping from wing and body motion, determining a linear dependence on translation and angular velocity and an increase in damping from increased wing beat frequency and amplitude [48],[49].

In actively controlled systems, averaging theory is widely used in control development, allowing approximation of the time-variant system with its time-invariant average [50]. In flapping systems, as long as flapping frequencies are sufficiently high and the wing generated forces sufficiently filtered by the body dynamics, a controller can be designed about the simplified system where wing forces are averaged over each wing beat. Using a linear approximation, the MFI design team was able to demonstrate controlled hover in simulation using linear-quadratic regulator (LQR) after parameterizing their fully controlled wing trajectory [42]. Khan et al. demonstrated longitudinal control in simulation of a flapping flight system using a differential flatness based nonlinear controller, with the assumption that mean lift force could be commanded [51]. A nonlinear robust controller was developed by Serrani for a robot in the longitudinal plane, allowing change in wing beat frequency and variable stroke plane angle [52].

The majority of previous work on flapping-wing system controllers has been performed in simulation due to the lack of platforms that are capable of both liftoff and controlling forces and torques. However, recently successful free flight control has been demonstrated in experiment with a variation of the Harvard Fly. In [53], a modified model free linear controller allowed controlled platform hovering. A performance improvement was demonstrated in [54] where an
adaptive controller was implemented.

## 1.3 Challenges and Open Issues

There remain a number of open issues that must be addressed in order for both existing and new flapping systems to achieve both the acrobatic agility of a hummingbird as well as autonomy. As weight and size constrained miniature systems, new sensors must be developed to meet small payload requirements and battery technology must be improved to allow increased flight time. Lift must improved with the design of new actuators, simplified lightweight structures and mechanisms, and new wings optimized for their application [55], [56], [57].

There have been a number researchers working on the development of small and biologically inspired sensors for miniature systems [58], [59], [60], [61]. The fly haltere and the moth antenna function as gyroscope sensors [62], [63], and efforts have been made to create devices that perform similar functions. Recent artificial halteres have been shown to well capture angular rates while maintaining a small device profile [64], [61]. Optical flow, or the direction of apparent motion of objects or edges in a field of view, has also been shown to be useful for both stable flight and navigation in insects [65], and has been developed and used for robot control [66], [67]. Optical flow is well suited for simple, light weight, low optical sensors, easing their incorporation into small platforms. Currently, both ocelli inspired optical sensors and magnetic field sensors have also been incorporated into to the Harvard Fly and used for system control [68], [69].

In robotic systems under one gram, there is currently no existing feasible on-board power source. Batteries continue to decrease in size, but due scaling limitations, cannot supply sufficient power for more than seconds of flight. Miniature fuel cells, supercapacitors, and radioactive thin films have been proposed as alternatives, but are yet unrealized [57]. In sub-ten gram systems performance is better, yet the Delfly Micro at 3 grams is still only capable of flights up to 3 minutes in length [70]. New actuators and light weight power electronics to decrease power
consumption and weight, and increase power output are also open areas of current research to address this constraint [71], [72], [73], [74].

In addition, precise manufacturing becomes critical in order for mechanisms to function as intended in platforms under one gram. The Smart Composite Microstructures (SCM) manufacturing technique has allows millimeter scale structures and actuators to be constructed with low error tolerance in sequential layering and laser cutting of materials [75], [76]. SCM works well for two dimensional, folded flexural mechanisms, but when considering components of more arbitrary three dimensional shape and material, challenges remain. Micro-forming, micro-assembly, and micro-molding are currently being investigated for continued reduction in size scale and increase in accuracy [77], [78].

Aerodynamic studies continue to be of interest in order to better understand and usefully apply the mechanisms of lift production on flapping wings. While many wing optimizations have been done, they are primarily platform specific and traditionally encompass the experimentally testing a variety of manufactured wings. Improvements in manufacturing, as well as better understanding the impact of shape, flexibility, and surface texture on lift production are important for improving overall platform lift generation [79], [80], [81], [82], [83].

1.4 Our Approach

While there are many challenges still ongoing in the development of flapping wing platforms, this work focuses on the fundamental problem of lift production. As previously mentioned, only two systems to date are capable of both vertical liftoff and controlled hovering, the Harvard Fly and the AeroVironment Nano Hummingbird [19], [20]. While there exist a number of flapping systems capable of vertical takeoff, most cannot generate controlling forces and torques. The additional actuators necessary typically increase total system mass to an extent liftoff is not possible. We work to balance mechanical complexity, system controllability, and weight for FMAVs, with the goal of creating a new system that is capable of liftoff and controlled flight.
Figure 1.2: Flapping frequency versus weight of selected orders and families of birds and insects. The figure is reprinted from [7] and augmented with the parameters of several constructed flapping-wing systems. To note, Trochilidae is the family of hummingbirds, while the other orders listed encompass insects including flies, moths, butterflies, dragonflies, etc. The constructed systems marked with † are described in this work.
Here, systems at two size scales are designed and described. In each system, both mechanical 
resonance and passive wing rotation is utilized to conserve expended power and reduce overall 
system weight. Two different approaches are taken: (1) a modular wing/actuator design and (2) 
coupled wing driving with small control actuators. In the modular wing/actuator approach, a 
single actuator is designed to drive the leading edge of a passively rotating wing. Each wing 
and actuator module is designed to be able to function independently, allowing control over each 
wing flapping angle. This design, which enables change in wing flapping amplitude and center of 
wing sweep, allows generation of both pitch and roll torques. In (2), an approach similar to Finio 
et al. [39] is envisioned where two wings are driven with a single actuator and small actuators 
create wing flapping asymmetries. A new control actuator made of shape memory polymer and 
designed as a multi-layer composite flexure is developed to modify wing stroke amplitude.

The final result is four main prototypes with various capabilities. The modeling and design 
of a 700 mg and 160 mg piezo-driven system are detailed, the larger of which is tested and 
controlled on a restricted degree of freedom rig. A two and four-wing motor driven system are 
also described, both of which are capable of controlling torques and liftoff. Though not explicitly 
considered in the design, each of the four developed platforms compare similarly to the trends 
seen in biological flapping-wing systems. An example of which can be seen in Fig. 1.2, where 
the relation of flapping frequency and mass of selected orders and families of hummingbirds and 
insects is plotted.

### 1.5 Thesis Outline

This work is divided into seven chapters. In Chapter 2, the aerodynamic model of a passively 
rotating, flapping wing is detailed. This model is then incorporated into a full dynamic simulation 
of the piezoelectric actuator-driven and motor-driven systems with a passively rotating wing. The 
dynamic model of the piezoelectric actuator-driven system was originally described in [84]. In 
Chapter 3, the design and construction of a large and small scale piezo-driven flapping-wing
system is described. Results from this work are adapted from [85]. In addition, the design and testing of a new shape memory polymer based control actuator is discussed, results of which were expanded from [86]. In Chapter 4, the design of a two and four-wing motor driven system is detailed. Results from this chapter are adapted from [87]. In Chapter 5, a linear and robust control method is developed and tested on each system respectively to help demonstrate controllability. Finally, in Chapter 6, thesis concluding remarks are made.

1.6 Contributions

The major contributions of this work are in the development of a two novel platforms based on passive wing pitch reversal that are capable of controlling torques. The larger, motor-driven platform is capable of liftoff. In the course of this research, contributions toward the following areas are expected:

- A new piezo-driven flapping-wing mechanism is designed and the capability of generating controlling torques is demonstrated.
- A new, simple, motor-driven flapping-wing mechanism is designed that is capable of lift-off and controlling torques. Physical parameters are experimentally varied to optimize lift production. A two and four-wing motor driven design is examined and characterized for system controllability.
- A full dynamic model is developed for use in system optimization for motor-driven passive wing rotation platforms. The aerodynamic model previously described by Arabagi [84] is verified for use on the large flapping platform.
- Robust and linear control methods are developed for the flapping-wing mechanisms and used to verify controllability.
- A new, lightweight, variable stiffness controlling actuator is developed for flexure based mechanisms and is tested on the piezo-driven flapping-wing system.
Chapter 2

Model of a Passively Rotating Wing

2.1 Introduction

In a hovering fly, bee, and hummingbird, wing motion is characterized by a forward and reversed flapping stroke where the wing is rotated such that positive lift force is produced over both directions of travel. In an artificial system this can be reproduced by relying upon passive wing rotation, which allows similar production of lift without an additional weighty actuator. Figure 2.1 shows an example of such wing motion where the top view of a single passively rotating wing can be seen.

Each of the developed platforms in this work use passive wing rotation to reduce overall platform weight. In this chapter, models of the flapping wing and aerodynamic forces are described to better understand system behavior and to better optimize performance. A simulation and design tool for a passively rotating wing driven by a piezoelectric actuator has been derived in [84]; here a description of the aerodynamic model and dynamic equation reviewed, and adapted to a motor-driven system.
Figure 2.1: Motion of a single, passively rotating wing flapping at 10 Hz. The top view of the wing is shown here with snap shots taken every 0.01 sec.
Figure 2.2: Schematic of the passive wing. Coordinate systems represent transformations by \( \theta \) (flapping angle), and \( \phi \) (wing rotation angle). The coordinate systems are shifted for clarity, while in simulation they are all centered at the point labeled ‘coordinate sets origin.’ The \( \vec{E}'' \) coordinate system is attached to the wing.

\section{2.2 Aerodynamic Model of Wing Lift and Drag Forces}

The aerodynamic forces on a flapping wing are composed of several components that can be broken down and modeled separately. Combined, the total force on the wing is:

\[ F_{\text{tot}} = F_t + F_{\text{rot}} + F_{\text{air}} + F_{\text{wc}}, \]  

(2.1)

where \( F_t \) is the translational force, \( F_{\text{rot}} \) is the force due to wing rotation, \( F_{\text{air}} \) is the force due to added air mass, and \( F_{\text{wc}} \) is the force created by wake capture. Force generated through the translational motion of the wing can be estimated from approximations adapted from thin airfoil theory. Wing rotational force, depicted in Fig. 2.3b represents the drag forces produced by the wing motion about its rotational joint. The added air mass forces are due to the additional air mass that accompanies the wing and are generated through unsteady wing motion [88]. \( F_t, F_{\text{rot}}, \) and \( F_{\text{air}} \) can be modeled quasistatically, however, the forces due to wake capture occur due
Figure 2.3: Wing profile illustrating the wing’s physical parameters can be seen in (a). The wing rotational lift coordinate system shown is fixed to the wing with the basis vector $E''_2$ normal to the wing’s surface. The wing center of gravity is marked CG. A schematic describing the relevant variables for calculation of wing rotational lift is shown in (b).
to an unsteady effect wherein the wing interacts with vortices produced in its previous stroke. Since it cannot be included in a closed form expression, and has so far only been well captured in computational fluid simulations, it is not included in the model. Thus, the expressions for aerodynamic forces acting on chord-wise wing strips, $dr$, as depicted in Fig. 2.3a take the form [84]:

$$dF_t = \frac{1}{2} \rho U^2 c(r)[C_l^2(\alpha) + C_d^2(\alpha)]^{1/2} dr, \quad (2.2)$$

$$dF_{rot} = -\frac{1}{2} C_{rot} \rho |\dot{\phi}| \int_{c(r)} z'' |z''| dz'' dr, \quad (2.3)$$

$$dF_{air} = -\frac{\rho \pi}{4} (\ddot{\theta} \cos \varphi - \dot{\theta} \dot{\varphi} \sin \varphi) rc(r)^2 dr - \frac{\rho \pi}{4} \ddot{\varphi} (z_{RA} - \frac{c(r)}{2}) c(r)^2 dr, \quad (2.4)$$

where $\rho$ is the air density, $r$ is the radial position of a wing-strip from the wing’s flapping axis, $c(r)$ is the chord length of the particular strip, $\theta$ and $\varphi$ are the flapping and rotation angles, respectively, $\alpha$ is the angle of attack of the wing, or $(\pi/2 - \varphi)$, and $z_{RA}$ is the location of the rotation axis measured from the wing’s leading edge. This formulation is taken from Arabagi et al. [84], which is adapted from [28], [89]. The translational force component is obtained by vector addition of mutually orthogonal lift and drag forces. The velocity $U$ is taken to be the velocity in the $\vec{E}'_2$ direction of each wing strip’s mid-chord point:

$$U(r) = \vec{v}_{mc} \cdot \vec{E}'_2 = r \dot{\theta} + \frac{c(r)}{2} \dot{\varphi} \cos \varphi, \quad (2.5)$$

where $\vec{v}_{mc}$ is the mid-chord velocity of each wing division. As shown in Fig. 2.2, the coordinate system $\vec{E}'$ is fixed to the wing flapping axis. In [89], a constant velocity field is used to obtain the translational lift coefficients. Here, the mid-chord velocity is used, as it is a good approximation to the incoming air stream velocity field which is linearly distributed along the wing chord. $C_l$, $C_d$ and $C_{rot}$ are the translational lift, drag and rotational force coefficients, respectively, where
the translational coefficient expressions were tabulated experimentally by Sane and Dickinson for a Reynolds number (Re) of 136 \([28]\):

\[
C_l(\alpha) = 0.225 + 1.58 \sin(2.13\alpha - 7.2), \tag{2.6}
\]

\[
C_d(\alpha) = 1.92 - 1.55 \cos(2.04\alpha - 9.82), \tag{2.7}
\]

where \(\alpha\) is in degrees. The value for the rotational drag coefficient \(C_{rot}\) was taken to be 2, as this is the theoretical result of rotational drag on a flat plate subjected to normal flow \([90], [91]\). To account for the radial variability of wing velocity, the above equations need to be integrated numerically to obtain the aerodynamic forces on the entire wing.

In nature, the Reynolds number in biological systems ranges from approximately 100 for houseflies, 10000 for hummingbirds, and 50000 for forward flying birds \([92], [93]\). In this work we examine systems of several sizes with several different wing lengths and operating frequencies, with correspondingly varying Reynolds numbers. For flapping-wing systems the effective Reynolds number can be calculated from the expression \(Re = \frac{\bar{c} U_{tip}}{\nu}\), where \(\bar{c}\) is the mean wing chord length, \(\nu\) is the kinematic viscosity of air, and \(U_{tip}\) is the mean wing tip velocity \((= 2\theta_{pp} f(R + d_w))\) with \(R\) as the wing length, \(d_w\) the horizontal offset of the wing base from the flapping axis, and \(\theta_{pp}\) the peak-to-peak wing flapping angle. The wing length and offset is shown defined for the motor-driven system in Fig. 4.1B. In the large scale piezoelectric driven systems the Reynolds number is \(Re = 1160\) and in the motor-driven system the Reynolds number is \(Re = 5660\).

A similar Reynolds number indicates that the fluid flow behavior is dynamically equivalent, independent of the specific physical system. Below \(10^3 - 10^4\), fluid flow is typically laminar, whereas turbulence becomes more common in higher regimes. The lift coefficients used in this work are were experimentally determined by Dickinson et al. using the constructed RoboFly, which operates at \(Re = 136 [28]\). While the manufactured systems here are similar in an aerodynamic sense to the RoboFly, it is worthwhile to verify both the aerodynamic model and lift
coefficients for the larger motor-driven system.

2.2.1 Instantaneous Lift Force Comparison of Wing Simulation and Experiments

In order to verify the aerodynamic model and confirm its validity at the higher Reynolds numbers of our flapping systems, wing instantaneous lift is measured experimentally while wing kinematics are simultaneously recorded. The measured wing kinematics are then used to simulate the expected lift using the previously described aerodynamic model.

A motor-driven wing of length of $R = 70$ mm, maximum chord length of $c_{\text{max}} = 25$ mm, and offset $d_w = 35$ mm was mounted to an 6 DOF force/torque sensor (ATI-Nano17 TI) and driven with a sinusoidal input voltage with a peak-to-peak amplitude of $V_{pp}$. The wing used is depicted in Fig. 2.3a. The wing is mounted to a flexure which allows rotation based on the flexural
stiffness, wing inertia, and airflow interacting with the wing. Specific experimental mounting
details can be found in Section 5.4.1 and prototype details in Section 4.2. Input voltage is set at
10 Hz and $V_{pp}$ is swept from 8 to 13 V. The lift is recorded at 50 kHz and is effectively reduced
to 5000 Hz by a block averaging filter. A low pass filter with a cutoff frequency of 50 Hz is
then applied. Wing kinematics are recorded at 500 fps with a high speed camera (pico.dimax)
positioned above the system. Three points are tracked on the wing, as shown in Fig. 2.4.

As the length of the wing, $R$, and the maximum chord length, $c_{max}$, is known, the wing
rotation angle can be calculated from the observed projected wing chord $c_{proj}$. Vectors defining
the wing leading edge and trailing edge are defined as $\mathbf{v}_{le} = p_2 - p_1$ and $\mathbf{v}_{te} = p_3 - p_1$ where
$p_i = [p_{ix}, p_{iy}, p_{iz}]$ are the tracked point positions in pixels as defined in Fig. 2.4. $p_{iz}$ are set to 0
to allow the cross product to be taken for the otherwise planar coordinates. The observed chord
length in millimeters can be calculated using the standard equation for the distance of a point
from a line with

$$
c_{proj} = \frac{|(p_{2x} - p_{1x})(p_{1y} - p_{3y}) - (p_{1x} - p_{3x})(p_{2y} - p_{1y})|}{|\mathbf{v}_{le}| R} \sqrt{(p_{2x} - p_{1x})^2 + (p_{2y} - p_{1y})^2} 
$$

(2.8)

(2.9)

where $\frac{|\mathbf{v}_{le}|}{R}$ is the conversion factor between pixels and millimeters. The wing rotation angle can
then be calculated as

$$
c_{dir} = \frac{\mathbf{v}_{le} \times \mathbf{v}_{te}}{|\mathbf{v}_{le}| |\mathbf{v}_{te}|} \quad \phi = \text{sign}(c_{dir,z}) \arcsin \left( \frac{c}{c_{max}} \right),
$$

(2.10)

(2.11)

where $\text{sign}(c_{dir,z})$ determines the direction of wing rotation. Wing flapping angle can be deter-
mined similarly, where \( \theta \) is relative to the nominal wing position defined as \( \vec{v}_{le,nom} \).

\[
R_{dir} = \frac{\vec{v}_{le,nom} \times \vec{v}_{le}}{|\vec{v}_{le,nom}| |\vec{v}_{le}|}
\]

\[
\theta = \text{sign}(R_{dir,z}) \arccos \left( \frac{\vec{v}_{le,nom} \cdot \vec{v}_{le}}{|\vec{v}_{le,nom}| |\vec{v}_{le}|} \right)
\]

Mean and instantaneous simulated and experimental lift is compared in Figs. 2.5 and 2.6. In Fig. 2.5, mean and standard deviation of mean lifted averaged over a wing stroke is plotted versus input voltage magnitude. Each data point includes at least 0.5 seconds of operation, or 5 complete wing strokes. In Fig. 2.6, two examples of experimentally measured wing kinematics and lift time series are shown and compared with the predicted lift.

Both the average lift over a wing stroke and instantaneous lift is well captured by the aerodynamic model. Predicted mean lift is within 10% of the measured value throughout the voltage amplitude sweep. Instantaneous lift is also well matched overall, though there exist some consistent discrepancies about the wing directional transition point where wing-wake interaction would
occur. As this effect is not explicitly modeled, these differences are not unexpected.

The two peaks in lift straddling wing reversal seen in both simulation and experiment are characteristic of the effects of wing rotational lift [89, 91]. As can be seen in Fig. 2.7, the contribution of rotational lift results in the distinctive shape mirrored in the experimental data. Though rotational lift does influence the system instantaneous lift, averaged over a wing stroke its contribution is almost 0. The phase of wing rotation relative to the flapping angle plays a significant role in this result. Maximum rotational lift, and typically maximum overall lift, occurs when the wing rotates in advance of stroke reversal [28]. In the system described here, wing rotation is symmetric to flapping angle (as we’ve defined $\phi$, this is where there is no phase difference between where $\phi = 0$ and where $\theta$ is at its maximum). Ideal wing rotation in a passive rotation system is difficult to achieve as the wing relies solely on rotational joint stiffness, drag, and inertial effects to rotate. In the Drosophila, a species related to the common housefly, rotational effects can contribute to roughly 35% of total lift [28]. However, the Drosophila maintains control over both the wing leading and trailing edges, allowing wing rotation to be actively maintained at a desired angle and an optimal phase shift achieved. By modifying the wing rotational flexure more ideal wing rotation can be produced, but translational lift is likely to remain the largest contributor to total lift production in the passive wing rotation system. This suggests, for an equivalent power expenditure, it is better to design a system to flap at a lower frequency with a larger peak-to-peak flapping angle than a higher frequency with a lower peak-to-peak flapping angle.
Figure 2.6: Kinematics and wing simulated and experimental instantaneous lift for two cases corresponding to labels (1) and (2) in Fig. 2.5. Results for $V_{pp} = 8$ V are in (a) and results for $V_{pp} = 13$ V are in (b). Included are, in order, wing kinematics, simulated and experimental lift, and the breakdown of the simulated lift components.
Figure 2.7: Instantaneous lift over a single wing stroke when $V_{pp} = 13$. In (a) total simulation and experimental lift is plotted. In (b), a breakdown of the simulated lift components is shown.
2.3 Equations of Motion of Passive Wing Rotation System

The flapping system model incorporates the actuator driving the wing leading edge and the passive wing rotational motion, and can be used to predict wing kinematics and produced lift for a given input voltage. The dynamic equations for the flapping system can be written in Lagrangian form with the variables $\theta$ and $\phi$ representing the wing flapping and rotation angle respectively.

\[
L = T_{\text{energy}} - V_{\text{energy}} = T_{\text{wing}} + T_{\text{act}} - V_{\text{wing}} - V_{\text{act}}
\]

\[
= \frac{1}{2} m_w \vec{\nu} \cdot \vec{\nu} + \frac{1}{2} J_w \vec{\omega} \cdot \vec{\omega} - \frac{1}{2} k_{\text{rot}} \phi^2 + T_{\text{act}} - V_{\text{act}}
\]

\[(2.14)\]

where $T_{\text{wing}}$ and $V_{\text{wing}}$ are the kinetic and potential energy of the wing. Depending on the choice of driving actuator, the kinetic and potential energy $T_{\text{act}}$ and $V_{\text{act}}$ will vary. The variable $\vec{\nu}$ is the velocity at the wing’s center of gravity. $J_w$ is the wing inertia matrix taken about the wing center of mass, $m_w$ is the wing mass, and $k_{\text{rot}}$ is the wing flexure rotational stiffness. The variable $\vec{\omega}$ is the angular velocity of the wing defined as

\[
\vec{\omega} = \dot{\theta} \vec{E}_3^\prime + \dot{\phi} \vec{E}_1^\prime
\]

\[
= \dot{\phi} \vec{E}_1^\prime + \dot{\theta} \sin(\phi) \vec{E}_2^\prime + \dot{\theta} \cos(\phi) \vec{E}_3^\prime
\]

\[(2.16)\]

expressed in terms of basis vectors of the coordinate frame attached to the wing as shown in Fig. 2.3a.

From eq. (2.15), the Lagrange equations describing the wing rotation and flapping dynamics
can be written as

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}} \right) - \frac{\partial L}{\partial \phi} = \vec{M}_{aero} \cdot \vec{E}_1'' - b_w \dot{\phi}, \quad (2.17)
\]

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = \vec{M}_{aero} \cdot \vec{E}_3 + M_{drive}, \quad (2.18)
\]

where \( \vec{M}_{aero} \) is the moment due to the aerodynamic forces and \( b_w \) is the wing rotational flexure damping. \( \vec{E}_3 \) denotes the basis vector about which the wing flaps, as shown in Fig. 2.2 \( M_{drive} \) is the torque provided by the actuator and defined as acting about the flapping axis. As the actuator drives the flapping angle of the system and does not directly influence the wing rotation, the dynamics of the wing rotation can be written as follows with the appropriate differentiations:

\[
\vec{M}_{aero} \cdot \vec{E}_1'' - b_w \dot{\phi} =
\dot{\theta}(m_w R_{CG} \beta_{CG} \cos(\phi) + J_{xz} \cos(\phi))
- \dot{\theta}^2 \cos(\phi) \sin(\phi) (m_w \beta_{CG}^2 + J_{yy} - J_{zz})
+ \frac{1}{2} m_w \beta_{CG}^2 + J_{xx}) + \dot{\phi} k_{rot}, \quad (2.19)
\]

and is independent of the choice of actuator. Note that for a thin wing, the wing inertial components \( J_{yz} \) and \( J_{xy} \) are very small and have been omitted from the equations.

The moment due to the wing aerodynamic forces \( \vec{M}_{aero} \) is defined as

\[
\vec{M}_{aero} = \vec{F}_t \times \vec{\beta}_t(\alpha) + \vec{F}_{rot} \times \vec{\beta}_{rot}(\alpha) \quad (2.20)
\]

where \( \vec{F}_t \) and \( \vec{F}_{rot} \) represent the translational and rotational aerodynamic forces, \( \vec{\beta}_t \) and \( \vec{\beta}_{rot} \) are
the respective positions of their centers of pressure from the wing rotation axis, defined as

$$|\vec{\beta}_t(\alpha)| = \left( \frac{0.82}{\pi} |\alpha| + 0.05 \right) c(r) \quad (2.21)$$

$$|\vec{\beta}_{rot}| = \frac{\int_{\text{span}} z'' dF_{rot}}{\int_{\text{span}} dF_{rot}} \quad (2.22)$$

where $\vec{\beta}_t$ is the position of the translational lift center as a function of angle of attach for each wing cord strip $c(r)$, as defined in [84]. The variable $z''$ is the distance from the wing rotational axis of the center of pressure for rotational forces over an individual wing strip while $\vec{\beta}_{rot}$ is the effective moment arm of the rotational force distribution over the wing in Fig. 2.3b.

Given that there is no analytical solution for the added air mass force on a wing planform moving in 3D fashion, $dF_{air}$ of (2.4) does not accurately describe the moments exerted by the added air mass on the flapping and rotation angles. As an approximation, the effect of added air mass forces was implemented in the form of virtual mass that augmented the physical mass of the wing, similar to the inertial implementation of [90, 94]. Emerging from the concept that the added mass effect can be estimated as $dm = \frac{\pi \rho}{4} c(r)^2 dr$ [7], the expression for $m$ emerges:

$$m = m_w + \int_{\text{span}} \frac{\pi \rho}{4} c(r)^2 dr, \quad (2.23)$$

where, $m_w$ is the physical mass of the wing. This approximation allows a simplified treatment of the added air mass effects that is sufficient for the development of a simulation tool able to predict general dynamics and lift force trends. Although this mass augmentation approach is used in the differential equations defining the system dynamics, the expression for added air mass force of (2.4) is used to estimate the aerodynamic lift generated by this effect in all lift plots presented in this work.
2.3.1 Model of Driving Actuator

Piezoelectric Cantilever Actuator

In the flapping system driven by a piezoelectrics, a piezoelectric actuator is used to drive the wing though a four-bar linkage transmission to amplify motion. The system Lagrangian from (2.15) can be updated with

\[ T_{act} = \sum \frac{1}{2} m_{Li} \bar{v}_{Li} \cdot \bar{v}_{Li} + \sum \frac{1}{2} J_{Li} \bar{\omega}_{Li} \cdot \bar{\omega}_{Li} + \frac{1}{2} m_{eff} \dot{\delta}_{act}^2 \]  
(2.24)

\[ V_{act} = \frac{1}{2} k_{act} \delta_{act}^2 \]  
(2.25)

The \( m_{Li}, J_{Li}, \bar{v}_{Li} \) and \( \bar{\omega}_{Li} \) are the masses, rotational inertias, linear and angular velocities of the four-bar links 1-3, respectively, \( m_{eff} \) is the effective mass of the actuator, \( k_{act} \) is the actuator stiffness, and \( \delta_{act} \) and \( \dot{\delta}_{act} \) are the horizontal displacement and velocity of the tip of the actuator (scalar). The effective mass of the actuator considers the dimensions of the cantilever actuator and is calculated as in [95].

The driving torque \( M_{drive} \) is dependent on the force generated at the tip of the cantilever
actuator as well as the transmission link lengths and linkage configuration which determine the system transmission ratio. $M_{\text{drive}}$ can be written as the function

$$M_{\text{drive}} = f(\text{links}_i, \text{config}, F_{\text{in}})$$

(2.26)

where $\text{links}_i$ are the lengths of the four-bar linkages, $\text{config}$ is the current displacement of the linkage, and $F_{\text{in}}$ is the force generated by the actuator as depicted in Fig. 2.8. The magnitude of $F_{\text{in}}$ is calculated using laminate plate theory as in [37]. It is defined to act about flapping axis $\vec{E}_3$ as depicted in Fig. 2.2.

The linear and angular velocities for the link lengths are complex in closed form and are instead substituted with numerically calculated trajectories parameterized by the flapping angle $\theta$ (i.e. for link $i$, $\vec{v}_{Li} = f_{Li}(\theta(t))$ and $\frac{dv_{Li}}{dt} = \dot{\theta}(t) \frac{df}{d\theta(t)}$). Defining the conventions $f' \equiv \frac{df}{d\theta(t)}$ and $\dot{f} \equiv \frac{df}{dt}$ and expanding (2.18) to determine the equation of motion for $\theta$, we have

$$M_{\text{drive}} + M_{\text{aero}} \cdot \vec{E}_3 - b_w \dot{\phi} =$$

$$\dot{\theta} \left( \sin^2(\phi) J_{yy} + \cos^2(\phi) J_{zz} + \left( R_{CG}^2 + \frac{\beta_{CG}^2}{2} \right) m_w \right)$$

$$- \frac{1}{2} \beta_{CG}^2 \cos(2\phi) m_w + \sum_{i=1}^{3} m_{Li} (x_{Li}' x_{Li}'' + y_{Li}' y_{Li}'') + m_{\text{eff}} \delta''^2$$

$$+ \sum_{i=1}^{2} J_{Li} \theta_{Li}'^2 + J_{Li} \right) + \tilde{\phi} \cos(\phi) \left( m_w R_{CG} \beta_{CG} + J_{xz} \right)$$

$$- \dot{\phi} \sin(\phi) \left( m_w R_{CG} \beta_{CG} + J_{xz} \right) + \dot{\theta} \phi \sin(2\phi) \left( m_w \beta_{CG}^2 + J_{yy} - J_{zz} \right)$$

$$+ \sum_{i=1}^{3} m_{Li} \dot{\theta}^2 (x_{Li}' x_{Li}' + y_{Li}' y_{Li}'') + \sum_{i=1}^{2} J_{Li} \theta_{Li}' \theta_{Li}' \dot{\theta}^2$$

$$+ m_{\text{eff}} \delta_{\text{act}}' \dot{\delta}_{\text{act}}''^2 - k_{\text{act}} \delta_{\text{act}} \dot{\delta}_{\text{act}}^2 - k_{\text{act}} \delta_{\text{act}}^2 \dot{\theta}.$$ (2.28)

Validation of this model is detailed in [84] and will not be discussed in this work.
DC Motor

On a larger scale, the passively driven wing can instead be driven by a motor in a reciprocating fashion. Unlike with the piezoelectric actuator, where its cantilever form functions as a spring and allows the system to resonate, an additional spring is added in parallel to the junction between wing and actuator. A simplified representation of the flapping system as a spring-mass-damper can be seen in Fig. 2.9. Motor damping, $b_0$, and the added torsional elastic element with stiffness $k_s$ are also assumed to act linearly. The driving torque $\tau_l$ is provided by a geared pager motor. The choice of the elastic element stiffness is critical; choosing $k_s$ such that system flapping resonates at the desired operating frequency will eliminate the energy necessary to oscillate the effective motor rotor and wing inertia, $J_0$ and $J_w$.

As in the previous case we can define

$$T_{act} = 0$$

$$(2.29)$$

$$L_{act} = -k_s\theta^2.$$  

$$(2.30)$$

Here, $T_{act} = 0$ as the contribution of motor rotor inertia is included in the definition of driving torque $M_{drive}$.

The electro-mechanical model of a gearless, brushed, DC motor can be written as follows:

$$V_{in} = R_0 I + k_t \omega_m$$

$$(2.31)$$

$$k_t I_{in} - \tau_m = J_m \alpha_m + b_m \omega_m$$

$$(2.32)$$

where $V_{in}$ and $I_{in}$ are the input voltage and current, respectively, $\omega_m$ and $\alpha_m$ are the rotor angular velocity and acceleration, respectively, and $R_0$ is armature electrical resistance. The motor constant is $k_t$, while $J_m$ is the rotor inertia and $b_m$ is the damping due to internal friction. An external torque applied to the system is represented with $\tau_m$. With the addition of a gearbox with
Figure 2.9: Simplified representation of the prototype as a spring-mass-damper system. The aerodynamic force on the wing, $F_{aero}$, acts as a nonlinear damping moment on both the wing rotational, $\phi$, and flapping, $\theta$, motion. The wing rotational flexure is assumed to have a linear torsion constant, $k_{rot}$, and damping, $b_w$, which is dependent on the wing flexure geometry. Ideally the torsional spring stiffness $k_s$ is chosen such that the system flapping resonates at the desired operating frequency.

With a DC motor driving the system, $M_{drive} = \tau_l$ and can be written as follows, considering (2.33) and (2.34):

$$V_{in} = R_0 I_{in} + k_t N_g \omega$$  \hspace{1cm} (2.33)

$$\eta_g N_g k_t I_{in} - \tau_l = J_0 \alpha + b_0 \omega$$  \hspace{1cm} (2.34)

where $b_0 = \eta_g N_g^2 b_m$, $J_0 = \eta_g N_g^2 J_m$ and $\tau_l$, $\omega$, and $\alpha$ are the external torque, angular velocity, and angular acceleration at the load, respectively.

With a DC motor driving the system, $M_{drive} = \tau_l$ and can be written as follows, considering (2.33) and (2.34):

$$\tau_l = \eta_g N_g k_t I_{in} - \eta_g N_g^2 J_m \dot{\theta} - b_0 \dot{\theta},$$  \hspace{1cm} (2.35)

$$= \frac{\eta_g N_g k_t}{R_0} (V_{in} - k_t N_g \dot{\theta}) - \eta_g N_g^2 J_m \ddot{\theta} - b_0 \dot{\theta},$$  \hspace{1cm} (2.36)

where in (2.36), $\tau_l$ is written as a function of input voltage.
Simplifying (2.18), the equation of motion for the wing flapping angle, \( \theta \), is:

\[
\vec{M}_{aero} \cdot \vec{E}_3 + \frac{\eta_g N_g k_t V_{in}}{R_0} = \\
\dot{\theta} \left( \sin^2(\phi) J_{22} + \cos^2(\phi) J_{33} + \left( R_{CG} + d_w \right)^2 + \frac{\beta_{CG}^2}{2} \right) m_w \\
- \frac{1}{2} \beta_{CG}^2 \cos(2\phi) m_w + \eta_g N_g^2 J_m \right) + \ddot{\phi} \cos(\phi) (m_w (R_{CG} + d_w) \beta_{CG} + J_{13}) \\
+ \dot{\theta} \left( \frac{\eta_g N_g^2 k_t^2}{R_0} + b_0 \right) - \dot{\phi}^2 \sin(\phi) (m_w (R_{CG} + d_w) \beta_{CG} + J_{13}) \\
+ \dot{\phi} \dot{\theta} \sin(2\phi) (m_w \beta_{CG}^2 + J_{22} - J_{33}) + k_s \theta
\]  

(2.37)

where \( \beta_{CG} \) and \( R_{CG} \) are defined in Fig. 2.3 and \( d_w \) is the wing horizontal offset between the base of the wing and the flapping axis. The terms \( J_{(1-3)(1-3)} \) are elements in the wing inertia matrix \( J_w \) aligned with the wing fixed bases \( E'' \).

Assuming a fixed, flat wing (\( \phi, \dot{\phi}, \ddot{\phi} = 0 \)), (2.37) resolves to

\[
\vec{M}_{aero} \cdot \vec{E}_3 + \frac{\eta_g N_g k_t V_{in}}{R_0} = \\
\left( J_{33} + (R_{CG} + d_w)^2 m_w + \eta_g N_g^2 J_m \right) \ddot{\theta} \\
+ \left( \frac{\eta_g N_g^2 k_t^2}{R_0} + b_0 \right) \dot{\theta} + k_s \theta,
\]  

(2.38)

which is the standard spring-mass-damper form, albeit with nonlinear damping due to the wing aerodynamic forces. Ideally, for maximum efficiency one would choose the elastic element stiffness, \( k_s \), such that the system resonates at the desired operating frequency. With a sinusoidal
input voltage, $k_{s,\text{ideal}}$ would be

$$k_{s,\text{ideal}} = (J_{33} + (R_{CG} + d_w)^2 m_w + \eta_g N^2 J_m)(2\pi f)^2,$$  

(2.39)

where $J_{33} + (R_{CG} + d_w)^2 m_w$ is the wing inertia about the flapping axis and $f$ is the operating frequency.

### 2.3.2 DC Motor-driven Model Validation

In Section 2.2.1 the aerodynamic model lift prediction is directly compared to the instantaneous lift measured in a motor-driven flapping wing system using the measured wing kinematics. Here, the full dynamic model of the system is similarly compared to the experientially measured lift. As in Section 2.2.1, the same motor-driven system prototype is used, with a wing of length of $R = 70 \text{ mm}$, offset $d_w = 35 \text{ mm}$, and maximum chord length of $c_{\text{max}} = 25 \text{ mm}$. A GM15A motor is used, with motor parameters can be found in Table 4.1, while wing mass and inertia are listed in Table 4.2. Stoppers are used to prevent wing rotation from exceeding 45 deg. The motor elastic element stiffness, $k_s$, was $2.5 \times 10^3 \text{ mN.mm/rad}$, which is less than $k_{s,\text{ideal}}$ at 10 Hz given structural limitations of the GM15A gearbox.

The system is driven with a sinusoidal input voltage with a peak-to-peak amplitude of $V_{pp}$. The input voltage sinusoid is set to a frequency of 10 Hz and $V_{pp}$ is swept from 8 to 13 V. The lift is recorded at 50 kHz with a 6 DOF force/torque sensor (ATI-Nano17 TI) and is effectively reduced to 5000 Hz by a block averaging filter. A low pass filter with a cutoff frequency of 50 Hz is then applied. Sensor mounting details can be found in Section 5.4.1.

Figure 2.11 shows both peak-to-peak wing flapping and rotation angle as well as mean lift for varying input voltages. Flapping angle tends to be underestimated, which leads to an underestimation of lift as well, though error in predicted lift remains within 10%. The use of wing
Figure 2.10: Peak wing flapping and rotation angles (a) and mean lift (b) for experimental system and full system dynamic simulation with an input voltage sweep from $V_{pp} = 8$ to $V_{pp} = 13$. Both average values and standard deviation is shown for experimental data.
stoppers helps prevent large deviations in wing rotation angle between simulation and experiment and, consequently, error in predicted lift. As the motor parameters are measured, aerodynamic force predictions and their effect on the passive wing motion remain the largest source of error. With a wing rotation angle predominately fixed by the stoppers, the damping experienced in both simulation and experiment is more likely to be the same, leading to similar peak flapping angles, wing speed, and lift. Section 2.2.1 demonstrates good lift prediction for a given set of wing kinematics, however, in the passive wing system, any deviations will result in differences in wing rotation which tend to have a cascading effect on simulation performance.

Figure 2.11 depicts instantaneous wing kinematics and lift for an input $V_{pp} = 13$. Here, a slight asymmetry can be seen in the wing flapping angle. This is most likely due to nonlinear spring behavior at large deflections. The helical spring used has a relatively small number of coils for its purpose, leading to possible behavior close to binding at large flapping amplitudes. The phase of predicted wing rotation also tends to be more advanced than seen in experiment, which could be contributed to aerodynamic force prediction error as well as error in the measured wing rotational flexure stiffness. As seen in Fig. 2.11, the instantaneous lift profile does not exactly match that
seen in experiment, which is also a consequence of imperfectly predicted wing rotation.

2.4 Summary

Closed form representations of wing translational, rotational, and added air mass forces have been presented and characterized in a number of previous works, including [28], [89], [91], [88]. In this chapter, these existing aerodynamic models are used in the development a full dynamic model of the passively rotating wing systems included in this work. The developed models consider the differing driving actuators and transmissions for each platform, including both the piezo-driven [84] as well as the motor-driven variations.

In order to verify the use of the aerodynamic model and dynamic model for the higher Reynolds number motor-driven system, simulation predictions were compared with experimentally measured wing lift and wing kinematics. With both the aerodynamic model and full dynamic model mean lift predictions remaining within 10%, the simulation serves as an adequate predictor on system performance.

Both the piezo-driven and motor-driven dynamic system model are used in the development of each respective platform and will be referenced and used in the chapters following.
Chapter 3

Piezo-driven Flapping Flight Platform

3.1 Introduction

In a passive rotation wing design, only the leading edge of the wing is controlled, leaving the trailing edge to passively rotate based on aerodynamic forces, inertial effects, and a flexural spring element at its rotation axis. While this reduces weight with fewer actuators, control over the produced wing forces is limited. Whereas a fully controlled wing can produced arbitrarily vectored forces over a wing stroke, a wing that passively rotates is primarily limited to changing the origin and magnitude of its upward mean lift force.

In light of this constraint, a two winged, two piezoelectric actuator system is designed and constructed. With one driving actuator per wing, the leading edge of each wing can be controlled independently, minimizing the number of actuators while maintaining high actuator bandwidth and the capacity of creating controlling torques. A mirrored rigid body structure for both the right and left wings ensures that they remain independent while a spherical 4-bar transmission allows an overall compact body design. Using Arabagi’s developed flapping flight system model [84], actuator, wing, and transmission dimensions are determined for a large and small scale system. Fabricated prototypes are tested for lift production. While the modular design proves too heavy for liftoff, both roll and pitch torques can be produced. The small scale system increase in lift-
to-weight ratio suggests that liftoff may be eventually possible with reduced sizes and improved manufacturing techniques.

As in alternative to the two piezoelectric actuator design, a new control actuator is also presented and tested on a single wing prototype. By driving both wings with a single actuator, and creating controlled wing asymmetries with a separate very lightweight actuator, platform weight could potentially be reduced even further.

As a reminder, the position of the wing leading edge is called the flapping angle, while the position of the trailing edge relative to the leading edge is called the wing rotation angle. These definitions, as well as defined roll, pitch, and yaw for the flapping platform can be seen in Figures 2.2 and 3.1.

### 3.2 Design Considerations

With the choice of a piezoelectric bending actuator to drive the system, there are two general robot designs possible, each utilizing resonance to increase maximum wing flapping angle and decrease power consumption. One is a resonant body design where the actuator is used to vibrate the body structure at resonance, resulting in large displacements used to drive the wings [97]. The second relies upon rigid mounting of the actuator and assumes certain points on the body
structure remain fixed \[^{38,41}\]. In this case actuator motion is amplified by a transmission, which is also mounted rigidly to the body. The cantilever bending actuator functions as the spring in the resonant system. Here, we take the second approach, creating a flapping robot with a rigid body, driven by two bimorph piezoelectric actuators, whose motion is amplified by a spherical 4-bar transmission, as can be seen in Figures 3.2 and 3.3.

As such we have a number of items considered in the system design:

- **Actuator Dimensions, Transmission Ratio, and Wing Size:** Arguably the most important components of the flapping-wing system. Together they must be selected such that the system resonates at the desired frequency and the actuator can drive the chosen wing at a sufficient flapping amplitude.

- **CG Position:** To improve system stability, the system CG should be below the wing stroke center of lift. This necessitates that the actuator and transmission be compactly arranged below the last output link of the 4-bar transmission.

- **Fixed Actuator and Transmission Mounting Points:** The use of an actuator and transmission requires that their mounting points to the manufactured airframe be rigid. Any deformation of the airframe or motion at the mounting points will result in a loss of energy and a loss of lift.

- **Minimal Coupling Between Wings:** As we desire that the flapping angle of each wing be individually controlled, there should be little or no coupling between each wing in the system.

To meet these requirements, the system is designed to have independent right and left halves; each wing is independently driven by a piezoelectric actuator, and the separately rigid body halves reduce any wing coupling caused by unwanted vibrations. This results in a robot that is underactuated, but still capable of producing body torques. The flat internal face of each half allows easy final body assembly after individual construction of each side. Each halve is constructed out of carbon fiber spars to maximize body stiffness while reducing weight. A spherical
4-bar is chosen and oriented such that the driving actuator is mounted below the flapping wing. A schematic of the platform halves can be seen in Fig. 3.2.

![Schematic of piezo-driven flapping robot’s separated right and left halves.](image)

**Figure 3.2**: Schematic of piezo-driven flapping robot’s separated right and left halves.

### 3.3 System Optimization

In light of the available models, the system actuator, transmission, and wing can be optimized to maximize lift. The system optimization is completed in three discrete steps: 1) meeting desired transmission kinematic motion, 2) maximizing lift generation with wing and flexure stiffness selection using the dynamic model, and 3) manufacturing and experimentally testing lift production of several local optima wings.

#### 3.3.1 Actuator and Spherical 4-bar Optimization

The flapping robot’s spherical 4-bar transmission both amplifies the driving actuators motion and helps centralize the robot’s body mass. The spherical 4-bar transmission, shown in Figure 3.3, functions like a traditional 4-bar, with the exception that all link rotational axes must cross at a single point, the ‘Axes Origin’, which is the center of an imaginary sphere. The points A, B, and C all move along the surface of this sphere. The tip of the actuator is attached to the
transmission through a slider crank, as both the actuator tip and transmission motion is nonlinear. The spherical 4-bar allows the actuator, the largest contributor of system mass, to be mounted in the interior of the body, centralizing the center of gravity and yielding a compact body design. Though the mounting position limits the maximum length of the actuator, the design benefits from compactness and the lack of complex components.

A set of numerical equations has been derived to model the kinematic motion of spherical 4-bar links L1-L3. The resulting flapping angle, $\theta$, which is the angular displacement of L3 about the linkage axis C, and the torque transmission can be written as

$$\theta = f(\gamma, L_{\text{links}}, \text{config})$$

(3.1)

$$\tau_{\text{drive}} = g(\theta, L_{\text{links}}, \text{config})$$

(3.2)
where $\gamma$ is the input angle to the first link $L_1$ and $L_{links}$ are the link lengths. The $config$ term refers to the relative link locations relative to the axes origin. The equations are highly nonlinear and are fully described in the Appendix of [98].

There are several requirements that must be considered in the optimization of the spherical 4-bar link lengths and actuator dimensions. The first is actuator stress level. The bimorph piezoelectric actuator is a sandwich of carbon fiber (Torayca M60J) and PZT-5H (Piezo Systems), of cross section shown in Figure 3.4. A passive extension of fiberglass (S-Glass) and carbon fiber is used to increase actuator displacement. To eliminate the possibility of depoling the actuator, it is driven in ‘simultaneous’ fashion, as described in [37]. Piezoelectric materials are sensitive to fracture, and, as such, the actuator is not driven at displacements exceeding its maximum stress of 60 MPa [99]. Actuator inputs are $V_a$ and $V_b$, an alternating and constant bias voltage, respectively.

The second requirement is achieving a reasonable flapping angle with the transmission. Lift is increased with an increase in flapping amplitude and frequency. A desired peak-to-peak flapping angle is chosen to be $100^\circ$.

The spherical 4-bar mechanism is inherently nonlinear and results in an unsymmetrical torque
transmission curve. The third consideration for spherical 4-bar optimization is symmetrical transmission motion about the nominal input of 0 volts, which will result in symmetrical wing motion.

Combined, these requirements can be used to create two quadratic cost functions for the selection of actuator and linkage dimensions:

\[
C_{\text{act}}(l_{PZT}, l_{CF}) = K_1 \| \sigma_{\text{act}} - 60 \|^2 + K_2 \| \delta_{\text{tip}} - \delta^* \|^2
\]
\[
C_{FB}(L_{\text{links}}, \text{config}) = K_3 \| \theta_{\text{max}} - 100^\circ \|^2 + K_4 \sum_i \| \tau(V_i) - \tau(-V_i) \|^2,
\]

where \( l_{PZT} \) and \( l_{CF} \) are the lengths of the actuator PZT and the carbon fiber respectively, \( K_i \) are the optimization weights, and \( \sigma_{\text{act}} \) is the actuator maximum stress in MPa. The actuator displacement at the tip and the desired displacement is \( \delta_{\text{tip}} \) and \( \delta^* \) respectively. The total displacement of the transmission output link is \( \theta_{\text{max}} \) while \( \tau(V_i) \) is the linkage output torque at \( V_i \), which is the maximum inputted actuator voltage.

A Nelder-Mead optimization routine was used to find the transmission and actuator geometry. Length scale was set by selecting a fixed L4 transmission link length. The resulting spherical 4-bar transmission ratio was \( \tau_{\text{in}}/\tau_{\text{out}} = 7 \), with the output toque curve depicted in Fig. 3.10a and actuator dimensions in Fig. 3.10b.

### 3.3.2 Wing Optimization

The determined actuator and linkage geometry was then entered into the full dynamic model of the passively rotating wing, described in Chapter 2. A series of optimizations were run with randomized initial wing shapes and wing flexure stiffnesses \( k_{\text{rot}} \) where the set reward function was a quadratic function of mean lift. Wing shape was assumed to be triangular for ease of later manufacturing, with three parameters defining the wing profile as shown in Fig. 3.5b.

Given the number of local optima, four of the best performing wings were then manufactured and tested experimentally on a single wing system. The single wing system, shown in Fig. 3.6 is
composed of the actuator, transmission, and wing mounted to a rigid fixture, attached to a lever arm and load cell setup. The rigid mount ensures there is no energy loss from potential body deformation. The lever arm and load cell allows mean lift of the flapping system to be measured.

![Manufactured Wings](image)

Figure 3.5: Manufactured wings (a) and varied wing parameters (b).

Each manufactured wing was tested on the system with a series of input voltage amplitude and frequency sweeps. The resulting maximum lift force, the frequency at which it occurs, and flexure stiffness used is listed in Table 3.1. Increasing wing size decreased the system resonance frequency significantly due to the increasing wing mass, and generally resulted in a decrease in lift. Wing A and B, however, produce similar amounts of lift albeit at differing flapping frequencies, indicating that the increase of wing area compensates for the reduced wing speeds. Wing D demonstrates an increase in lift over C at similar flapping frequencies, which may be due to the increased wing area located further from the flapping axis. Due to their performance, either wing A or B would be a reasonable choice for use in the full body prototype. Wing B is selected here and incorporated into the tested systems.
Figure 3.6: Single-wing piezo-driven system mounted to mean lift measurement setup. The actuator and transmission are mounted to a 3D printed rigid structure.

<table>
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<tr>
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<th>Wing A</th>
<th>Wing B</th>
<th>Wing C</th>
<th>Wing D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Lift (mN)</td>
<td>0.9</td>
<td>0.85</td>
<td>0.5</td>
<td>0.65</td>
</tr>
<tr>
<td>Maximum Lift Frequency (Hz)</td>
<td>43</td>
<td>35</td>
<td>23</td>
<td>22</td>
</tr>
<tr>
<td>Experimental Flexure Stiffness (mN.mm)</td>
<td>16</td>
<td>20</td>
<td>22</td>
<td>22</td>
</tr>
</tbody>
</table>

### 3.4 Fabrication

The wing, actuator, and transmission of the flapping flight system are fabricated following the ‘Smart Composite Microstructures’ technique, as described in [38][100], an example of which can be seen in Fig. 3.7. The transmission and wing are composed of layered carbon fiber and polyimide film. With the interior layer of polyimide film functioning as the bending sheet flexural hinges, both linkages and folded rigid structures can be designed as desired. First, flexures are laser cut (New Wave Laser Machining System) from two mirrored layers of carbon fiber. Next a sandwich of carbon fiber layers and polyimide film (Kapton, Dupont) are assembled. The layup is then cured for 2 hours at 180 °C. The part is then released with a final laser cut of the exterior
Figure 3.7: Piezo-driven system fabrication process with transmission and slider crank depicted: (a) flexures are laser cut from two mirrored sheets of carbon fiber, (b) a layup of carbon fiber layers and polymide film are assembled, (c) the layup is cured for 2 hours at 180 °C after which the part contours are laser cut. The final released part is shown in (d).

The body structure is constructed from solid carbon fiber spars to reduce system weight and increase rigidity. A rig is used to align the spars together before being bonded with cyanoacrylate glue. Each system halve is constructed individually then also secured together with cyanoacrylate.

3.5 Body Analysis

To ensure that no weaknesses in the body design were overlooked, both modal and stress/displacement Finite Element Analysis were performed on the full body design in ANSYS. The stress and deformation analysis were performed by applying the forces and moments predicted by the equivalent system in simulation as described in Chapter 2 for the large scale prototype. The resulting Von Mises stresses are depicted in Fig. 3.8a where maximum stress occurs at the actuator mounts. Maximum deformation is depicted in Fig. 3.8b. While a rigid constraint at the base of the body is used, maximum deformation at the wing mounts is still only
0.2 mm. The modal analysis is performed both with a rigid body constraint and a loose spring constraint. In both cases, the lowest natural frequency mode was over 1000 Hz, significantly higher than any expected operating frequency of the platform.

![FEA Analysis](image)

Figure 3.8: FEA Ansys body analysis: (a) Maximum displacement distribution throughout the body and (b) Von Mises stress levels. The body was rigidly constrained by a surface at the base, as illustrated by the black triangle. The applied forces and moments for the structural analysis are portrayed in (a).

### 3.6 Flapping Flight System Prototypes

#### 3.6.1 Large Scale System

Several large scale prototypes, with transmission dimensions of \{L1, L2, L3\} = \{32.9, 13.8, 5.8\} mm, were manufactured with physical and operating parameters listed in Table 3.2. Each had a lift-to-weight ratio between \~1/5 and \~1/6. An image of Prototype 1 can be seen in Fig. 3.9, with actuator and transmission torque curves shown in Fig. 3.10.
Table 3.2: MAV Physical Parameters and Operating Information

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prototype 1</th>
<th>Prototype 2</th>
<th>Prototype 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Mass (mg)</td>
<td>650</td>
<td>705</td>
<td>690</td>
</tr>
<tr>
<td>Inertia* (g mm$^2$)</td>
<td>[58.5, 31.8, 27.1]</td>
<td>[63.4, 34.5, 29.4]</td>
<td>[62.0, 33.7, 28.8]</td>
</tr>
<tr>
<td>Horz. CL to CG, $R$ (mm)</td>
<td>23</td>
<td>23.5</td>
<td>23</td>
</tr>
<tr>
<td>Vert. CL to CG, $H$ (mm)</td>
<td>26</td>
<td>34</td>
<td>26</td>
</tr>
<tr>
<td>Actuator Length (mm)</td>
<td>18.5 (7.5 ext.)</td>
<td>18.5 (7.5 ext.)</td>
<td>18.5 (7.5 ext.)</td>
</tr>
<tr>
<td>Actuator Width (mm)</td>
<td>5 (taper to 4)</td>
<td>5 (taper to 4)</td>
<td>5 (taper to 4)</td>
</tr>
<tr>
<td>Actuator Mass (mg)</td>
<td>110</td>
<td>130</td>
<td>130</td>
</tr>
<tr>
<td>Wing Dimensions, $[w_1, w_2, w_3]$ (mm)</td>
<td>[27, 14, 14]</td>
<td>[21, 12, 14]</td>
<td>[27, 14, 14]</td>
</tr>
<tr>
<td>Resonant Frequency (Hz)</td>
<td>26</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>Max Wing Flapping Angle (deg)</td>
<td>100</td>
<td>100</td>
<td>90</td>
</tr>
<tr>
<td>Lift-to-Weight Ratio</td>
<td>0.16</td>
<td>0.2</td>
<td>0.15</td>
</tr>
</tbody>
</table>

*Predicted from Solidworks model

Figure 3.9: Fully assembled body with principal dimensions (experimental prototype 1 shown). System mass is 0.65 g.
Figure 3.10: (a) Overlaid kinematic angular displacement (solid) and torque transmission (dashed) curves for the optimized spherical four-bar system in the large scale FWMAV. $\tau_{out}$ is the torque output in the last link of the four-bar, where the wing attaches. The lengths of the four-bar links are $\{L1, L2, L3\} = \{32.9, 13.8, 5.8\}$ mm, and the radius of the operating sphere is 33 mm. (b) Photograph of bimorph actuator and labeled dimensions.

Figure 3.11: Large scale Prototype 1 mounted to lift measurement setup
Figure 3.12: Experimental frequency responses of the Prototype 1 carbon composite body at 180 V peak-to-peak with left wing flexure rotational stiffness, $k_{rot}$, of 15.5 mN.mm, damping, $b_w$, of 20.8 $\mu$mm.s, and right wing flexure $k_{rot}$ of 18.4 mN.mm and $d$ of 19 $\mu$N.mm.s. The mean and standard deviation of each lift measurement was obtained from a sample pool of 3000 points.

The mean lift of each prototype was measured using a single axis 30 g load cell (Transducer Techniques, Temecula, CA) coupled to a lever arm with a contact connection to amplify the generated force. The lift measurement setup can be seen in Fig. 3.11, while Prototype 1 lift for varying input frequencies is shown in Fig. 3.12. Commercially available load cells generally are subjected to the trade-off between bandwidth and sensitivity due to their construction principle. Thus when one desires to measure high frequency, low magnitude forces, as for the flapping wing robot in question here, there is no simple solution. The load cell balance system allows amplification of the produced lift forces, however the increased inertia places the bandwidth cutoff frequency of this second order system at 5 Hz, thus making measurements of higher frequency lift component signals impossible. Measurements of instantaneous small lift forces are possible but require more expensive torque sensors or custom built sensors [101]. Therefore, due to the limitations of the load cell/lever arm setup, only mean lift, not instantaneous lift, could be captured.
Figure 3.13: Experimental vibration amplitudes of the right and left wing when the other wing is actively driven with parameters: input 180 V peak-to-peak, left wing flexure stiffness: $k_{rot} = 17$ mN.mm, damping: $b_w = 20$ N.mm.s, right wing flexure $k_{rot} = 17$ mN.mm, $d = 20$ N.mm.s.

Wing Coupling

Coupling between the right and left wings could inhibit the ability to create independent left and right wing flapping amplitudes and, consequently, roll torques. One wing of Prototype 2 was driven at a time with the parasitic motion of the opposite wing captured with a camera. Maximum peak-to-peak flapping amplitude due to coupling is 10 deg, depicted in Figure 3.13, which is significantly less than the ∼100 deg flapping amplitude achieved when actively driven, demonstrating minimal wing coupling.

3.6.2 Half Scale System

A prototype of approximately half the size of the large system was fabricated and is shown in Fig. 3.14. The small scale system had a mass of 0.16 grams and produced a maximum of 0.58 mN of lift, having a lift-to-weight ratio of ∼3/8. System resonant frequency was 37 Hz with maximum lift occurring at 60 Hz.

Generally, a decrease in piezoelectric actuator size results in improved power delivery due
Figure 3.14: Large and small scale experimental prototypes. The small scale system massed 0.16 g.

Figure 3.15: (a) Overlayed transmission output angle (solid) and torque (dashed) curves, and (b), half scale system actuator dimensions. Transmission link lengths are \( \{L_1, L_2, L_3\} = \{19, 6.72, 2\} \) mm.
Figure 3.16: Frequency response of half scale piezo-driven prototype. Mean lift was measured for both wing individually and for both wings flapping concurrently. The predicted lift by the system simulation for a single wing is also shown. Experimental parameters: \( V_{pp} = 340 \, \text{V} \), \( k_{rot,left} = 4.2 \, \text{mN.mm} \), \( k_{rot,right} = 3.2 \, \text{mN.mm} \), \( d=10 \, \mu\text{N.mm.s} \). Mean and standard deviation is calculated from measured mean lift from a total of 2000 experimental data points filtered with a Butterworth filter with a 4 Hz cutoff frequency.

Correspondingly, a decrease in flapping flight platform size yields a greater platform lift-to-weight ratio. In [85], Arabagi investigates system scaling, relating power density, wing size, weight, and lift. This smaller prototype demonstrates the benefits of system scaling with a lift-to-weight ratio of \( \sim 3/8 \) versus \( \sim 1/6 \) for the large scale system. While neither prototype is capable of lift-off, the improvement given a change in size scale suggests the existence of a system capable of free flight even if current manufacturing capabilities are not adequate for fabrication. As size decreases, alignment and assembly become more difficult, effectively negating the benefits gained with smaller systems.

Currently over 60% of mass in the small scale system is contributed by airframe and actuator mass, the remaining contributed mostly from transmission mass, glue, and solder. With
a single piezoelectric actuator design, total actuator and supporting structure weight can be reduced, bringing the platform closer to free-flight when accompanied with alternative controlling actuators.

### 3.7 Shape Memory Polymer Flexures for Platform Control

Both the piezo-driven and motor-driven flapping-wing platforms described previously were designed based on the concept of wing modularity. Each wing flapping angle is independently controlled, allowing the generation of controlling torques. An alternative approach relies upon the differentiation between what is used to drive the wing flapping stroke and what is used to create small flapping asymmetries to allow system control. Whereas previously each wing was driven by an individual actuator, a single actuator could be used instead as depicted in Fig. 3.17 nominally coupling the wings but potentially reducing overall system weight. Additional lightweight actuators could be then added to allow for controlled wing asymmetry for control torque generation.

While the small scale two piezoelectric actuator prototype was shown to have an improved lift-to-weight ratio of $\sim 3/8$ over the $\sim 1/6$ of the larger prototypes, the precision necessary for manufacturing even smaller systems to achieve liftoff prevents the design from being successful at this time. In this chapter, we aim to develop a way to reduce the system weight further yet maintain controllability through the use of alternative actuators other than piezoelectric.

At smaller scales ($< 1$ gram), body components are manufactured from carbon fiber for increased weight reduction and are typically driven by piezoelectric actuators. Carbon fiber linkage transmissions are well suited for these platforms, being lightweight, scalable, and easily manufacturable. Linkage joints are simple flexural hinges that act as near frictionless pin joints and are typically constructed from a passive polyimide film. The Harvard MicroFly, Berkeley MFI, and the prototypes described here are all constructed following this theme. The behavior of linkage joints are highly dependent on material geometry and material elastic modulus; if one can control
Figure 3.17: Two actuator (a) and single actuator (b) half scale piezoelectric flapping system. The two actuator system has a mass of 160 mg, while the single actuator prototypes have a mass of 125 mg and 110 mg respectively.
the stiffness of these joints, one can effectively influence the behavior of the mechanism they are a component of.

Shape memory polymers (SMPs) belong to a specific class of materials that exhibit significant property changes when exposed to external stimuli such as light, heat, or magnetic fields [103]. More specifically, they are capable of ‘remembering’ one or more shapes and are able to recover after being deformed. In part with their shape changing ability, SMPs can demonstrate large changes in elastic modulus. Previously, SMPs have been used to create thermally changing microstructures [104], [105] as well as to create composite beams of varying stiffness [106],[107]. If added to a flexural hinge, the stiffness of the hinge could be tuned depending on the particular SMP external stimuli. In a transmission mechanism, a flexural hinge with tunable stiffness can be used to increase overall mechanism stiffness as well as couple and decouple links. As a wing flexural hinge, the SMP flexure will affect wing rotation, directly controlling lift production.

Following, we discuss the fabrication and testing of developed SMP flexural hinges. The SMP flexures are added to both the transmission and wing of a piezoelectric actuator driven system and control of mean lift though the application of heat is demonstrated.

3.7.1 Flexure Design Considerations

The tunable stiffness flexure can be envisioned to function in one of two ways:

1. A pin joint that is capable of controllable stiffness change between its fully warmed and cooled state. It should always function as a pin joint no matter its current stiffness.

2. A flexure that functions as a pin joint while cool and a loose link coupling when warmed.

Concept one could be used in the flapping transmission, allowing a change in mechanism stiffness and experimental tuning for increase in system lift. Using an active flexure to replace the normally passive wing flexure would allow control of the wing rotation and consequently system lift as well. Concept two, when used in the transmission, would interfere with normal
transmission operation when the active flexure was warmed. With a flexure between links acting as a loose coupling instead of a pin joint, force transfer through the transmission would be affected, resulting in a decrease in flapping angle and reduction in wing lift for control.

A typical flexural joint used in microdevices is the leaf type hinge, here created with appropriate layering of carbon fiber and polyimide film, a depiction of which can be seen in Fig. 3.18. The hinge is characterized by a thin material clamped at each end by two rigid bodies. As with any flexure design, there are several key features to consider when determining actual hinge geometry and composition for our desired application:

- **Joint bending stiffness**

- **Joint rotational axis:** Unlike notch hinges, the pivot point of the leaf spring is dependent upon joint deflection. With small deflections or short hinges (small $L$, see Fig. 3.20), this does not vary significantly. However, as deflections increase, flexure behavior can deviate from pin joint behavior.

- **Fatigue:** A flexure in a flapping mechanism will be continually oscillated at frequencies ranging from 30 to 100 Hz and at varying deflections depending upon location in the mechanism.

With the addition of the smart memory polymer, several additional issues must be considered:

- **Range of achievable stiffnesses**

- **Controllability:** Can the flexure stiffness be controlled and held to a desired value? How fast can the stiffness change?
• **Actuation method:** A large range of smart memory polymers exist, actuated by various external stimuli. The actuating device must be easy to integrate into a final flapping mechanism. Power consumption of the actuation method is also a concern.

In consideration of issues 4 and 6, an epoxy SMP of EPON 826 (Hexion), Jeffamine D230 (Huntsman), and Neopentyl glycol diglycidyl ether (TCI America) from the work of Xie *et al.* in 108 was chosen to create the prototype active flexures. Heat activated, the flexure stiffness can be changed easily with an off board infrared lamp or with an embedded high resistance heating wire. Depending on the component mixture, the glass transition temperature can be chosen to be from values ranging from approximately 30 °C to 95 °C. Here the formulation was chosen to have a glass transition temperature of 50 °C, above the ambient air temperature but easily producible by a heat source. The SMP storage modulus change for a change in temperature is depicted in Fig. 3.19 taken from 108. At its fully cooled state, the maximum SMP storage modulus is 2 GPa; when warmed, the minimum modulus is 9 MPa.
Flexure Modeling

Flexure bending and torsional stiffness can be modeled using laminate plate theory. A leaf flexural hinge can be approximated with a cantilever beam, as depicted in Fig. 3.20. For small displacements in a beam constructed from a single material, angular bending stiffness and torsional stiffness are:

\[
k_{\theta_x,F} = \frac{F}{\theta_x} = \frac{2EI_a}{L^2},
\]
\[
k_{\theta_x,T} = \frac{\tau}{\theta_x} = \frac{\gamma Gt^3d}{3L},
\]

(3.5)

(3.6)

where \(t\) is beam thickness, \(d\) is beam width, \(E\) is the material elastic modulus, \(G\) is the material shear modulus, \(I_a\) is the second moment of area, and \(\gamma\) is a constant dependent on the ratio \(d/t\) (see [109]). The variables \(F\) and \(\tau\) is the force and torque applied to the tip, respectively. In multilayer materials, with only a force or a twisting moment applied, this translates to

\[
k_{\theta_x,F} = \frac{2}{C_{44}L^2},
\]
\[
k_{\theta_x,T} = \frac{1}{C_{66}L},
\]

(3.7)

(3.8)

where \(C_{44}\) and \(C_{66}\) are elements in a matrix dependent on layer thickness and material properties (see [37],[110]) and are derived from laminate beam theory. The pivot point of the flexure, \(s\), is given by

\[
s = \frac{\delta_y}{\tan(\theta_x)}.
\]

(3.9)

Fabrication

Active flexure fabrication follows the ‘Smart Composite Microstructures’ technique, with the addition of several steps of SMP application and curing. A full mechanism or stand alone flexures
can be fabricated separately as desired. See Fig. 3.21 for a pictorial depiction of the process.

To fabricate a flexure, first the flexure sheet area is cut from two mirrored carbon fiber sheets. A layup of sandwiched polymide film and carbon fiber is assembled and aligned. The layup is then cured for 2 hours at 180 °C. The epoxy SMP was mixed according to the process described in Xie et al. [108], with equal mass ratios of EPON 826, Jeffamine D230, and Neopentyl glycol diglycidyl ether used. The SMP is then applied by hand to one side of the flexure area of the layup. The assembly is then thermally cured at 100 °C for 1.5 hours and post-cured at 130 °C for 1 hour. The final shape is released with cutting of the flexure or mechanism contours with the New Wave Laser Machining System. A photograph of an SMP flexural hinge can be seen in Fig. 3.21h.

The low viscosity of the uncured SMP formulation precludes precise SMP layer thickness in this fabrication process. A portion of the SMP is wicked into the carbon fiber layers before fully curing. When applied, the flexural sheet is overfilled to achieve an adequate layering effect.
Figure 3.21: Process steps for composite SMP flexure fabrication: a) flexure area is cut from two mirrored carbon fiber sheets, using laser cutter; b) carbon fiber and polyimide film are layered; c) layup is cured for 2 hours at 180 °C; d) SMP is mixed, applied to the layup, and cured according to SMP formulation specifications; e) flexure is released with exterior contours cut by laser cutter; f) final released composite SMP flexure. Images of a frontal and side view of a manufactured flexure is shown in (g)
Experimental Testing of Flexure Stiffness and Damping

The cool and warm state bending stiffness of manufactured flexures was measured by applying an impulse and recording the response using a laser micrometer (Keyence, LS-3100) at room temperature and at approximately 70 °C, above the SMP transition temperature. An extension of known mass was added to amplify motion and make the system underdamped, the complete setup of which can be seen in Fig. 3.22.

Several flexures of various geometries have been manufactured. Table 3.3 includes several that were incorporated into the flapping system; Table A.1 in the appendix contains data on additional flexures. The SMP flexure used in the transmission will be called SMP flexure 1. The SMP flexure used as the wing rotational hinge will be called SMP flexure 2. Each flexure’s weight is inflated by 3 mm long carbon fiber tabs necessary in standalone hinges; when fully integrated to the flapping mechanism, additional weight would be due only to SMP layer mass. SMP layer thickness is measured under the microscope from a side view of the hinge and due
Table 3.3: Manufactured Flexure Measured Stiffness and Damping

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Flexure Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Geometry (width × length) (mm)</td>
<td>3×0.75</td>
</tr>
<tr>
<td>SMP layer thickness (µm)</td>
<td>68</td>
</tr>
<tr>
<td>Experimental bending stiffness at 22 °C and 70 °C (mN.mm)</td>
<td>572, 11</td>
</tr>
<tr>
<td>Experimental flexure damping at 22 °C and 70 °C (µN.mm.s)</td>
<td>400, 30</td>
</tr>
<tr>
<td>Mass (mg)</td>
<td>4</td>
</tr>
<tr>
<td>Embedded Wire (Y/N)</td>
<td>No</td>
</tr>
</tbody>
</table>

Thickness variations along the width of the flexure, is only approximate. The bending stiffness of SMP flexure 2 is not experimentally tested due to failure wing lift testing.

3.7.2 Flexures for Control

Transmission Flexures

With an SMP flexure incorporated into the flapping transmission, wing lift can conceivably be varying without modifying the input of the main driving piezoelectric actuator. In this section this application is tested using the single wing piezoelectric system as originally described in Chapter 3. Due to the flexure variability introduced during the manufacturing process, flexures were first manufactured and their stiffness tested before a select number were chosen to be added to the flapping-wing transmission.

As minimizing power consumption of the mechanism is of particular importance to miniature robots and devices, the ideal flexures should be able to achieve nominal rotational joint motion with no external heating applied and large deflections when warmed. Once incorporated into the transmission of the flapping mechanism, this allows the system to operate nominally at maximal lift with no additional power expenditure.

The SMP flexures were incorporated into a rigidly mounted single wing flapping prototype at the base of first transmission link L1 as depicted in 3.3. The test setup is shown in Fig. 3.23.
where the mechanism had a passive wing flexure of rotational stiffness 11 mN.mm. Unless otherwise noted, heat was applied to the flexure through an external heating element constructed from coiled NiCr wire (40 AWG, Newtons Third Rocketry). As the temperature of the SMP itself cannot be taken directly without affecting system performance, the air temperature near the flexure was measured with a thermocouple. With an input of 2 W, air temperature 1 mm from the heating element reaches 50 °C in less than 5 sec and a maximum temperature of approximately 110 °C after 15 sec. Air temperature remains below 30 °C at the piezoelectric actuator.

The system was mounted to a mechanical balance coupled to a 30 g load cell (Transducer Techniques) for measurement of average lift. Recorded lift data at 1000 Hz was filtered with a simple moving average filter of length 100 samples. Wing kinematics and mean lift of the test system were captured concurrently with a high speed camera (pco.dimax, PCO AG, Kelheim, Germany) at 900 fps, while the SMP flexure was heated beyond its transition temperature and then allowed to cool. The camera captured the system from its top view and was able to capture both the wing flapping angle and the wing rotation angle. Wing kinematics were measured with point tracking in post-processing with After Effects CS4.
Figure 3.24: Lift and wing maximum flapping and rotation angles of the prototype single wing flapping mechanism over time with warmed and cooled SMP integrated flexure. The system was driven at 36 Hz and 200 V peak-to-peak with 2 W inputted to the heating element at times indicated with the red shaded area. Raw and filtered lift data are colored gray and black, respectively. Rotation angles are indicated with (+) and (-) for positive and negative wing rotation about the nominal position since rotation was not symmetric.
Figure 3.25: (a) System lift change versus temperature for SMP integrated flexure heating and cooling. Each data point represents at least 15 sec of captured data. (b) Photographs (1) and (2) depict side views of the SMP flexure when fully cooled and fully warmed, respectively. Two images are superimposed at the time of maximum flexure deformation with white dotted lines marking the position of the first transmission link.
Figure 3.26: Control of mean lift with external NiCr heating element. Piezoelectric actuator input voltage is held constant at 200 V peak-to-peak. A P-only controller is used with P = 10. The controller started at the 12 sec mark and the system is turned off at 52 sec.

Figure 3.24 shows the system wing flapping and rotation angles with changing lift over time for SMP flexure 1, properties of which can be seen in Table 3.3. Power of 2 W was applied to the resistance heater while the piezoelectric actuator was driven with a sinusoid of 36 Hz and 200 V peak-to-peak amplitude. With the described heating element, the system transitioned from a maximum lift of 0.52 mN to minimum lift of 0.1 mN in 3.5 secs, a loss of 80%. Wing flapping angle decreased significantly, being 80 deg peak-to-peak when cool and dropping to 50 deg peak-to-peak when warmed.

The decrease in lift is caused by a combination of decrease in system resonant frequency and transmission displacement loss due to the heated, highly deformable flexure. With the cool SMP flexure, the maximum lift of 0.52 mN peaks at 36 Hz, as can be seen in Figure 3.29b. When the flexure is heated, maximum lift is 0.25 mN and occurs at 28 Hz. Holding the driving frequency of the system constant accentuates the lift change. Figure 3.25(b) images (1) and (2) depict flexure behavior while both cool and warm. In both (1) and (2) a side view of the
system and SMP flexure is illustrated, in which two images of maximum flexure deformation are superimposed. While cool, the flexure operates as a typical, albeit stiff, rotational flexure, leaving the transmission to function as originally designed. When heated, the flexure deforms significantly and allows translational motion between the fixed base and first transmission link, resulting in a loss of flapping amplitude.

To produce maximal lift, ideally the transmission links would function as perfect rotational joints with no stiffness, resulting in amplification of actuator motion without loss or change in resonant frequency. Though the described cool SMP flexure is significantly stiffer than normal Kapton only flexures, the chosen transmission joint experiences only minimal rotational motion, minimizing the effect of additional flexure stiffness. Indeed, with a replaced passive Kapton only flexure at the first transmission link, maximum lift was 0.46 mN and occurred at 36 Hz. The small difference in maximum lift can be attributed to transmission misalignment which can occur when completely removing and replacing joints. Overall system lift is lower than what was seen in the prototypes described in Chapter 3 due to the repaired passive wing flexure which is stiffer than optimal and asymmetrically rotates, as seen in Fig. 3.24; resonance and flapping amplitude between systems is comparable.

To ensure that the flexure stiffness tuning is precisely controllable, system lift was recorded with discrete steps in the inputed power into the resistance heater. Figure 3.25a depicts change in lift with change in air temperature. Each input power was held constant at least 15 sec to ensure that both the temperature and lift stabilized. While transitioning, change in lift is almost linear with temperature, with little hysteresis. With the use of Matlab’s Control Desk dSpace board, closed loop control of lift was applied, the result of which can be seen in Fig. 3.26. Piezoelectric input voltage was held constant at 200 V peak-to-peak while voltage to the NiCr heating element was varied based on a P-only controller. While slow, the result shows that the SMP flexure can be used to control mean lift generation.
Transmission Modeling

The first transmission link behavior can be modeled with a simplified two degree of freedom dynamic system as shown in eq. (3.11). Fig. 3.27 depicts a free body diagram of L1 link driven by the actuator vertically and free to rotate about the connection point, \( P \). Newton’s equations of motion are listed below:

\[
\sum F_y = m_{\text{tot}}\ddot{y} = F_{\text{act}} - k_f y - k_{\text{act}} (y + \theta \ell_1) - d_f \dot{y} - d_w (\dot{y} + \dot{\theta} \ell_1 + \ell_2),
\]

\[
\sum M = I_{\text{tot}} \ddot{\theta} = F_{\text{act}} \ell_1 - k_{\text{act}} (y + \theta \ell_1) \ell_1 - d_w (\dot{y} + \dot{\theta} \ell_1 + \ell_2) (\ell_1 + \ell_2).
\]

Here the link rotation angle \( \theta \) is assumed to be small, which is not unreasonable given that the first transmission linkage typically rotates a maximum of no more than 15 degrees. The cantilever piezoelectric actuator is represented as an applied force \( F_{\text{act}} \), spring \( k_{\text{act}} \), and effective mass \( m_{\text{eff}} \). The active flexure is attached to the link at \( P \) with \( F_f \) the total force due to the flexure effective translational stiffness \( k_{ft} \) and damping \( d_{ft} \). The effects of wing damping and the mass of the wing and remaining spherical 4-bar links are \( F_w \) and \( m_{\text{spf}} \) respectively. For the purposes of this
simplified model, the wing force is assumed to be linear and equal to 

\[ F_w = d_w(\dot{y} + \dot{\theta}_\ell(\ell_1 + \ell_2)) \]

where

\[ d_w = \gamma \frac{\rho C_d a}{2}. \]  

(3.12)

The variable \( \rho \) is the density of air, \( C_d \) coefficient of damping taken from [28] with a wing of 45 deg, and \( a \) is the area of the wing. The variable \( \gamma \) relates the velocity of the right tip of link \( L_1 \) to the velocity of the wing and, given the transmission ratio of the system, is approximately 1.3.

Taking the Laplacian of the system of equations, we can examine how the relative magnitude of the output of link \( L_1 \) changes with a change in input driving frequency and active flexure stiffness. The transfer function relating the input force \( F_{\text{act}} \) and the angle of \( L_1 \) is

\[ \frac{\theta_\ell(s)}{F_{\text{act}}(s)} = \frac{h(s)}{g(s)} \]  

(3.13)

\[ h(s) = \ell_1 m_{\text{tot}} s^2 + (d_f \ell_1 - d_w \ell_2) s + k_f \ell_1 \]  

(3.14)

\[ g(s) = I_{\text{tot}} m_{\text{tot}} s^4 + (d_f I_{\text{tot}} + d(I_{\text{tot}} + (\ell_1 + \ell_2)^2 m_{\text{tot}})) s^3 \]

\[ + (I_{\text{tot}} k_{\text{act}} + I_{\text{tot}} k_f + d_f d_w (\ell_1 + \ell_2)^2 + k_{\text{act}} \ell_1^2 m_{\text{tot}}) s^2 \]

\[ + (d_f k_{\text{act}} \ell_1^2 + d_w k_{\text{act}} \ell_2^2 + d_w k_f (\ell_1 + \ell_2)^2) s + k_{\text{act}} k_f \ell_1^2 \]  

(3.15)

As can be seen in the plotted frequency response in Fig. 3.28, as the flexure stiffness increases, the behavior of the system approaches the behavior of the transmission with a traditional pin joint at \( P \). Nominally the simulated system resonates at approximately 55 Hz; with a lower \( k_f \), the resonance frequency decreases. Maintaining a constant operating frequency of 55 Hz results in a lower flapping amplitude with a decreasing \( k_f \), achieving the desired lift control.

A difference in maximum and minimum flexure stiffness will result in correspondingly different ranges of lift variation. Lift plotted verses input driving frequency for flexure 1, 11, and 12 can be seen in Fig. 3.29. Flexure 12, has a smaller difference in flexural stiffness between
Figure 3.28: Simulated frequency response of system transfer function 3.15. The response plotted in a dotted line is the actuator and linkage with a pin joint at $P$. As the stiffness $k_f$ increases, the behavior of the linkage with the active flexure approaches ideal operation.

In the case of flexure 12, the warmed flexure produces a comparatively small change on the system behavior. While this results in only a small change in lift with the operating frequency set at 46 Hz, changing the operating frequency can allow lift to be actually increased when heat is applied due to the broader lift peak in Fig. 3.29c. Mean lift over time with heat applied in both cases is shown in Fig. A.1 in the Appendix.

**Active Wing Flexures**

The other critical flexural joint in the flapping system is the wing hinge, which whose stiffness, along with aerodynamic forces and wing inertia, determines the wing rotation angle. The normally passive wing flexural joint of the rigidly mounted system was replaced by SMP flexure 2. To prevent over-rotation of the wing, stoppers are added to the gamma transmission link,
Figure 3.29: Flapping flight prototype frequency response with piezoelectric actuator input voltage 200 V peak-to-peak. The frequency response is taken when both the incorporated SMP flexure is cool (22 °C, room temperature) and warm (70 °C, above SMP glass transition temperature). Flexure 11 is shown in (a), Flexure 1 in (b), and Flexure 12 in (c), details of which can be found in Table 3.3.
Figure 3.30: Experimental Setup for active wing flexural hinge.

Heat, in this case, was applied by a 150 W infrared lamp placed 3 inches from the wing flexure. The lamp was capable of bringing the ambient air temperature about the flexure up to \( \sim 70 \, ^\circ C \) in 5 sec. Alike in the active transmission tests, lift and wing motion is captured concurrently with a high speed camera and load cell. The experimental setup is shown in Fig. 3.30.

In order to increase data processing speed, the wing rotation angles are measured in post-processing using a routine utilizing edge detection, Gaussian clustering, and linear regression. As the wing trailing and leading edges are well defined by single carbon fiber spars, the angle of these spars can be extracted from the captured high speed camera video. A description of the algorithm steps is given below.

For each frame:

1. Use Canny edge detection to identify image edges
2. Smooth edges and extract contours
Figure 3.31: Three stages of automatic wing tracking: (a) single image from high speed camera, (b) edge detection after smoothing, (c) point clustering with two Gaussians and linear regression.

3. Eliminate frame background with comparison to previous frame

4. Find the best fit of the remaining edges to two Gaussian distributions

5. Cluster edge points based on fitted distributions

6. Use linear regression to line fit each cluster

Example stages of a processed frame can be seen in Fig. 3.31. Existing MATLAB implementations of Canny edge detection, fitting Gaussian distributions, and clustering were used. Edge smoothing and contour extraction was implemented by He Xiaochen, HKU EEE Dept., and distributed through MATLAB Central. Peak-to-peak wing rotation angle measurement accuracy, compared to hand tracked points, is within approximately 5 deg.

System lift and peak-to-peak wing rotation angles over time with the flexure heated and cooled is shown in Figure 3.32. As opposed to the case of the active flexure in the transmission, maximum lift occurs when heat is applied to the system. The wing flexure allows only minimal wing rotation when cooled, producing little lift. When heated, the wing flexure softens and allows the wing to rotate to its maximum allowed angle by the stoppers.

While the active wing flexure does allow lift force variation, the maximum lift produced is
Figure 3.32: Peak-to-peak wing rotation angle and lift over time. Maximum wing rotation was limited with a stopper. Piezoelectric actuator input voltage was 190 V peak-to-peak at 30 Hz. Heater is on during time indicated with shaded red band.
Figure 3.33: High speed camera snap shots of wing motion taken in intervals of 4 ms. Each row corresponds to time region marked in Fig. 3.32 where (2) denotes the wing flapping with a fully warmed flexure.

significantly less than one might expect from the system. This is primarily due to undesirable wing torsion when the flexure is warmed. As can be seen in Fig. 3.34, the peak-to-peak gamma transmission link and wing flapping angles are significantly different. The typical passive wing flexure is designed to be highly resistant to torsion, allowing only bending about its pivot point, leaving the gamma transmission link angle and wing flapping angle to be the same. In the warmed SMP flexure, however, the wing drag forces overcome the flexure torsional stiffness, expending energy on wing twist at the expense of lift production.

While a reduction of flexural torsion could be achieved with optimization of the flexure dimensions, the large, necessary deflections for wing rotation introduce additional problems due to fatigue. Unlike the trials involving the SMP flexures in the transmission, wing SMP flexures failed quickly and often. The large deflections and unconstrained motion resulted in cracks along the SMP layer and could occur in as little as 200 flapping cycles.

**Transition Time and Power Consumption**

While the original SMP flexures were heated with an external resistance or infrared heater, internal heating is possible, though with increased manufacturing complexity. The external resistance
Figure 3.34: In (a) hand-tracked wing rotation, flapping, and gamma transmission link angles are shown. In (b) the wing leading edge, position of which determines the system flapping angle, as well as the gamma transmission link are labeled.

heater uses a little under 2 W of power to induce a full transition of the flexure; embedding NiCr wire into the SMP flexure layer can result in a more compact system and significantly reduce power expenditure necessary for a stiffness change. Transition speed is also a predominate concern for practical use in control of a miniature flapping wing based flying robot. The fastest demonstrated change in lift in the externally heated flexure 1 is 0.12 mN/sec, which may or may not be sufficient considering current flapping wing controllers rely upon a per wing stroke update frequency. Relying upon the temperature differential between a flexure and room normal will be unlikely to result in very fast transition times, but a change in heating element can help both reduce both that transition time and total power consumption.

In the manufacturing steps described in Fig. 3.21 a length of NiCr wire is inlaid into the flexure before the SMP is applied. The use of a wire instead of an added heating element patterned on the kapton itself precludes the use of thin SMP layers as the wire should not detach. Details of flexures 10 - 12 are described in Table 3.3 and each have an internal heating element. Each of the three flexures is tested individually in regards to power consumption and transition time.

Where as the external NiCr heating element required power above that required to drive the device flapping (2 W), internal heating required a small fraction of that value. Figure 3.35 depicts
Figure 3.35: Average stiffness of three SMP flexures with NiCr embedded heating elements over a range of inputted power. Error bars represent the standard deviation of 5 measurements at each point. Flexure dimensions can be found in Table 3.3. The flexures are held at the set input power for at least 30 seconds to allow temperature to stabilize.

Flexure stiffness versus input power. Each flexure was held fully transitioned with 50 mW. While this is still a source of significant power consumption, it is now less than the approximately 100 mW required for wing flapping.

Figure 3.36 contains several runs of each flexures’ stiffness as it is heated and cooled to room temperature. Each achieves 75% of its cool to warm transition in between 1 and 1.3 seconds. Flexures 11 and 12 cooled to 75% of their maximum stiffness in 2.5 seconds, while flexure 10 achieved the same transition in 3.5 seconds. When used in the flapping-wing system transmission this translates to a significant improvement in time for maximum lift variation. Examining flexure 1 and 11 directly, flexure 11 reaches 75% of its minimum lift in 1.5 seconds when warmed at a similar NiCr material heating limit and cooled to 75% of its maximum lift in 2.5 secs, as depicted in Fig. A.2. Flexure 1 experiences a much greater delay, requiring 4.5 seconds to reach 75% of its minimum lift from when the heater is first turned on. Similarly, the flexure requires more than 4 seconds to reach 75% of its maximum lift from when it begins to transition.
Figure 3.36: Transition time for three SMP flexures with embedded heating elements. Each line denotes a separate experimental trial. In (a) flexures are heated from room to their transition temperature, where time = 0 denotes when a current is applied to the heating element. In (b) flexures are cooled to room temperature, where time = 0 denotes when the flexure begins to drop below their transition temperature.
3.8 Summary

In this chapter the design and fabrication of a piezo-driven flapping wing system is described. In small, flapping wing systems there exists an inherent trade off between liftoff and platform controllability. Additional actuators are needed in order to generate controlling forces and torques, but actuators tend to be heavy and contribute significantly to overall platform weight, decreasing the lift-to-weight ratio. In the majority of existing flapping wing platforms one is achieved at the expense of the other. Here both a two piezoelectric actuator design is detailed and a new controlling actuator is proposed. The two piezoelectric actuator system uses passive wing rotation and underactuation to reduce the platform weight while remaining control of individual flapping angle for generation of controlling forces and torques. The alternative controlling actuator allows joint stiffness control in flexural mechanisms; in the piezo-driven flapping system it allows lift control in a lighter single piezoelectric actuator system with the addition of very little additional system mass.

Several large scale two piezoelectric actuator prototypes were manufactured, with lift-to-weight ratios no greater than \( \sim \frac{1}{5} \). A smaller prototype was constructed with an improved lift-to-weight ratio of \( \sim \frac{3}{8} \). Though none of the constructed piezoelectric prototypes were capable of liftoff, with further system scaling and improved manufacturing techniques the design has the potential to reach a lift-to-weight ratio of above one. Currently they can function as testbeds for control experiments and provide useful insight into the design of an eventual controllable autonomous flapping wing robot. Chapter ?? details the verification of platform roll and torque generation as well as experimental control testing on a restricted degree of freedom rig.

A series of control actuators, or active flexures, are described and experimentally tested. Composed of polyimide film and shape memory polymer, the active flexures demonstrate lift control when placed in two positions in the piezoelectric actuator driven system without changing the main driving actuator input. Though even with an embedded heating element that reduces power consumption and transition time, response time is relatively slow, regulating SMP flex-
ures in more dynamic flapping mechanisms to tasks such as steering. However, while large lift variations cannot be achieved on the order of a single wing beat, the SMP flexures could be incorporated into any existing flexural mechanism and be well suited to other micro-scale devices.
Chapter 4

Motor-driven Flapping Flight Platform

4.1 Introduction

The reciprocating wing motion in the flapping system can be quite power intensive as it is not only aerodynamic drag that must be overcome, but also the inertia of the constantly oscillating wing and accompanying mechanisms. Insects such as fruit flies and blowflies drive their wings at mechanical resonance to combat this effect [112]. Similarly, in small scale flapping wing micro aerial vehicles (FMAVs) (< 1 g), piezoelectric actuator driven transmissions have been designed to resonate, utilizing the actuator itself as the elastic element as with the system described in Chapter 2 and in [113], [114], [38] with various actuator configurations for control [39], [115], [86]. This has successfully led to liftoff and stable hovering as demonstrated by Teoh et al. [116].

Piezoelectric actuators are particularly well suited for small scale flapping systems due to their capability for high power densities and high efficiencies [95], [37]. Motor power is significantly less at similar sizes as electromagnetic forces scale poorly with a decrease of characteristic length, $L$, and are proportional to $L^4$. For larger systems with a higher desired payload, the use of motors is more common due to the corresponding increase in available power and limits of piezoelectric actuator displacement. Resonance, however, is not much utilized. Slider cranks or more complicated transmissions, such as those seen in the motor-driven Delfly [31] or the
AeroVironment’s Nano Hummingbird [20], are used to ensure a desired oscillating wing stroke but are ill suited to fully exploit resonance. Previous works have incorporated elastic elements into similar transmissions for energy recovery [117], [118], [119], [120], [121], but the kinematic nonlinearity of the mechanisms inhibits the full benefits one would get with resonance operation.

Traditional slider crank mechanisms have additional limitations when considering system control. There exist several small, motor-driven FMAVs capable of vertical liftoff but are either unable to achieve controlled hover due to symmetrically driven wings or rely upon system passive stability with the use of added dampers [32], [46], [45]. Modifying the system to allow control can have significant trade-offs. In general, a desired flapping stroke can be designed with a slider crank, but the amplitude of that stroke cannot be varied. Elastic elements can be added in series to induce varying wing amplitudes with a change in flapping frequency in a single motor-driven system, but this coupled relationship can make control over both total lift generation and body torques difficult. Wing flapping frequency can be easily changed, but independent wings driven at differing frequencies can create undesirable body oscillations. Additional mechanisms and actuators can be added, either to shift fins, body center of mass, or wing rotation angles, but add to overall system weight, reducing the lift-to-weight ratio.

In this chapter we describe a simple motor-driven flapping prototype capable of liftoff and controlling torques. Based on the concepts developed by Campolo et al. [122], [123], with an initial prototype presented in [124], the design relies upon an elastic element placed in parallel to a motor output shaft and wing. In the current implementation, a gearbox is used to increase motor torque which maintains the linear relationship between motor and wing motion. Because of this linear transmission and added spring, the system can resonate, eliminating the need to expend power on motor rotor and wing motion. Resonance frequency can be easily chosen as desired as, unlike in piezoelectric-driven systems, the system’s elastic element is not a property of the actuator itself. In addition, wing flapping angle is not restricted, allowing change in flapping amplitude and mean flapping angle. In comparison to the Nano Hummingbird [20] it is a
Figure 4.1: Magnified view of CAD single wing module (a) and photo of prototype composed of two modules (b). The basis vectors $x$, $y$, and $z$ denote the prototype body coordinate system which is fixed at the center of mass, and are used to define roll, pitch, and yaw body rotations.
very minimalist design, yet among the first to be constructed to operate about hover and allow independent wing motion for control.

Note that due to restrictions from the current motor gearbox, relatively low stiffness elastic elements are used in the constructed systems, leading maximum lift to occur beyond system resonance. Though the full power savings available from this design are not completely taken advantage of in the current prototypes, a lift-to-weight ratio greater than 1 is still achieved.

Following, the design of single actuator and wing for lift generation is described with several physical parameters varied experimentally. Wing placement in multi-wing systems is also explored considering implications on system stability and controllability.

4.2 Design of Single-wing Module for Lift Generation

4.2.1 System Description

The FMAV design, shown in Fig. 4.1, is composed of three main components: the driving motor, elastic element, and wing. Two brushed DC pager motors (GM15A Solarbotics) directly drive each wing. Unlike most motor-driven systems, no additional transmission is added; motor torque is increased only with the GM15A’s 25:1 planetary gearbox. The main consequence of this direct connection is that in order to generate wing flapping, each motor undergoes a reciprocating motion rather than continuous spinning. An elastic element, a helical spring, is attached to both the gearbox output shaft as well as the motor casing, as can be seen in Fig. 4.1a. A 3D printed cap is used to secure the end of the spring, output shaft, and wing spar. Cyanoacrylate (Loctite 498) is used to secure the connections between each component. The maximum stiffness of the elastic element is limited by the gear strength in the current gearbox. The GM15A is not back-drivable and will fail if the load on the output shaft is too high.

The direct attachment of the wings to the output shaft allows change in wing flapping amplitude and mean flapping angle. Flapping motion is induced with a sinusoidal input voltage
The input voltage to each motor, where $V_{in} = \left( \frac{V_{pp}}{2} \right) \sin(2\pi ft) + V_b$ to each motor, where $f$ is the driving frequency, $V_{pp}$ is the peak-to-peak voltage magnitude, and $V_b$ is the constant voltage bias. The torque required to induce a rolling motion as shown in Fig. 4.1b can be created with a difference in $V_{pp}$ between motors. This results in a difference in flapping amplitude and mean lift between each wing. A pitching motion, or rotation about the $y$ axis, can be created by changing $V_{bias}$ which shifts the wing flapping angle. This effectively shifts the aerodynamic center of lift over a wing stroke either in front of or behind the prototype center of mass.

Two wings are mounted to the system a distance $d_w$ offset from their respective motor output shaft. While motors directly drive wing flapping, wing rotation is passively achieved through a rotational flexure. Relying on passive wing rotation reduces the number of actuators required for wing motion and reduces overall system mass significantly, a strategy which was also used in the piezoelectric design described in Chapter 3.

As there is no additional transmission that requires fixed mounting points, the extent of the prototype’s rigid body structure is the motor casings, leading to a system mass of 2.7 g. Battery, sensors, and additional electronics are not considered here, but will be integrated into the system in future work.

### 4.2.2 Impedance Matching and Motor Selection

In the simplest sense, designing a liftoff capable, hovering FMAV can be divided into two main questions: (1) What wing and wing stroke will produce the lift necessary for liftoff?, and (2) Can the driving actuator supply enough power in order to achieve this wing stroke? Due to the contribution of actuator mass to overall system weight, the questions are coupled and the design space large. While running an optimization on a full dynamic model of the system is an option, a better alternative for initial motor and wing selection is a simplified analysis based on our knowledge of power transfer between our motor source and our flapping load.

According to the maximum power transfer theorem for linear networks, for a given input...
voltage the maximum mechanical power delivered to the load equals the electrical losses. Maximum power is delivered when the resistance of the source is equal to the resistance of the load, which is called the *impedance matching* condition. To gain a better understanding of how the impedance ratio influences our system performance, we can use the approach described in the work of Campolo *et al.* \[122, 123\], where impedance matching is examined for motor selection when driving similar damped resonant loads. The simplified geared motor and wing model from \[122\] is

\[
V_{in} = R_0 I_{in} + k_t N_g \omega_l \tag{4.1}
\]

\[
\eta_g N_g k_a I_{in} = J_{tot} \alpha_l + (b_0 + B_0 \omega_l \text{sign}(\omega_l)) \omega_l + k_s \theta_l \tag{4.2}
\]

where \(\omega_l\) and \(\alpha_l\) are the angular velocity and acceleration of the wing and \(I_{in}\) and \(V_{in}\) are the input current and voltage. The combined inertia of the motor rotor and wing is \(J_{tot}\), \(N_g\) is the gearbox ratio, and \(\eta_g\) is the gearbox efficiency. The armature resistance and motor constant are \(R_0\) and \(k_t\) respectively. The motor spring constant is \(k_s\). The effective motor damping, \(b_0\) is a function of the rotor damping \(b_m\) where \(b_0 = \eta_g N_g^2 b_m\). The effect of wing damping is estimated with a coefficient \(B_0\) which is defined as

\[
B_0 = \frac{1}{2} \rho C_d \int_{d_w}^{R + d_w} r^3 c(r) \, dr, \tag{4.3}
\]

where \(\rho\) is the density of air, \(R\) is the length of the wing, \(d_w\) is the wing offset, \(r\) is the distance from the flapping axis, and \(c(r)\) is the wing cord length at \(r\). The coefficient of drag, \(C_d\), is assumed to be 1.7 for a wing with a rotation angle of 45 deg \[28\].

From the simplified system model in eqs. \[4.1\] and \[4.2\], the instantaneous mechanical power balance can be derived as in \[122\]:

\[
\eta_g \eta_x \frac{1}{2} \left( \frac{V_{pp}}{2} \right)^2 = \frac{1}{\mu} P_{drag,p}, \tag{4.4}
\]
where it is assumed that the input voltage is sinusoidal such that \( V = \frac{V_{pp}}{2} \sin(2\pi ft) \) and the power expended due to drag \( P_{\text{drag}} \) is \( P_{\text{drag}} = P_{\text{drag,p}} \sin(2\pi ft) \). The motor spring \( k_s \) is assumed to be chosen such that the system resonates at the flapping frequency \( f \) where \( k_s = (2\pi f)^2 J_{\text{tot}} \). The efficiency term \( \eta_x \) is

\[
\eta_x = \left(1 + \frac{b_0}{\frac{s}{3\pi} B_0 \Omega_0}\right)^{-1},
\]

(4.5)

and accounts the mechanical power dissipated against internal motor friction, while

\[
\mu = 4 \frac{R_{\text{mech}}/R_0}{(1 + R_{\text{mech}}/R_0)^2}
\]

(4.6)
is the impedance-mismatch factor. The variable \( \Omega_0 \) is dependent on wing speed and is equal to \( \pi f \theta_{pp} \) where \( \theta_{pp} \) is the peak to peak flapping angle. The peak power dissipated against aerodynamic drag as derived in [122] is

\[
P_{\text{drag,p}} = B_0 \frac{8}{3\pi} \Omega_0^2,
\]

(4.7)

and the equivalent electrical impedance of the mechanical damping is

\[
R_{\text{mech}} = \eta_g \eta_x \frac{k_a^2}{\frac{s}{3\pi} B_0 \Omega_0}.
\]

(4.8)

Equation 4.4 represents a relation between any desired flapping amplitude \( \theta_{pp} \) and the resulting necessary peak-to-peak input voltage \( V_{pp} \) for a given motor. However, its current form lends to a nice graphical interpretation for evaluation of motor suitability. The right side of eq. 4.4 can be viewed as actuator independent, and is the required power to flap a desired wing at set kinematics given a system impedance ratio \( R_{\text{mech}}/R_0 \). The left side is a representation of the maximum power that can be supplied by a motor given a maximum \( V_{pp} \) as defined by motor
specifications. We can then consider two definitions

\[ P_{\text{available}} = \eta_p \eta_x \frac{1}{2} \frac{(V_{pp,max}/2)^2}{2R_0} \]  \hspace{1cm} (4.9)

\[ P_{\text{req}} = \frac{1}{\mu} B_0 \frac{8}{3\pi} \Omega_0^3. \]  \hspace{1cm} (4.10)

For an assumed wing shape and wing flapping amplitude, the difference between the available and required power for a chosen motor can be plotted as flapping frequency and wing length varies. In order to achieve the desired flapping motion, \( P_{\text{available}} - P_{\text{req}} > 0 \) must hold.

However, being able to achieve the desired flapping amplitude is not sufficient. It is necessary to include a prediction on system lift-to-weight ratio to verify that a chosen motor can achieve liftoff at some selection of wing size and flapping frequency. With wing flapping and rotation angle assumed to be

\[ \theta(t) = \frac{\theta_{pp}}{2} \sin(2\pi ft), \]  \hspace{1cm} (4.11)

\[ \phi(t) = 0.8 \sin(2\pi ft - 1.1), \]  \hspace{1cm} (4.12)

the mean lift over a wing stroke can be calculated using the model described in Chapter 2. This results in a system lift to weight ratio of

\[ LW = \frac{\int_0^{1/f} F_z dt}{m_{\text{motor}} g} \]  \hspace{1cm} (4.13)

where \( F_z \) is the predicted instantaneous wing lift force and \( m_{\text{motor}} \) is the mass of the motor. For a motor to be a reasonable option for use in the flapping MAV, the two conditions \( P_{\text{available}} - P_{\text{req}} > 0 \) and \( LW > 1 \) must hold.

When evaluating motors, wing peak-to-peak flapping angle \( \theta_{pp} \) was set to 120 deg and wing shape is assumed to be constant. The normalized wing shape used is depicted in Fig. 4.2 where a change in wing length results in an appropriate scaling of wing shape. An additional offset of
35 mm from the flapping axis is added to each wing. Over 60 brushed DC motor and gearbox pairs were evaluated using this process, the full list of which can be found in Tables B.1-B.3.

A graphical representation of the two required conditions is shown for three motors in Fig. 4.3. The difference between available and required power is shown given varying wing length and flapping frequency. The motor is expected to be able to lift its own weight in the area right of the decision boundary marked by $LW > 1$. Ideally the conditions where $P_{\text{available}} - P_{\text{req}}$ is maximized will occur well within the region $LW > 1$ as the lift-to-weight ratio increases significantly with increases of wing size and flapping frequency. Of the motors tested the Solarbotics GM15A was among the best performing, though maximum performance was predicted at a wing length and frequency where expected lift-to-weight ratio is around one. The two other motors shown in Fig. 4.3b and 4.3c cannot supply enough power to lift their own weight. Of the motors tested, most un-geared motors tended to supply significantly less power than needed. The majority of small, geared motors however, added a detrimental amount of mass with their increase in torque. The GM15A with a plastic gearbox, low weight, and high torque output proves to be a good compromise at, perhaps, the expense of durability.

As would be expected, Fig. 4.3a shows that there are a range of wing lengths and flapping frequencies that could be selected for the flapping MAV. A choice of a flapping frequency of 20 Hz and wing length of 40 mm should be as suitable as a frequency of 10 Hz and length of 70 mm as long as the stiffness of the motor spring $k_s$ is chosen such that the system resonates in each case. At these points the effective load the motor experiences, and consequently the system
Figure 4.3: Difference between the maximum power that can be supplied by a motor and the power required to flap the wing with a wing stroke of 120 deg peak-to-peak. The difference in power is plotted as flapping frequency and wing size are varied. The wing shape used is defined in Fig. 4.2. The lined area in the figure denotes the area where the predicted lift is greater than the weight of the motor. The results of three motors are shown: (a) Solarbotics GM15A, (b) Vigor Precision BO-P1A/B, and (c) Precision Micromotor 104-001. Only in (a) is the motor expected to be capable of lifting its own weight.
impedance ratio, is the same. Due to restrictions on the maximum spring stiffness used and the load on the gearbox, however, it is beneficial in this case to choose a lowing flapping resonance.

4.2.3 Experimental Lift Generation and Physical Parameter Variation

There are several key prototype parameters that affect overall system performance. They include the driving motor choice (and attached gearbox ratio), wing size and shape, wing offset from the flapping axis, wing flexure stiffness, elastic element stiffness, and flapping frequency. Here we restrict ourselves to experimentally testing change in wing offset, elastic element stiffness, and operating frequency. By changing the wing offset, the effective damping force experienced by the motor is changed, allowing motor specific limitations such as torque generation and power transmission to be examined. Likewise, a change in the stiffness of the elastic element changes the system resonance frequency, affecting the frequency at which maximum flapping amplitudes occur and efficiency at a set operating point. Driving motor choice, wing size and shape, and wing flexure stiffness remain constant in the following experiments.

The selection of commercially available pager motors suitable for this application is limited; in this work we test only the GM15A which has both a reasonable output torque, price, and weight. While the wing size is kept constant at 70 mm in length, increasing the wing offset also effectively increases the damping force experienced for the same swept flapping angle, which can be limited given the maximum motor torque. Three wing offsets are tested: 38, 26, and 12 mm.

Ideally, one would choose the prototype elastic stiffness such that it follows \( (2.39) \) and resonates at the desired operating frequency \( f \). In this case, with a wing offset of 38 mm and desired operating frequency of 10 Hz, \( k_{s,\text{ideal}} = 4.7 \times 10^3 \text{ mN.mm/rad} \). However, the GM15A is not backdriveable, and the plastic gears are prone to failure at high loads, limiting the elastic element stiffness feasible in this particular case. To prevent gearbox failure, the stiffnesses tested are typically less than the ideal at the tested wing flapping frequency and are \( 2.8 \times 10^3, 1.6 \times 10^3, \) and
$1.1e3$ mN.mm/rad. While the tested systems could be more efficient at the desired operating frequency, the elastic element should still show a benefit. Later improvements are possible with better gearboxes and motors.

**Materials and Methods**

In order to compare system performance with varied physical and driving parameters, an experimental setup, illustrated in Fig. 4.4, was constructed in order to measure the mean lift, flapping amplitude, and input power of a motor driving a single passively rotating wing. Multiple of these single wing modules are envisioned in a complete flapping system, so total expected system lift would simply be a multiple of the lift measured in experiment.

A single motor and wing was attached to a lever arm and load cell (GSO-30, Transducer Techniques), a photograph of which can be seen in Fig. 4.5. The lever arm in conjunction with the load cell amplifies lift forces but allows only mean lift to be measured. The system was driven with a PWM approximated sinusoidal voltage generated by a motor driver (SyRen 10, Dimension Engineering) which took input from a function generator (33120A, Hewlett Packard). A video camera (HDR-SR11, Sony) was placed above the system to capture the wing swept area. Peak-to-peak flapping angles were measured in video post processing where successive frames were combined and peak wing positions were captured. Lift, input voltage, and input current were recorded with dSpace DAQ board (DS1104 Controller Board) at 10 kHz. Input impedance of the DAQ board was increased with additional resistors in series for the motor input voltage measurement. Input current was measured with a Hall-effect-based current sensor (ACS712-30A, Allegro MicroSystems).

Motor and general system physical parameters can be found in Table 4.1 and 4.2. The GM15A (Solarbotics) DC motor, with a planetary gear box and a gear ratio 25:1, was used in the prototypes. Several parameters, as noted, are taken from a 3D CAD model of the component. The wing flexure stiffness and damping was calculated using the logarithmic decrement
Figure 4.4: Diagram of experimental Setup A. Input voltage, current, system lift, and flapping amplitude are all recorded.

Figure 4.5: Close-up view of half prototype mounted to the lever arm load cell setup used in the mean lift measurement experiments.
Table 4.1: GM15A Motor Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor mass</td>
<td>1.21</td>
<td>g</td>
</tr>
<tr>
<td>$k_a$ motor constant</td>
<td>1000</td>
<td>mN.mm/A</td>
</tr>
<tr>
<td>$R_0$ armature resistance</td>
<td>16</td>
<td>Ohm</td>
</tr>
<tr>
<td>$J_m$ rotor inertia$^\dagger$</td>
<td>1.6</td>
<td>g.mm$^2$</td>
</tr>
<tr>
<td>$b_m$ rotor damping</td>
<td>7</td>
<td>$\mu$N.mm.sec/rad</td>
</tr>
<tr>
<td>$N_g$ gear ratio</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>$\eta_g$ gearbox efficiency</td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>

$^\dagger$Predicted from 3D CAD model

method after adding a known weight to the flexure and recording the resulting motion after an impulse was applied. The elastic element stiffness was calculated from a torque measured with the Transducer Techniques load cell for a known displacement. Gearbox efficiency was taken to be 60% based on blocking force measurements of the motor with and without the gearbox. In reality this value may be even less due to the additional loads on the gearbox during flapping. The GM15A is rated to a maximum DC voltage of 6 V, which translates to an effective upper bound of 16.8 $V_{pp}$ considering the input voltage RMS. An image of the wing used in the experiments can be seen in Fig. 4.1b.

Results

To determine the resonant frequency of each system and the frequency at which the maximum lift occurs, each of the prototype variations were mounted as described previously with the input voltage frequency swept from 2 to 20 Hz. Figure 4.6 depicts the average and standard deviation of peak-to-peak flapping angles, mean lift, and input power for the range of input frequencies. DAQ recorded data is filtered with a moving average filter dependent on flapping frequency, encompassing 2 wing strokes. Data average and standard deviation are calculated for a time segment of at least 2 seconds where the input voltage is held at the desired amplitude and frequency. Over each time segment, peak-to-peak flapping angles are measured at least 4 times.
Table 4.2: FMAV Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body mass</td>
<td>2.7 g</td>
<td></td>
</tr>
<tr>
<td>Max lift-to-weight ratio</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>$\vec{J}$ body inertia, $[J_x, J_y, J_z]^\dagger$</td>
<td>$[1.3e3, 0.3e3, 1.0e3]$</td>
<td>g.mm$^2$</td>
</tr>
<tr>
<td>$K_s$ elastic element stiffness, varies</td>
<td>$2.8e3$, $1.6e3$, $1.1e3$</td>
<td>mN.mm/rad</td>
</tr>
<tr>
<td>$d_w$ wing offset, varies</td>
<td>38, 26, 12</td>
<td>mm</td>
</tr>
<tr>
<td>$m_w$ wing mass</td>
<td>0.08 g</td>
<td></td>
</tr>
</tbody>
</table>
| $J_w$ wing inertia matrix$^\dagger$ | \[
\begin{bmatrix}
4.50 & 0 & 0.76 \\
0 & 38.95 & 0 \\
0.76 & 0 & 34.46 \\
\end{bmatrix}
\] | g.mm$^2$ |
| $R_{CG}$ wing CG horz. distance$^\dagger$ | 29.5 mm | |
| $\beta_{CG}$ wing CG vert. distance$^\dagger$ | 6.7 mm | |
| Wing rotation axis distance from leading edge | 1 mm | |
| Wing length        | 70 mm  |           |
| $k_{rot}$ wing flexure stiffness | 170 mN.mm/rad | |
| $b_w$ wing flexure damping | 0.3 mN.mm.s/rad | |

$^\dagger$Predicted from 3D CAD model

For the system input voltage frequency sweeps, voltage magnitude was held at $5.1 V_{pp}$, limited in magnitude due to the high flapping angles seen in the prototype with the least stiff elastic element. In Figure 4.6, each color denotes a different elastic element stiffness while each symbol type denotes a different wing offset. For sake of figure clarity, the simulated system results based on the model described in Chapter 2 are included as solid lines and are only shown for the wing offset of 38 mm.

As can be seen in Fig. 4.6a, resonance for each system, as indicated by a peak in flapping angles, occurs below 10 Hz as expected. The $1.1e3$ mN.mm/rad system peak is barely measurable, $1.6e3$ mN.mm/rad system peaks at $\sim 4$ Hz, and the $2.8e3$ mN.mm/rad system peaks at $\sim 7$ Hz. Maximum lift, however, occurs beyond flapping resonance, as can be seen in Fig. 4.6b. At very low flapping frequencies, wing speed is simply too low to produce appreciable lift or wing rotation. This results in maximum lift at frequencies above flapping resonance at a balance of
Figure 4.6: Peak to peak flapping angle (a), mean lift (b), and input power (c) for various input voltage frequencies, elastic element stiffness, and wing offset. Both data mean and standard deviation is shown. Input voltage is set at 5.1 $V_{pp}$. The simulated system results for the 38 mm wing offset prototypes with various elastic element stiffnesses are included as solid lines.
wing speed, peak-to-peak flapping angles, and wing rotation. Here, in the order of increasing stiffness, maximum mean lift occurs at \( \sim 6 \) Hz, \( \sim 8 \) Hz, and \( \sim 10 \) Hz. Input power, as shown in Fig. 4.6c decreases around the peaks of flapping angle and mean lift.

The simulated system results show a slightly low if similarly increasing peak flapping angle and mean lift frequencies for increasing elastic element stiffnesses. Flapping angles in simulation peak at magnitudes significantly greater than what was seen in experiment which may be due to additional unmodeled frictional effects of a loaded gearbox, nonlinear spring behavior at large deflections, and imperfect damping influencing resonance operation.

The change in wing offset tested does not significantly change the frequency of the peak flapping angle in the majority of the prototypes shown here. The difference in system inertia and effective wing damping between 38 mm and 12 mm offset does result in slightly larger peak flapping angles in prototypes with the same elastic element stiffness.

A sweep of input voltage amplitudes was also applied to the prototypes, the results of which can be seen in Fig. 4.7. Each prototype was run at the frequency where the maximum lift was observed in the previous frequency sweep. Three prototypes in Fig. 4.7b are able to achieve a lift-to-weight ratio of greater than one, indicated with a horizontal dotted line at the weight of the half FMAV (13.5 mN). These are the \( 2.8e3 \) mN.mm/rad systems with a wing offset of 38 and 26 mm as well as the \( 1.6e3 \) mN.mm/rad system with a wing offset of 38 mm. In general, the prototypes with the greater wing offset exhibited a greater mean lift, albeit with smaller flapping amplitudes, compared to other systems with the same elastic element stiffness.

Figure 4.7 well illustrates a practical constraint on the FMAV design. The FMAV is limited to a maximum peak-to-peak flapping angle of 180 deg to prevent wing interference in the full system. While the prototypes with the least stiff elastic element may exhibit a reasonable lift peak at 6 Hz, the large flapping angles limit the maximum achievable lift for a swept input voltage magnitude. Each of the \( 1.1e3 \) mN.mm/rad systems and the 12 mm offset \( 1.6e3 \) mN.mm/rad system have low maximum lift, results of which overlay each other in Fig. 4.7.
Figure 4.7: Peak to peak flapping angle (a) and mean lift (b) for various input peak to peak input voltages, elastic element stiffness, and wing offset. Both data mean and standard deviation is shown. With each elastic element stiffness the system was run at the frequency where the highest lift value was observed during an input voltage frequency sweep: $2.8e3$ mN.mm/rad at 10 Hz, $1.6e3$ mN.mm/rad at 8 Hz, and $1.1e3$ mN.mm/rad at 6 Hz. The horizontal dotted line indicates the weight of the half FMAV, where the lift-to-weight of the system is equal to one. The simulated system results for the 38 mm wing offset prototypes with various elastic element stiffnesses are included as solid lines.
Figure 4.8: Peak to peak flapping angle (a) and mean lift (b) plotted against input power. Both data mean and standard deviation is shown. Input voltage amplitude is swept while frequency is held constant at 10 Hz. Prototypes with various elastic element stiffness and wing offset are included. The horizontal dotted line indicates the weight of the half FMAV, where the lift-to-weight of the system is equal to one. The simulated system results for the 38 mm wing offset prototypes with various elastic element stiffnesses are included as solid lines.
To compare prototype performance at our desired operating frequency, a sweep of input voltages was repeated at 10 Hz. Figure 4.8 depicts peak-to-peak flapping angles and mean lift plotted over electrical input power. Each variation of the prototype with the exception of the 12 mm wing offset is able to achieve lift-to-weight of greater than one. It is clear, however, that the stiffest system with the longest wing offset performs the best, achieving the greatest lift with the least input power.

Though the system simulation tends to under predict flapping amplitudes at higher input voltages, it does well in capturing trends between prototypes. The difference in lift and flapping amplitude is matched with a change in input frequency in Fig. 4.6. A lift-to-weight ratio of greater than one is correctly predicted for the relevant systems in Fig. 4.7b and 4.8b. The same distribution of system performance is captured by the system simulation in Fig. 4.8b where greater lift is achieved at a lower input power for increasing elastic element stiffnesses.

4.2.4 Discussion

Effect of Wing Offset and Elastic Element Variation

Operating at resonance reduces necessary power to flap by eliminating the inertial power spent on wing motion. However, power transmission between the motor source and the wing load also plays a roll, affecting the percentage of motor power that is available to move the wing. With variation in the two physical parameters of elastic element stiffness and wing offset we are able to see the effects of both. A change in elastic element stiffness moves system resonance closer or farther away from our desired operating frequency. An increase in wing offset increases the load on the motor and influences power transmission.

As the electrical resistance of the armature, $R_0$, remains constant for any given motor, the impedance ratio varies due to changes in the load resistance $R_{\text{mech}}$. For the prototype described in this work, the system impedance ratio is changed with a change in wing offset. Assuming that the system operates and resonates at 10 Hz, prototype motor power (solid blue line) and required
Figure 4.9: Prototype aerodynamic power and motor power over impedance ratio. The system is assumed to resonate at 10 Hz. Wing offsets of 38 mm (a), 26 mm (b), and 12 mm (c) are shown.
aerodynamic power for various values of $\theta_{pp}$ (dashed red lines) are plotted in Fig. 4.9 for the wing offsets of 38, 26, and 12 mm. As the wing offset decreases from 38 to 12 mm, the impedance ratio of the system approaches one. Though power transmission improves at with a wing offset of 12 mm, improved maximum system lift does not necessarily follow. While a large flapping amplitude is easier to achieve, the smaller offset results in a smaller wing velocity for the same peak-to-peak flapping angle. As seen experimentally in Figs. 4.7 and 4.8, any gain that might be seen from an improved power transmission is negated by the physical limit of $\theta_{pp} \leq 180$ deg.

Operation at resonance is the ideal when considering power expenditure. In the flapping system with a wing offset of 38 mm and a wing stroke of $\theta_{pp} = 120$ deg, the ratio of inertial to aerodynamic torque can be quickly estimated to be

\[
\frac{\tau_{\text{inertial}}}{\tau_{\text{aero}}} = \frac{J_{\text{tot}}(2\pi f)^2(\theta_{pp}/2)}{B_0(2\pi f(\theta_{pp}/2))^2} = \frac{J_{\text{tot}}}{B_0(\theta_{pp}/2)} \approx 1.6
\]  

(4.14)

which means power expended on the oscillatory wing motion is roughly equivalent to the power expended on wing drag. At resonance, none of this power is wasted on inertial torques which do not contribute to overall mean lift. Accordingly, with the prototypes examined in this work, one would expect the system with the closest resonance frequency to the desired 10 Hz to show a clear benefit when considering the same inputted voltage parameters.

Preferably, we would examine overall efficiency to help compare prototype performance as the effects of passive wing motion, which can differ between systems, are reduced. However, we do not have direct measurement of output power which would require instantaneous wing drag force and velocity. As our ultimate concern is maximal lift production, we can instead examine system lift with various input power which functions as a similar measure. As seen in Fig. 4.8b, at 10 Hz prototype mean lift is relatively closely grouped at lower input values, but there is a clear differentiation in the 38 mm wing offset systems as input voltage and peak-to-peak flapping angles increase. Here it is clear that as expected, the prototype with the stiffest elastic element and a resonance frequency closest to 10 Hz can achieve the greatest mean lift for
the lowest input power, with the 1.6e3 and 1.1e3 mN.mm/rad prototypes following.

**Prototype Performance and Possible Lift Improvements**

Overall prototype performance is judged on total lift generation and power consumption at our desired operating frequency of 10 Hz. Of the prototypes tested, the system which can produce the most lift for the least input power had an elastic element of stiffness of 2.8e3 mN.mm/rad and a wing offset of 38 mm, which can be seen in Fig. 4.8b. While they may have been less efficient, each system with a wing offset of 38 and 26 mm was able to achieve a lift-to-weight ratio above 1 at 10 Hz. System mass varied insignificantly between prototypes. The majority of system weight is contributed by the driving motors; at 1.21 g each this is 90% of the total system mass. At the current maximum performance, the 1.4 lift-to-weight ratio leaves approximately 1 g to additional sensors, electronics, and battery.

Practical constraints on the prototype include the motor torque, motor breakdown voltage, gear breaking strength, and peak flapping amplitude. The elastic element stiffness was limited here because of gear failure, but further utilizing the maximum motor torque should be explored. Increasing wing size, wing offset, and input voltage $V_{pp}$ further can increase mean lift while maintaining a maximum peak-to-peak flapping amplitude of less than 180 deg. As the dynamic model of the prototype described in this work captures system behavior across both the tested input frequency and amplitude sweeps, it could be used for further prototype optimization and improvement.

Without the constraint on gearbox forces to prevent failure, the elastic element stiffness should be chosen such that the system resonates at the desired operating frequency. The best current overall efficiency, or the ratio of power spent on aerodynamic loads to electrical input power, has been roughly estimated as a little over 10%. This estimation was calculated from observed peak-to-peak flapping angle, assumed wing rotational motion, and measured lift. In a similar resonating system Campolo et al. was able to achieve an efficiency of approximately
Recent tests suggest that the GM15, also supplied by Solarbotics, demonstrates better performance though the provided specifications are the same as the GM15A. The GM15 has an output shaft shaped for pulleys and belts, but, more significantly, has thicker shafts on the inner planetary gearbox components. The end result is a gearbox that is more efficient and more robust, which allows a stiffer spring to be used. Figure 4.10 shows the peak-to-peak flapping angle of a prototype using a GM15 with an elastic element of 7.7e3 mN.m/rad calculated. Wing length, $R$, was 70 mm, with a horizontal wing offset, $d_w$, of 35 mm and vertical offset, $h_w$, of 40 mm. The additional inertia of the structure offsetting the wings increases the necessary stiffness to achieve the desired resonance of 10 Hz. Power consumption has not yet been measured, though initial lift generation is promising, exceeding previous results at similar input voltage magnitudes.

4.3 Multi-wing System Design

While the single wing and single motor is capable of producing lift greater than its own weight, this only remains true when the motor is mounted rigidly to a fixture. In attempted free flight, the mean and instantaneous body forces and torques, or body wrench, produced by the flapping wing would lead to both reduced flapping amplitude and continued spinning. To make an effective
free-fight system, additional motors and wings must be added to counteract this instantaneous wrench.

The placement of these wing and motor pairs, which we will refer to as individual modules, has implications on both system dynamics, control inputs, and control authority. Following, we will discuss the placement of modules in regards to instantaneous wing force and torque cancellation and describe two prototypes, with two and four wing modules respectively, and their design implications on system control.

4.3.1 Module Placement for Instantaneous Torque Cancellation

In a flapping wing system, the wings generate nonlinear, time-varying forces and torques on the body. In hovering insects, flapping occurs at high frequencies in respect to the body dynamics, which serves to filter the resulting body oscillations. This significantly simplifies control when this effect is recreated in constructed flappers, where only the averaged force and torque over a wing stroke needs to be examined.

To gain a better understanding whether this is satisfied for a particular system, we can examine simplified single degree of freedom body dynamics. For each of the six degrees of freedom (DOF) we can write a simple relation between the body mass or inertia, wing produced forces and torques, and body damping. For the system pitching angle, for example, is 

\[ J_y \ddot{\theta}_y = \tau_y - d_{\theta_y} \dot{\theta}_y \]

where \( J_y \) is the pitch body inertia, \( d_{\theta_y} \) is the body damping coefficient, \( g \) is the acceleration due to gravity, and \( \tau_y \) is the wing applied instantaneous torque. The transfer function between \( \tau_y(t) \) and \( \theta_y \) is therefore

\[ \frac{\theta_y(s)}{\tau_y(s)} = \frac{1}{J_y s^2 + d_{\theta_y} s} \]

which is a low pass filter. Considering a typical wing module of our design, with no additional body modifications, and flapping at 10 Hz, the instantaneous body torques will not be well filtered for the system rotational DOFs. This is unfortunate, but there are several ways to address this problem. The first is to increase the body inertia or damping to slow the system dynamics. With limited lift, however, increasing mass directly or adding additional structures comes at the possible consequence of decreasing lift-to-weight ratio and preventing
Figure 4.11: Coordinate system defined for a single wing module. Side (a) and top view (b) are shown. The world coordinate system is $E_1$, the coordinate system $E_b$ is fixed to the CG of the single module system, and $E_w$ is fixed to the wing center of lift.
lift off. The second is to reduce the magnitude of the instantaneous wrench on the body. With careful placement of multiple wing modules and selection of wing flapping phase, the wrench will be mostly canceled, reducing the oscillations of the body at the flapping frequency. Here we examine the configuration of modules to create an ideal system design where both the average and instantaneous torques are zero at nominal hovering flight.

To determine whether a certain configuration of motors and wings would be suitable, we can generalize module placement and calculate the total torques generated about the resulting center of mass. Each module is assumed to have the same wing offset with a center of lift location designated by $h_{zoff,m}$ and $h_{yoff,m}$ depicted in Fig. 4.11. Each module has three individual parameters that determine its configuration: a position $\vec{r} = [\delta_x, \delta_y, \delta_z]^T$ and orientation $\theta_{nom}$ in the coordinate system $E_1$ and the phase $\phi$ of its input driving signal and resulting wing flapping angle $\theta_{flap}(t)$.

There are three relevant coordinate systems used to define the needed system parameters. The world coordinate system $E_1$ is used to position each module relative to each other. Each module has an individual body-fixed coordinate system $E_b$ and wing fixed coordinate system $E_w$. $E_b$ is fixed to the module center of mass. $E_w$ is fixed to the wing center of lift, and for the purposes of calculating its position relative to $E_1$, the wing is assumed not to rotate.

Given that each module cannot be guaranteed to supply much lift beyond its own weight, it is assumed that in a configuration each module will remain orientated ‘upwards’ as depicted in Fig. 4.11 and is free only to be positioned with a rotation about the axis denoted by $E_{bz}$. Forces generated by the wing including lift, drag, and centripetal effects can be defined in the wing fixed coordinate system $E_w$ and approximated with:

$$\vec{f}_x = F_0 \dot{\theta}_{flap}(t)^2 \text{sign}(\dot{\theta}_{flap}(t)) \vec{E}_{wx}$$  \hspace{1cm} (4.15)

$$\vec{f}_y = m_w h_{cg} \dot{\theta}_{flap}(t) \vec{E}_{wy}$$  \hspace{1cm} (4.16)

$$\vec{f}_z = F_L \dot{\theta}_{flap}(t) \vec{E}_{wz}$$  \hspace{1cm} (4.17)
where $m_w$ is the wing mass and $h_{cg}$ is horizontal offset of the wing CG from the module CG. The wing force and damping coefficients, $F_L$ and $F_0$ respectively, are calculated from the contribution from translational wing forces in (2.2) with a fixed wing rotation angle,

$$F_L = \frac{1}{2} \rho C_l \int_{d_w}^{R+d_w} r^2 c(r)dr$$

$$F_0 = \frac{1}{2} \rho C_d \int_{d_w}^{R+d_w} r^2 c(r)dr$$

where $C_l$ and $C_d$ are taken to be 1.7 and 1.5, respectively for a wing rotation angle of 45 deg \[28\]. $R$ is the wing length, and $d_w$ is the wing offset. The flapping angle of the wing while being actively driven relative to the module fixed coordinate system $E_b$ can be approximated as

$$\theta_{flap}(t) = \frac{\theta_{pp}}{2} \sin(2\pi ft + \phi),$$

where $f$ is the flapping frequency, $\phi$ is the phase shift, and $\theta_{pp}$ is the peak-to-peak flapping amplitude. For purposes of module positioning, $t = 0$ where $\theta_{flap}(0) = 0$ which corresponds to no displacement of the system flapping elastic element. With $R$ defined as the traditional single axis rotation matrix, the transformation from the wing fixed coordinate system $E_w$ to the world coordinate system $E_1$ is $T = R(\theta_{nom})R(\theta_{flap}(t))$.

The position of the system center of mass $\vec{r}_{cm}$ which contains $n$ total modules can be calculated as

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{n} \vec{r}_i m_i}{\sum_{i=1}^{n} m_i},$$

(4.20)

where $\vec{r}_i$ and $m_i$ are the $i^{th}$ module’s position and mass, respectively. The position of $E_w$ for module $i$ in the world coordinate system relative to the center of mass is therefore

$$T_i \begin{bmatrix} 0 \\ h_{yoff,m} \\ h_{zoff,m} \end{bmatrix} + \vec{r}_i - \vec{r}_{cm} = \vec{h}_i.$$  

(4.21)
Consequently, the total torque about the system center of mass given the chosen module configuration can be written as

\[
\vec{\tau}(t) = \begin{bmatrix}
\tau_x(t) \\
\tau_y(t) \\
\tau_z(t)
\end{bmatrix} = \sum_{i=0}^{n} \vec{h}_i \times T_i \vec{f}_{xi} + \vec{h}_i \times T_i \vec{f}_{yi} + \vec{h}_i \times T_i \vec{f}_{zi}.
\] (4.22)

which is a summation of the cross products of the wing forces and the appropriate lever arm for each of \( n \) modules.

In the traditional two-winged, biologically-inspired systems, the module configuration parameters would be as described in Fig. 4.13b. Here, the modules are placed next to one another and the wings are driven 180 deg out of phase. When the body torques are totaled, the result is

\[
\begin{bmatrix}
\tau_x(t) \\
\tau_y(t) \\
\tau_z(t)
\end{bmatrix}
= \begin{bmatrix}
0 \\
\left(2 f^2 \pi^2 \theta_{pp}^2 \cos(2 f \pi t)^2 (F_0 h_{zoff,m} \cos(\frac{\theta_{pp}}{2} \sin(2 f \pi t)) \text{sign}(\cos(2 f \pi t)))
\\
-(F_L h_{yoff,m} + h_{cg} h_{zoff,m} m_w) \sin(\frac{\theta_{pp}}{2} \sin(2 f \pi t))))
\\
0
\end{bmatrix}
\] (4.23)

Though over a complete wing stroke \( \frac{1}{f} \int_{0}^{1/f} \tau_y(t) dt = 0 \), the fact that instantaneously \( \tau_y \) is nonzero will result in body oscillations about \( \vec{E}_{1y} \) in free flight. The magnitude of the body oscillations is dependent on the system body and flapping parameters. In a two-wing system with GM15A motors, 70 mm wings, \( y_{off}=5 \) mm, \( h_{zoff,m}=30 \) mm, \( h_{yoff,m}=76 \) mm, and \( f=10 \) Hz, the effect of \( \tau_y \) is significant. Neglecting the effects of body and wing damping on system oscillation, our estimation of \( \tau_y \) predicts a pitching motion about \( \vec{E}_{1y} \) with an amplitude of as much as 90 deg. Though damping induced by the wings will serve to reduce the magnitude of these oscillations further, this suggests that system free flight behavior can be significantly effected.

With two additional modules the oscillating torque \( \tau_y(t) \) can be resolved. Figure 4.13 depicts a system configuration of four modules. The top pair of wings are driven in phase to one another
Figure 4.12: System configuration with two modules. In (a) a depiction of the module location is shown. In (b), a table of each modules’ parameters is shown. Here, the wings are driven out of phase from one another, canceling the wing produced instantaneous $\tau_z(t)$.
Figure 4.13: System configuration with four modules. In (a) a depiction of the module location is shown. In (b) a table of each modules’ parameters is shown. Here, the top wing pair are driven in phase with the bottom pair driven in reverse.
while the bottom pair is reversed. The total torque about the center of mass is

\[
\begin{bmatrix}
\tau_x(t) \\
\tau_y(t) \\
\tau_z(t)
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
-2 f^2 \pi^2 \theta_{pp}^2 (x_{off} - y_{off}) \cos(2 f \pi t) \frac{F_0 \cos(\theta_{pp} \sin(2 f \pi t)) \text{sign}(\cos(2 f \pi t))}{2} \\
-h_{cg} m_w \sin(\theta_{pp} \sin(2 f \pi t))
\end{bmatrix},
\]

\text{(4.24)}

where \( \tau_z \) is equal to zero when the module offsets are set such that \( x_{off} = y_{off} \).

There are many possible module configurations that will satisfy \( \vec{\tau}(t) = 0 \). Given the nonlinear and time varying nature of eq. (4.22), an analytical solution for all the possible configurations of \( n \) modules is difficult. Here, symmetry is used to find a single solution for a 4 module system which is then verified, but (4.22) could be incorporated larger numerical system optimization if so desired.

While manufacturing errors and changes in \( \theta_{pp} \) will prevent the instantaneous body torques from ever canceling completely, choosing a design where they are canceled in the ideal case will reduce their effect significantly. There are many module configurations that will satisfy this requirement, but in this work we examine only the two and four winged configurations described above. Choosing between them and between other configurations comes down to a combination of practicality and system control authority. A greater number of modules will increase the total lift-to-weight ratio of the system and allow greater controlling torques to be produced, but constraints on module orientation prevent any additional degrees of freedom to be controlled and the increase in overall system size can be unwieldy. The four-wing configuration described above, however, is the simplest arrangement that allows torque cancellation and also has marked advantages over the two-wing system in the generation of controlling torques, which will be detailed in the following sections.
Figure 4.14: In (a) the system coordinate system is defined along with roll, pitch, and yaw motion. The coordinate system shown is fixed to the body center of mass. In (b) a diagram of how pitch and roll torques can be produced is shown. The top view of the two-wing system is depicted with the swept area for each wing shown. The relative magnitude of the lift produced over a wing stroke is shown by the crossed circles of varying radii. Three cases are included: typical symmetrical wing motion, differing wing amplitudes to create a roll torque, and a shifted wing sweep center of lift to create a pitch torque.

4.3.2 Two-wing System

Here, the equations of motion for a two-wing flapping system are described. The eigenvalues of the linearized set of system equations are then examined to predict system open loop stability with the modification of two system parameters. Free flight experiments are performed.

While the equations of motion for the two and four-wing systems are similar, differences are reflected in the choice of system control inputs. Here roll and pitch torques are be created with varying wing amplitudes and offsets of wing stroke center of lift. Fig. 4.14 depicts a top view of a two-wing system and varying peak wing positions in a flapping stroke.
Equations of Motion

In the body fixed coordinate frame, the equations for motion for the flapping system are

\[
\begin{bmatrix}
m I & 0 \\
0 & J
\end{bmatrix}
\begin{bmatrix}
\dot{\nu} \\
\dot{\omega}
\end{bmatrix}
+ \begin{bmatrix}
\omega \times m \nu \\
\omega \times J \omega
\end{bmatrix}
= \begin{bmatrix}
f \\
\tau
\end{bmatrix}
\]  \hspace{1cm} (4.25)

where \( m \) is the body mass, \( I \) is the \( 3 \times 3 \) identity matrix, and \( J \) is the body inertia matrix. The translational and angular velocity vectors in the body frame are \( \vec{\nu} = [\nu_x, \nu_y, \nu_z]^T \) and \( \vec{\omega} = [\omega_x, \omega_y, \omega_z]^T \) respectively. The wrench applied to the system \( W = [f, \tau]^T \) is a vector encompassing the external forces and torques about each axis. Both the wrench and inertia matrix \( J \) are time varying and depend on the flapping wing motion.

The body wrench can be broken down into several components including the forces and torques produced by the flapping wing, the damping due to body translational and angular motion, and the effect of gravitational acceleration.

\[
\begin{bmatrix}
f \\
\tau
\end{bmatrix}
= \begin{bmatrix}
R_{WB}mg \\
0
\end{bmatrix}
+ \begin{bmatrix}
f_w \\
\tau_w
\end{bmatrix}
\]  \hspace{1cm} (4.26)

The matrix \( R_{WB} \) is the rotation matrix transformation between the ground or world fixed coordinate system and the coordinate system that is fixed to the body. The gravity vector, \( g \), is \([0, 0, -9.8]\) m/s. The wing forces are calculated as in (2.2)-(2.4), but with wing cord velocity and wing angle of attack updated considering the velocity of the body. As the system translates and rotates in space, the airflow over the wing surfaces changes, leading to a change in lift in drag forces between the forward and reverse wing stroke as well as between left and right wings [49]. The velocity of the wing’s mid-chord point in the body fixed frame when the body velocity is
Table 4.3: Two-wing System Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ mass</td>
<td>3.1</td>
<td>g</td>
</tr>
<tr>
<td>$\vec{J}$ body inertia, $[J_x, J_y, J_z]^\dagger$</td>
<td>$[1.5e3, .45e3, 1.13e3]$</td>
<td>g.mm$^2$</td>
</tr>
<tr>
<td>$h_w$ offset between CG and wing leading edge</td>
<td>40</td>
<td>mm</td>
</tr>
<tr>
<td>$k_s$ elastic element stiffness</td>
<td>2.8e3</td>
<td>mN.mm/rad</td>
</tr>
<tr>
<td>$d_w$ wing offset</td>
<td>35</td>
<td>mm</td>
</tr>
<tr>
<td>$R$ wing length</td>
<td>70</td>
<td>mm</td>
</tr>
<tr>
<td>$R_{CL}$ wing CL horz. distance</td>
<td>42</td>
<td>mm</td>
</tr>
<tr>
<td>$\beta_{CL}$ wing CL vert. distance</td>
<td>10</td>
<td>mm</td>
</tr>
</tbody>
</table>

$^\dagger$Predicted from 3D CAD model

Nonzero can be written as

$$\vec{v}_{mw} = R_\phi R_\theta (\dot{\phi} \times c(r)) + R_\theta (\dot{\theta} \times r) + (\omega \times R_\phi R_\theta \vec{p}_c) + \nu$$

(4.27)

where $\vec{p}_c$ is dependent upon the system parameters, and in this case is $[0, r, h_w - c(r)/2]^\top$. The matrices $R_\theta$ and $R_\phi$ represent the coordinate system transformations between the body fixed frame and $\vec{E}'$ and $\vec{E}''$ frames respectively, which have been defined in Fig. 2.2. With the total wing force for the left and right wings, $F_L$ and $F_R$, calculated in their respective $\vec{E}'$ frames and acting on the wing center of lift, the wing forces and torques on the body can be written as

$$\begin{bmatrix} f_w \\ \tau_w \end{bmatrix} = \begin{bmatrix} R_{\theta,L} F_L + R_{\theta,R} F_R \\ R_{\theta,L} (\vec{p}_w \times F_L) + R_{\theta,R} (\vec{p}_w \times F_R) \end{bmatrix}$$

(4.28)

where $\vec{p}_w$ is dependent on the wing center of lift position and is in this case $[0, w_{off} + R_{CL}, h_w - \beta_{CL}]$.

This system of equations is nonlinear and time varying, which does not facilitate either system analysis or later control. As the wing forces and torques are periodic, they can be approximated with their average value resulting in a system approximation that is time-invariant. This
approximation is accurate only when the wing flapping period frequency is considerably higher than the bandwidth of the system, or the magnitude of the applied wrench is low. Following Cheng et. al, the effective body damping over a complete wing stroke due to body motion can be calculated for each DOF \[49\], and body wrench rewritten as

\[
\begin{bmatrix}
 f \\
 \tau 
\end{bmatrix} = 
\begin{bmatrix}
 R_{WB}mg \\
 0 
\end{bmatrix} - 
\begin{bmatrix}
 \xi_1 & \xi_1p_1 \\
 \xi_2 & \xi_2p_2 + \xi_3 
\end{bmatrix}
\begin{bmatrix}
 \dot{\nu} \\
 \dot{\omega} 
\end{bmatrix} + 
\begin{bmatrix}
 f_{w0} \\
 \tau_{w0} 
\end{bmatrix}
\]

(4.29)

where \(\xi_1-\xi_3\) are diagonal matrices with the diagonal terms \([\bar{X}, \bar{Y}, \bar{Z}], [-\bar{L}_y, \bar{M}_x, 0], \) and \([\bar{L}, \bar{M}, \bar{N}]\) respectively. The terms \(\bar{X}, \bar{Y}, \) etc. are the damping coefficients for the wing pair for each DOF and are dependent on the wing flapping amplitude and frequency. The matrices \(p_1\) and \(p_2\) are diagonal matrices with the diagonal terms \([h_{zoff}, h_{yoff}, 0]\) and \([-h_{zoff}, h_{yoff}, 0]\), respectively. The wing produced forces, averaged over a wing stroke, with no body motion, are \(\bar{f}_{w0}\) and \(\bar{\tau}_{w0}\) and are written as

\[
\begin{bmatrix}
 \bar{f}_{w0} \\
 \bar{\tau}_{w0} 
\end{bmatrix} = 
\begin{bmatrix}
 R_{\bar{\theta}_L}F_L + R_{\bar{\theta}_R}F_R \\
 R_{\bar{\theta}_L}(\bar{p}_w \times F_L) + R_{\bar{\theta}_R}(\bar{p}_w \times F_R) 
\end{bmatrix}
\]

(4.30)

For typical sinusoidal wing motion, wing stroke center of lift will occur at the average wing flapping angle, which is defined as \(\bar{\theta}_L\) and \(\bar{\theta}_R\) for the left and right wing respectively. Assuming that the left and right wing are symmetric, there are no manufacturing imperfections, and \(|\bar{\theta}_L| = \)}
\[ \theta_R = \bar{\theta}, \] the body wrench can be simplified further to

\[
\begin{bmatrix}
\bar{f}_{w0,x} \\
\bar{f}_{w0,y} \\
\bar{f}_{w0,z} \\
\bar{r}_{w0,x} \\
\bar{r}_{w0,y} \\
\bar{r}_{w0,z}
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
\bar{F}_{L,z} + \bar{F}_{R,z} \\
h_{yoff} \cos(\bar{\theta})(\bar{F}_{L,z} - \bar{F}_{R,z}) \\
h_{yoff} \sin(\bar{\theta})(\bar{F}_{L,z} + \bar{F}_{R,z}) \\
0
\end{bmatrix}.
\] (4.31)

To better understand the effect of physical parameter change on the open loop system behavior, and for later control development, a linear approximation can be found for averaged model of the form \( \dot{\bar{x}} = A\bar{x} + Bu \) where \( A \) and \( B \) are found assuming small perturbations about the nominal operating point of hover where \( \bar{x} = \bar{u} \). Here, \( A \) and \( B \) are

\[
A =
\begin{bmatrix}
0_{6 \times 6} & I_{6 \times 6} \\
0 & g & 0 & 0 & 0 & 0 & -\frac{\bar{X}}{m} & -\frac{h_{yoff} \bar{X}}{m} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\bar{M}_x}{J_2} & -\frac{\bar{M}}{J_2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -g & 0 & 0 & 0 & -\frac{\bar{Y}}{m} & -\frac{h_{yoff} \bar{Y}}{m} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\bar{L}_x}{J_1} & -\frac{\bar{L}}{J_1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\bar{Z}}{m} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\bar{N}}{J_3}
\end{bmatrix}
\] (4.32)

\[
B =
\begin{bmatrix}
0_{7 \times 3} \\
0 & 0 & \bar{E}_{trim} & J_2 \\
0 & 0 & 0 \\
0 & h_{yoff} & 0 \\
\frac{1}{m} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\] (4.33)
where $\vec{x} = [x, \theta, y, \bar{\theta}, y, \omega, \nu, x, \nu, z, \bar{\omega}, z, \nu]$ and the system inputs, $\vec{u}$, are $[\bar{F}_{L,z} + \bar{F}_{R,z}, \bar{F}_{L,z} - \bar{F}_{R,z}, \bar{\theta}]$. The system trim inputs are chosen as $\bar{F}_{L,z} + \bar{F}_{R,z} = \bar{F}_{z}^{\text{trim}} = mg$, $\bar{F}_{L,z} - \bar{F}_{R,z} = 0$, and $\bar{\theta} = 0$. While the inertia matrix is also time varying due to the constant wing motion, it can be approximated as a constant matrix where the wing flapping angle is zero. Due to the body shape, non-diagonal inertial terms are small, so $\mathbf{J}$ can be written as the diagonal matrix with diagonal terms $[J_1, J_2, J_3]$.

**Translational Damping Parameter Determination**

In a flapping wing system, body damping is primarily dependent on the wing due to the relative area of the wing surface to the body structure. However, as the wing is constantly oscillating and moving through space, determining the effective damping it has on the system is non-trivial. As the system translates and rotates, the airflow over the wing changes, resulting in an effective change in wing kinematics. In [49], Cheng et. al describes the consequential change in wing wrench, and derives system translational and rotational damping as a function of wing shape, flapping frequency, and flapping amplitude. From [49] we can write:

$$\bar{X} = \rho R^2 c \Phi n \bar{r}_1(S) C_D(\alpha_0) \cos^2(\phi) \left| \frac{d\phi}{dt} \right|$$ (4.34)

$$\bar{Y} = \rho R^2 c \Phi n \bar{r}_1(S) C_D(\alpha_0) \sin^2(\phi) \left| \frac{d\phi}{dt} \right|$$ (4.35)

$$Z = \frac{1}{2} \rho R^2 c \Phi n \bar{r}_1(S) \frac{dC_N(\alpha)}{d\alpha} \mid_{\alpha = \alpha_0} \cos(\alpha_0) \left| \frac{d\phi}{dt} \right|$$ (4.36)

$$\bar{L} = \frac{1}{2} \rho R^4 c \Phi n \bar{r}_3(S) \frac{dC_N(\alpha)}{d\alpha} \mid_{\alpha = \alpha_0} \cos(\alpha_0) \cos^2(\phi) \left| \frac{d\phi}{dt} \right|$$ (4.37)

$$\bar{M} = \frac{1}{2} \rho R^4 c \Phi n \bar{r}_3(S) \frac{dC_N(\alpha)}{d\alpha} \mid_{\alpha = \alpha_0} \cos(\alpha_0) \sin^2(\phi) \left| \frac{d\phi}{dt} \right|$$ (4.38)

$$\bar{N} = \rho R^4 c \Phi n \bar{r}_3(S) C_D(\alpha_0) \left| \frac{d\phi}{dt} \right|$$ (4.39)
where here $\bar{X}, \bar{Y}, \bar{Z}, \bar{L}, \bar{M}, \bar{N}$ are defined as the damping coefficients averaged over a wing stroke, rather than the averaged drag forces as in [49]. $\bar{X}, \bar{Y}, \bar{Z}$ are the drag coefficients for system motion in the $x$, $y$, and $z$ direction as defined in Fig. 4.14a. The damping coefficients for system roll, pitch, and yaw motion are $\bar{L}, \bar{M}, \bar{N}$ respectively. The additional damping terms shown in eq. (4.32), which are $\bar{M}_x$ and $\bar{L}_y$, are simply $\bar{X} \ell$ and $\bar{Y} \ell$ respectively where $\ell$ is the lever arm between the wing and system center of gravity. These terms represent the additional angular damping the system experiences due to an offset of the wing from the CG. Above, $\rho$ is the air density, $R$ is the wing length, $\bar{c}$ is the average chord length, $\Phi$ is the wing flapping amplitude, $n$ is the flapping frequency, $\hat{r}_{1}^{1}(S)$ and $\hat{r}_{3}^{3}(S)$ are the first and third nondimensional moments of wing area. The term $\alpha$ is the effective angle of attack due to the change in airflow and $\alpha_0$ is the geometric angle of attack. $C_D$ is the drag coefficient and $d\hat{\phi}/d\hat{t}$ is the nondimensional wing flapping velocity. Nondimensional time, $\hat{t}$, is defined as $\hat{t} = tn$. Wing drag coefficients are taken from [28], and wing area is nondimensionalized according to [125].

Given the current wing offsets, $\bar{M}_x \gg \bar{M}$ and $\bar{L}_y \gg \bar{L}$, leaving $\bar{X}, \bar{Y}, \bar{Z}$ the dominant terms in the system degrees of freedom that can be controlled. System inertia, mass, and wing offsets can be accurately estimated using a Solidworks model of the full system assembly. In order to increase the accuracy of the dynamic model, however, the system translational damping terms can be experimentally measured. Following Parks et. al, the system can be mounted to a pendulum and the translational damping terms calculated from the swinging pendulum motion [126].

As the system is nonlinear, the logarithmic decrement method to determine the damping coefficient will not be accurate here, so instead frictional and damping coefficients are fitted such that a model of the pendulum matches the observed system response using an optimization routine. The pendulum can be modeled as

$$\ddot{\theta} = \frac{1}{J}(mgL \sin(\theta) - c_0 \text{sign}(\dot{\theta}) - c_1 \dot{\theta}),$$

(4.40)
Figure 4.15: Experimental setup used for the translational damping characterization. The system is mounted to a pendulum and pendulum swing motion is recorded for varying motor inputs voltages.

Figure 4.16: The two-wing system was oriented in three positions to measure translational drag in the x direction (a), y direction (b), and in the z direction (c) as defined in the body fixed coordinate system.
Table 4.4: Pendulum Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>174 g</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>9.35 g.m²</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>160 mm</td>
<td></td>
</tr>
<tr>
<td>L_sys</td>
<td>500 mm</td>
<td></td>
</tr>
</tbody>
</table>

where \( m \) is the total pendulum and platform mass, \( J \) is the total inertia, \( g \) is the gravitational constant, \( c_0 \) is the pendulum coefficient of friction and \( c_1 \) is the pendulum coefficient of damping. The pendulum angle, \( \theta \), and CG offset, \( L \), are defined in Fig. 4.15b. Pendulum parameters used in the simulation can be found in Table 4.4.

A two-wing platform prototype is mounted to the pendulum, shown in Fig. 4.15 in three different orientations as displayed in 4.16. Pendulum motion is measured using the Vicon Bonita motion tracking setup. Flapping frequency is held constant at 10 Hz and motor voltage input is incremented between trials. For each experimental trial, the pendulum model is simulated and \( c_0 \) and \( c_1 \) iterated such that the cost \( C = \int (f_{\text{exp}}(\theta_{\text{peaks,exp}}) - f_{\text{sim}}(\theta_{\text{peaks,sim}}))^2 dt \) is minimized. The function \( f(\theta_{\text{peaks}}) \) is a piecewise linear function defined by the positive peak values of \( \theta \) observed. Figure 4.17 depicts an example simulated system with fitted frictional and damping coefficients. The system time response well matches the observed experimental damping envelope, though phase is not perfectly captured.

A total of 36 trials were performed with varying input voltage amplitude and with varying system orientation. In Fig. 4.18 the system translational damping coefficients, which are equal to \( (c_1 - c_{1,\text{rig}})/L_{\text{sys}} \), are plotted over motor input voltage. The term \( c_{1,\text{rig}} \) represents the damping due to the moving structure of the rig and was determined by recording the pendulum motion without the platform attached. The average and standard deviation of the fitted fiction coefficient over all the trials was 0.12 ± 0.06 mN/m/s.

While the relation between damping and a change in motor input is depicted, the amplitude
Figure 4.17: Example experimental pendulum motion and simulation with fitted friction and damping coefficients.

Figure 4.18: Measured damping coefficients for varying input voltage amplitudes.
of the input sinusoid is linearly related to the system flapping amplitude, an example of which can be seen in Fig. 4.7a. Given eqs. (4.35)-(4.37), we would expect linearly increasing damping as the input signal is increased. Both \( \bar{Y} \) and \( \bar{Z} \) follow this trend, reaching their maximum at the highest input voltage. Interestingly, \( \bar{X} \) remains almost constant with changing flapping amplitude and input voltage. A contributing factor may be due to the system’s passive wing rotation. The derivation of the system damping terms assumes that though external air flow changes the effective angle of attack, the actual geometrical wing kinematics are not affected. With a passively rotating wing, translational motion in the \( x \) direction would increase the force of air acting about the wing rotational axis, directly affecting wing rotation. Additional experimental trials would be necessary to confirm this result with various prototypes and wing flexure stiffnesses along with directly measured wing kinematics.

In regards to characterizing the system damping coefficient, we wish to understand how system damping changes with motor input, as flapping amplitude will not be directly observed in experimental free flight. The motion of the pendulum prevents flapping angle to be measured during the trials, but for purposes of comparison to the derived model we can use the flapping angle measured on a fixed mount for the same input voltage. In general, for the platform tested, the model over predicts the damping coefficient measured. Specifically we examine can a single point where the system produces the lift required to support its own weight. At a 130 peak-to-peak flapping amplitude, which corresponds to approximately 9.5 \( V_{pp} \) for both wings, the predicted coefficients are \( \bar{X} = 8.3 \), \( \bar{Y} = 7.1 \), and \( \bar{Z} = 4.2 \) mN/m/s. At \( \bar{X} = 3 \) mN/m/s the experimentally measured translational damping in the \( x \) direction is less than half what is expected. Error in the \( y \) and \( z \) direction is less, with experimentally measured damping \( \bar{Y} = 4.5 \) and \( \bar{Z} = 2.9 \) mN/m/s, a respective percent error of 36% and 30%.
Table 4.5: System Wing Damping Parameters at Trim Inputs

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{X}$</td>
<td>3.0</td>
<td>mN/m/s</td>
</tr>
<tr>
<td>$\bar{Y}$</td>
<td>4.5</td>
<td>mN/m/s</td>
</tr>
<tr>
<td>$\bar{Z}$</td>
<td>2.9</td>
<td>mN/m/s</td>
</tr>
<tr>
<td>$\bar{L}$</td>
<td>$4.7e^{-3}$ †</td>
<td>mN.m/rad/sec</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>$4.0e^{-3}$ †</td>
<td>mN.m/rad/sec</td>
</tr>
<tr>
<td>$\bar{N}$</td>
<td>$3.2e^{-2}$ †</td>
<td>mN.m/rad/sec</td>
</tr>
</tbody>
</table>

† Calculated following [49]

Open Loop Stability Analysis

As seen in (4.32) and (4.33), the linearized system dynamics show that the system lateral dynamics (side-to-side motion, $\nu_x$, $\omega_y$), longitudinal dynamics (forward-backward motion, $\nu_y$, $\omega_x$), heave dynamics (up-and-down motion, $\nu_z$), and yaw dynamics ($\omega_z$) are decoupled about hover. Each of these subsystems can be examined individually to better understand overall system behavior. The lateral and longitudinal dynamics are the more complicated and benefit from a closer analysis.

While a number of system physical parameters are directly related to lift production and should not be modified, the wing vertical offset $h_w$ and system inertia can be changed with little consequence to influence open loop behavior. Ideally, system parameters could be chosen such that the plant is stable to ease system control. Both the lateral and longitudinal dynamics have the same form albeit with a differing inertial constant and body damping. Only the longitudinal dynamics are detailed here.

From (4.32), the open loop longitudinal dynamics of the system linearized about hover can
Figure 4.19: Diagram defining physical parameters for nominal two wing-system without (a) and with (b) an added damping surface.
be written as:

\[
\dot{v}_x = -\frac{\bar{X}}{m} v_x - \frac{h_w - \beta_{CL}}{m} \bar{X} \dot{\theta}_y + g \theta_y, \tag{4.41}
\]

\[
\dot{\theta}_y = \omega_y, \tag{4.42}
\]

\[
\dot{\omega}_y = -\frac{\bar{X}}{J_2} (h_w - \beta_{CL}) (v_x + (h_w - \beta_{CL}) \omega_y) - \frac{-\bar{M}}{J_2} \omega_y, \tag{4.43}
\]

where \( \bar{M}_x = h_{zoff} \bar{X} \). In matrix form,

\[
\begin{bmatrix}
\dot{v}_x \\
\dot{\theta}_y \\
\dot{\omega}_y
\end{bmatrix} =
\begin{bmatrix}
\frac{-\bar{X}}{m} & g & -\frac{h_w - \beta_{CL}}{m} \bar{X} \\
0 & 0 & 1 \\
\frac{-\bar{X}}{J_2} (h_w - \beta_{CL}) & 0 & \frac{-\bar{X}}{J_2} (h_w - \beta_{CL})^2 - \frac{\bar{M}}{J_2}
\end{bmatrix}
\begin{bmatrix}
v_x \\
\theta_y \\
\omega_y
\end{bmatrix}, \tag{4.44}
\]

including forward speed, pitch, and pitch rate. As the system is at hover, lift produced by the flapping wings is set to \( mg \). The criteria for system open loop stability can be formally defined by finding the eigenvalues, \( \lambda \), of this system of equations. The eigenvalue polynomial here is

\[
\lambda^3 + \lambda^2 \frac{\bar{M}m + J_2 \bar{X} + (h_w - \beta_{CL})^2 m \bar{X}}{J_2 m} + \lambda \frac{M \bar{X}}{J_2 m} + \frac{g(h_w - \beta_{CL}) \bar{X}}{J_2} = 0. \tag{4.45}
\]

According to the Routh-Hurwitz condition, in order for eigenvalues to be greater than zero, there are four conditions that must be met:

\[
\frac{\bar{M}m + J_2 \bar{X} + (h_w - \beta_{CL})^2 m \bar{X}}{J_2 m} > 0 \tag{4.46}
\]

\[
\frac{M \bar{X}}{J_2 m} > 0 \tag{4.47}
\]

\[
g(h_w - \beta_{CL}) \bar{X} > 0 \tag{4.48}
\]

\[
\frac{\bar{M}m + J_2 \bar{X} + (h_w - \beta_{CL})^2 m \bar{X} \bar{M} \bar{X}}{J_2 m} > \frac{g(h_w - \beta_{CL}) \bar{X}}{J_2}. \tag{4.49}
\]

Conditions (4.46) and (4.47) are always satisfied due to the system physical properties; mass,
Figure 4.20: Stable and unstable regions for the two wing system longitudinal dynamics. Prototypes with three CG offsets are included in the figure, and reference the images shown in Fig. 4.21. The dotted line denotes the relationship between change in CG offset and inertia for the two wing system design.

inertia, and damping coefficients will always be positive. Condition (4.48) simply requires that system center of mass must be below the wing center of lift. The final condition is second order function of \( h_w \) and is more difficult to intuitively understand. Figure 4.20 depicts the stability of the system bounded by these final two conditions over variation of inertia and CG offset \( h_w \).

For the two-wing system these parameters are coupled, and their possible values are plotted as a dotted line in the figure.

The inertia and CG offset of three constructed prototypes are included in Fig. 4.20 which correspond to the photographs seen in Fig. 4.21. Though the first prototype, Fig. 4.21a, should be stable according to our predictions, the small size of the stable region results in a high sensitivity to manufacturing and modeling errors. The first prototype proves to be divergent on liftoff tests. The second and third prototypes with offsets of 40 and 65 mm oscillate in free flight as expected before flying into the ground or reaching the end of the safety tether. The small range of CG offsets that result in a stable system make construction of such a system difficult. However,
even within this region the system is expected to be underdamped when $h_w > \beta_{CL}$. Eigenvalues range from $\lambda = (-32.2, -2.5, 0)$ when $h_w = \beta_{CL} = 11$ mm to $\lambda = (-35.9, \pm 9.1i)$ when $h_w = 14.8$ mm.

With the addition of another surface to increase body damping, the longitudinal dynamics can be slowed and the stable region can be increased. Similar to the passively stable systems shown in [32] and [33], a foam damping surface is considered. Here a damper of width and height $a$, as shown in Fig. 4.19b is added to the top of the system. The system equations of motion are
Figure 4.22: Two-wing system with added damper to reduce pitch oscillations.
augmented with an additional damping term $D$ and can be written as

$$
v_x = -\frac{\ddot{X}}{m}v_x - \frac{(h_w - \beta_{CL})}{m}\ddot{X} + bD\dot{\theta} + g\dot{\theta},$$

$$\dot{\theta} = \omega_y,$$

$$\dot{\omega}_y = \frac{-\ddot{X}(h_w - \beta_{CL}) + bD}{J_2}v_x - \frac{\ddot{X}(h_w - \beta_{CL})^2 + b^2D + \ddot{M}}{J_2}\omega_y,$$

where body inertia, mass, and body damping change with a change in damper length $a$. The changing system parameters are defined as

$$b = \frac{a}{2} + b_{nom},$$

$$J_2 = J_m + J_w + m_w h_w^2 + J_D,$$

$$J_D = \frac{1}{12}\rho_{foam}a^4 + (b + h_w)^2\rho_{foam}a^2 + \frac{1}{3}\rho_{cf}r^2(b + \frac{a}{2} + h_w)^3$$

$$+\rho_{cf}\pi r^2a\left(\frac{1}{2}r^2 + (b - a/2)^2\right) + \rho_{cf}\pi r^2a\left(\frac{1}{2}r^2 + (b + a/2)^2\right),$$

$$m = \rho_{cf}(2\pi r^2a + \pi r^2b) + \rho_{foam}a^2 + m_{nom},$$

$$D = 0.5C_d\rho a^2.$$

where the damper is constructed from carbon fiber rods and a lightweight foam. The new eigenvalue polynomial can be written as

$$\lambda^3 + \lambda^2\frac{DJ_2 + b^2Dm + \ddot{M}m + J_2\ddot{X} + (h_w - \beta_{CL})^2m\ddot{X}}{J_2m}$$

$$+ \lambda \frac{\ddot{M}\ddot{X} + D(M + (b - (h_w - \beta_{CL})^2\ddot{X})}{J_2m} + \frac{g(bD + h_w\ddot{X})}{J_2} = 0$$

which result in similar stability conditions. System longitudinal stability for varying inertia, CG offset, and damper size is plotted in Fig. 4.23. System physical parameters used in the prediction are listed in Table 4.6.

As the damper size increases, the region over which the system is stable also increases.
Figure 4.23: Stable and unstable regions for the two wing system with an added damper. Three damper sizes are included with dimensions $a = 150, 200, \text{ and } 250 \text{ mm}$. Each dotted line corresponds to the relationship between inertia and CG offset for each system. As the damper increases in size, the possible CG offsets that will allow the system to remain stable increase.
Table 4.6: Parameters of Longitudinal Dynamic Simulation

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_m$ motor inertia about $\vec{y}$</td>
<td>1.10</td>
<td>g.cm$^2$</td>
</tr>
<tr>
<td>$J_w$ wing inertia about $\vec{y}$</td>
<td>0.38</td>
<td>g.cm$^2$</td>
</tr>
<tr>
<td>$m_w$ wing and wing support mass</td>
<td>0.2</td>
<td>g</td>
</tr>
<tr>
<td>$m_{nom}$ nominal system mass</td>
<td>3.1</td>
<td>g</td>
</tr>
<tr>
<td>$b_{nom}$ damper offset</td>
<td>10</td>
<td>mm</td>
</tr>
<tr>
<td>$r$ carbon fiber rod radius</td>
<td>0.25</td>
<td>mm</td>
</tr>
<tr>
<td>$\rho_{cf}$ carbon fiber density</td>
<td>1810</td>
<td>kg/m$^2$</td>
</tr>
<tr>
<td>$\rho_{foam}$ foam density</td>
<td>0.001</td>
<td>g/mm$^2$</td>
</tr>
</tbody>
</table>

However, at $a = 200$ mm and above, the additional mass of the damper results in a marginal lift-to-weight ratio, necessitating a damper made of a less dense foam to make the modification feasible.

Open Loop Flight

A series of open loop flights were performed with the prototype with a CG offset of 40 mm, seen in Fig. 4.21b. As constructing an open loop stable system is difficult without the addition of weighty dampers, the offset of 40 mm was chosen to allow longer uncontrolled free flights and to reduce the weight of additional material and supports that become necessary as the offset is increased. Overlaid snapshots of the system in free flight taken at intervals of 0.1 sec are shown in Fig. 4.24.

Six free flight runs tracked using the Bonita Vicon motion tracker are plotted in Fig. 4.25. Data is truncated where the system reaches the end of its safety tether. Three trials (blue, cyan, and green) use the same system with the same constant wing voltage inputs and flapping at 10 Hz. The remaining three trails are taken with a repaired system with recalibrated inputs. For all cases, input voltages are chosen such that averaged roll and pitch torques are canceled while lift is greater than the system weight, allowing upward flight. Though each system is released from the same location and with nominally the same calibrated state, there is a substantial amount of
Figure 4.24: Snapshots of two-wing system in free flight taken in intervals of 0.1 sec.
variation in flight trajectory.

As expected from the initial predictions in subsection 4.3.1 there are large oscillations about the pitching axis, $\theta_y$, though they are not enough to force a system crash in open loop flight. As shown in Fig. 4.26a, the system pitch oscillations are as much as 50 deg peak-to-peak at the flapping frequency. The system also experiences a yaw torque due to manufacturing imperfections and in this case makes two complete revolutions about $\theta_z$ in 10 wing strokes or 1 sec.

Though a damper large enough to make the system stable is too heavy for current prototype, a smaller damper can be added to slow system pitch dynamics, eliminating the large pitch oscillations seen in the unmodified system. In Fig. 4.22, a small damper is added to the two wing systems. In Fig. 4.26b, the altitude and body angles of the damped system in free flight is shown. The 10 Hz pitch oscillations are completely eliminated.
Figure 4.25: Experimental free flight trajectories of the two-winged system. In (a) six flight trajectories are plotted in three dimensions. In (b) the trajectories are projected onto the $x$-$y$ plane. Each trajectory is truncated when the system reaches the end of the tether.
Figure 4.26: Experimental altitude and body angles in free flight for the undamped (a) and damped (b) two-wing motor-driven system. Colors in (a) correspond to the trajectories shown in Fig. 4.25. As the system approaches the limit of the Vicon tracking area, the tracked points attached to the system are harder to distinguish, leading to higher errors.
Model Accuracy

In order to understand the accuracy of the nonlinear and linearized system models described above, we can examine both their frequency response as well as time history.

In Fig. 4.27 the simulated nonlinear and linear system frequency response to each of three force and torque inputs is shown. In Fig. 4.27a, heave response $z$ is plotted over a change in lift force $F_z$ about the nominal trim equal to the weight of the system. In Fig. 4.27b and 4.27c, roll, $\theta_x$, and pitch, $\theta_y$, response is plotted over a change in inputted $\tau_x$ and $\tau_y$ respectively.

The linearized system model reasonably well captures the simulated nonlinear system in the frequency domain. Peaks in the response in both roll and pitch motion occur at approximately 2 Hz. While a full frequency response characterization cannot be done on the free-flight system in experiment due to lift limitations on flapping frequency, a similar resonant behavior is observed at a slightly higher 3 Hz, which Fig. 4.26a clearly depicts. The magnitude of the simulated roll and pitch frequency responses in Fig. 4.27 however, clearly demonstrate the lack of sufficient filtering at the system flapping frequency. At 10 Hz, the magnitude of the heave response is $\sim 0$ dB, while roll and pitch remain at or above $\sim 50$ dB. In free flight this results in angular body oscillations at 10 Hz, which due to the magnitude of the instantaneous pitch torques are significant about $\theta_y$.

The large oscillations about the pitching axis in the undamped, two-wing system poise problems for both the averaging assumption and linearization for both predicting body motion and control development. While our desired operating point is at hover, uncontrollable large deviations from body angles of zero at the flapping frequency contribute to modeling and control error.

In Fig. 4.28, both the nonlinear and linear model of the flapping system is simulated in free flight. The wing generated wrench applied to both systems was measured using the ATI sensor setup described in Section 5.4.1 and balanced such that average body torques were approximately 0. While there are significant differences in the exact roll and pitch angle between the two
Figure 4.27: Nonlinear and linear motor-driven, two-wing system frequency response to force and torque inputs along the $z$ axis (a), about $\theta_x$ (b), and about $\theta_y$ (c).
models, both the magnitude of 10 Hz pitch and roll oscillations and the low frequency system behavior is well captured between the simulation and what is observed in experiment. Due to the large angular motions of the system, however, the small angle assumption no longer holds, contributing to the error between models over time.

The addition of a damper is one method to improve the system response and the validity of our assumptions. As shown in Fig. 4.26b, an added damper, though not large enough to make the system stable, slows the system dynamics such that the 10 Hz pitching oscillations are eliminated completely.
Table 4.7: Two-wing System Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ mass</td>
<td>6.9</td>
<td>g</td>
</tr>
<tr>
<td>$\vec{J}$ body inertia, $[J_x, J_y, J_z]$†</td>
<td>$[4.1e^3, 4.1e^3, 2.2e^3]$</td>
<td>g.mm²</td>
</tr>
<tr>
<td>$z_{off}$ CG offset</td>
<td>40</td>
<td>mm</td>
</tr>
<tr>
<td>$h_1$ offset 1</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td>$h_2$ offset 2</td>
<td>10</td>
<td>mm</td>
</tr>
</tbody>
</table>

†Predicted from 3D CAD model

4.3.3 Four-wing System

Here, the equations of motion for a four-wing flapping system are described. The eigenvalues of the linearized set of system equations are then examined to predict system open loop stability with the modification of two system parameters. Free flight experiments are performed.

The position of the wings in the four-wing system do not allow control over additional degrees of freedom, but do better decouple control inputs. Roll and pitch torques are be created with varying only wing amplitudes. Figure 4.29b depicts a top view of a two-wing system and varying peak wing positions in a flapping stroke. Physical parameters, as depicted in Fig. 4.29a are listed in Table ?? unless otherwise noted, individual module parameters are the same as in the two-wing platform.

Equations of Motion

The equations of motion for the four module system are similar to those of the system with two modules. From (4.29), the average body wrench over a wing stroke is now

$$
\begin{bmatrix}
\vec{f} \\
\vec{\tau}
\end{bmatrix} =
\begin{bmatrix}
R_{WB}mg \\
0
\end{bmatrix} -
\begin{bmatrix}
\alpha_1 & \alpha_2 \\
\alpha_3 & \alpha_4
\end{bmatrix}
\begin{bmatrix}
\dot{\nu} \\
\dot{\omega}
\end{bmatrix} +
\begin{bmatrix}
\vec{f}_{w0} \\
\vec{\tau}_{w0}
\end{bmatrix}
$$

(4.60)
Figure 4.29: In (a) physical parameters of the four-wing system are shown. A diagram depicting the top view of the four-wing system with wing swept area is shown in (b). The relative magnitude of the lift produced over a wing stroke is shown by the crossed circles of varying radii. In the four-wing system modifying wing amplitudes allows the generation of both roll and pitch torques.
where instead $\alpha_1 - \alpha_4$ are diagonal matrices where the diagonal elements $j\bar{j}$ are $\alpha_{1,jj} = [\bar{X}_1 + \bar{Y}_2, \bar{Y}_1 + \bar{X}_2, \bar{Z}_1 + \bar{Z}_2], \alpha_{2,jj} = [\bar{X}_1 h_1 + \bar{Y}_2 h_2, \bar{Y}_1 h_2 + \bar{X}_2 h_2, \bar{Z}_1 + \bar{Z}_2], \alpha_{3,jj} = [-(\bar{L}_y + \bar{M}_x), \bar{M}_x + \bar{L}_y, 0], \alpha_{4,jj} = [\bar{L}_y h_1 + \bar{M}_x h_2, \bar{Y}_1 h_2 + \bar{X}_2 h_2, \bar{Z}_1 + \bar{Z}_2]$.

The damping terms correspond to the damping effect of the top and bottom wing pairs. Since the top and bottom modules are symmetric, $\bar{X}_1 = \bar{X}_2$, etc. Because of the module orientation, in the system’s forward direction the damping is a function of the top module $\bar{Y}_2$ and bottom module $\bar{X}_1$. A diagram depicting the system parameters can be seen in Fig. 4.29a. The wing center lift vertical offsets from the system center of mass are $h_1 = h_{zoff} + 0.5 z_{off}$ and $h_2 = h_{zoff} - 0.5 z_{off}$, where $z_{off}$ is the vertical offset between the two wing module pairs.

The total wing contribution to body wrench is

\[
\begin{bmatrix}
\bar{f}_{w0} \\
\bar{\tau}_{w0}
\end{bmatrix} = \begin{bmatrix}
\sum_n \mathbf{R}_{\tilde{\theta}_n} \bar{F}_n \\
\sum_n \mathbf{R}_{\tilde{\theta}_n} (\bar{p}_{w,n} \times \bar{F}_n)
\end{bmatrix},
\]

(4.61)

where $\bar{p}_{w,n}$ is the position vector defining the nominal position of the center of lift of wing $n$ from the system center of mass. With our previous assumptions such that the modules are symmetric, there are no manufacturing imperfections, and the wing stroke center of lift is the same for the wings in each pair ($|\tilde{\theta}_1| = |\tilde{\theta}_2| = \tilde{\theta}_{12}$ and $|\tilde{\theta}_3| = |\tilde{\theta}_4| = \tilde{\theta}_{34}$), the mean wrench can be simplified to

\[
\begin{bmatrix}
\bar{f}_{w0,x} \\
\bar{f}_{w0,y} \\
\bar{f}_{w0,z} \\
\bar{\tau}_{w0,x} \\
\bar{\tau}_{w0,y} \\
\bar{\tau}_{w0,z}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
\sum_n \bar{F}_{n,z} \\
h_{yoff} \cos(\tilde{\theta}_{12}) (\bar{F}_{1,z} - \bar{F}_{2,z}) + h_{yoff} \sin(\tilde{\theta}_{34}) (\bar{F}_{3,z} + \bar{F}_{4,z}) \\
h_{yoff} \sin(\tilde{\theta}_{12}) (\bar{F}_{1,z} + \bar{F}_{2,z}) + h_{yoff} \cos(\tilde{\theta}_{34}) (\bar{F}_{3,z} - \bar{F}_{4,z}) \\
0
\end{bmatrix}.
\]

(4.62)
A linear approximation of the form \( \dot{x} = Ax + Bu \) can also be found for the four-winged system. Here, \( A \) is

\[
A = \begin{bmatrix}
A_1 & A_2
\end{bmatrix}
\]

\[
A_1 = \begin{bmatrix}
0_{6\times6} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -g & 0 \\
0_{3\times6}
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
-\frac{X_1 + Y_2}{m} & -\frac{X_1 h_1 + Y_2 h_2}{m} & 0 & 0 & 0 & 0 \\
-\frac{M_{x1} + L_{y2}}{J_2} & -\frac{M_{x1} h_1 + L_{y2} h_2 + M_{x1} + L_{x2}}{J_2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{Y_1 + X_2}{m} & -\frac{Y_1 h_1 + X_2 h_2}{m} & 0 & 0 \\
0 & 0 & 0 & -\frac{L_{y1} + M_{x2}}{J_1} & -\frac{L_{y1} h_1 + M_{x2} h_2 + L_1 + M_2}{J_1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{Z_1 + Z_2}{m} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{N_1 + N_2}{J_3}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0_{7\times4} \\
0 & \frac{\theta_{y2} h_{off}}{J_2} & \frac{h_{yoff}}{J_2} & 0 \\
0 & 0 & 0 & 0 \\
\frac{\theta_{y2} h_{off}}{J_1} & 0 & 0 & \frac{h_{yoff}}{J_1} \\
\frac{1}{m} & \frac{1}{m} & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
Figure 4.30: Stable and unstable regions for the four-wing system longitudinal and lateral dynamics. Stability is plotted in respect to changes in two physical parameters $h_w$ and $z_{off}$. Each region is labeled with a roman numeral I-V. In I the lateral and longitudinal dynamics are divergent. In II the lateral dynamics are stable. In III the lateral dynamics unstably oscillate and the longitudinal dynamics are divergent. In IV the longitudinal dynamics are stable. In V both the longitudinal and lateral dynamics unstably oscillate. Only where II and IV overlap are both stable. This however occurs in a region where this combination of physical parameters is impossible due to wing interference, marked with a hatched pattern. Otherwise in II the longitudinal dynamics are divergent and in IV the lateral dynamics unstably oscillate.

where $\bar{x} = [x, \theta_y, y, \theta_z, z, \nu_x, \nu_y, \omega_x, \omega_y, \omega_z, \nu_z, \omega_z]$ and the system inputs, $\bar{u}$, are $[\bar{F}_1 + \bar{F}_2, \bar{F}_3 + \bar{F}_4, \bar{F}_1 - \bar{F}_2, \bar{F}_3 - \bar{F}_4]$. Given the module orientation in the four-wing system, changing the wing stroke center of lift becomes unnecessary. The control inputs are functions the lift force of each wing. With the wing lift stroke centers of lift $\bar{\theta}_{12}$ and $\bar{\theta}_{34}$ held constant and nominally equal to zero, the inputs become nicely decoupled.

**Open Loop Stability Analysis**

As with the two-wing system the linearized dynamics can be examined to gain a better understanding on how system open loop stability changes with a change in physical parameters. Here,
both the module pair offset \( z_{off} \) and the wing vertical offset \( h_w \) can be modified without effecting lift generation.

The eigenvalue polynomial of the four-wing system longitudinal dynamics are as follows:

\[
\lambda^3 + \lambda^2 \left( \frac{L_2m + J_2(\bar{X}_1 + \bar{Y}_2) + m(\bar{M}_1 + h_1^2\bar{X}_1 + h_2^2\bar{Y}_2)}{J_2m} \right) \\
+ \lambda \left( \frac{J_2m}{4}(h_1 - h_2)^2\bar{X}_1\bar{Y}_2 + J_2(\bar{X}_1 + \bar{Y}_2) + \bar{M}_1(\bar{X}_1 + \bar{Y}_2) \right) + \left( \frac{g(h_1\bar{X}_1 + h_2\bar{Y}_2)}{J_2} \right) > 0
\]

(4.67)

\[
\frac{L_2m + J_2(\bar{X}_1 + \bar{Y}_2) + m(\bar{M}_1 + h_1^2\bar{X}_1 + h_2^2\bar{Y}_2)}{J_2m} > 0
\]

(4.68)

\[
\frac{(h_1 - h_2)^2\bar{X}_1\bar{Y}_2 + L_2(\bar{X}_1 + \bar{Y}_2) + \bar{M}_1(\bar{X}_1 + \bar{Y}_2)}{J_2m} > 0
\]

(4.69)

\[
\frac{g(h_1\bar{X}_1 + h_2\bar{Y}_2)}{J_2} > 0
\]

(4.70)

\[
\frac{((L_2 + \bar{M}_1)\bar{X}_1 + (L_2 + \bar{M}_1 + (h_1 - h_2)^2\bar{X}_1)\bar{Y}_2)(J_2(\bar{X}_1 + \bar{Y}_2) + m(\bar{L}_2 + \bar{M}_1 + h_1^2\bar{X}_1 + h_2^2\bar{Y}_2))}{J_2^2m^2} > \frac{g(h_1\bar{X}_1 + h_2\bar{Y}_2)}{J_2},
\]

(4.71)

where \( h_1 = h_{zoff} + z_{off} \) and \( h_2 = h_{zoff} - z_{off} \) assuming the wing offsets \( h_w \) of each module are equal. Like the two-wing system, the first two conditions are always true. The third condition, however, is now dependent on the relative values of \( \bar{X} \) and \( \bar{Y} \) in addition to \( h_1 \) and \( h_2 \). In terms of the wing offset and vertical distance between modules, the third condition from eq. (4.70) requires that

\[
\frac{z_{off}}{h_w - \beta_{CL}} > \frac{2(\bar{X}_1 + \bar{Y}_2)}{\bar{X}_1 - \bar{Y}_2}.
\]

(4.72)
The eigenvalue polynomial of the four-wing system lateral dynamics can be written as follows:

\[ \lambda^3 + \lambda^2 \left( \frac{\bar{L}_1 m + J_1(\bar{X}_2 + \bar{Y}_1) + m(\bar{M}_2 + h_2^2 \bar{X}_2 + h_1^2 \bar{Y}_1)}{(J_1 m)} \right) + \\
+ \lambda \left( \frac{(h_1 - h_2)^2 \bar{X}_2 \bar{Y}_1 + \bar{L}_1(\bar{X}_2 + \bar{Y}_1) + \bar{M}_2(\bar{X}_2 + \bar{Y}_1)}{J_1 m} \right) \frac{g(h_2 \bar{X}_2 h_1 \bar{Y}_1)}{J_1} = 0, \tag{4.73} \]

with similar conditions which will not be written out completely here.

The stable regions of both the lateral and longitudinal dynamics are plotted in Fig. 4.30. Due to the relative magnitude of the body damping terms, there is no selection of \( z_{off} \) and \( h_w \) that will result in a stable system with the current assumptions on system design.

**Open Loop Flight**

As with the two-winged system, a series of experimental free-flight runs were performed and tracked using the Vicon Motion Tracker. Snap shots of the system in flight are shown in Fig. 4.31, where high speed camera images are overlaid at intervals of 0.1 sec. The translational motion of three runs are plotted in Fig. 4.32a, with the altitude and body angles plotted for a single run in Fig. 4.32b. With the wing flapping phase driven as described in Section 4.3.1, the desired result of body oscillations at the flapping frequency of 10 Hz is achieved.

**4.4 Summary**

Reciprocating, flapping wing motion can be power intensive due to the need to not only overcome aerodynamic drag but also the inertia of the oscillating wing and mechanisms. Piezo-driven systems are typically driven at resonance to reduce power consumption, but due to the typical nonlinear transmissions used in motor-driven systems this is not an option for larger platforms. In this chapter, a novel motor-driven flapping platform is described. The simplicity of the design keeps weight to a minimum while allowing independent wing flapping motion and generation of
Figure 4.31: Overlaid images of four-wing system taken in intervals of 0.1 sec. The system is in open loop free flight and flapping at 10 Hz.
Figure 4.32: Experimental free flight trajectories of the four-winged system. In (a) three flight trajectories are plotted in three dimensions. In (b) the altitude and body angles of a single trajectory are plotted over time.
controlling torques. As a linear transmission is used and a spring is added, the system is capable of resonance allowing a greater percentage of power to be expended on drag and increasing lift production. A lift-to-weight ratio of 1.4 is achieved through careful motor, wing and flapping frequency selection along with a series of constructed prototypes tested with lift experiments varying wing offset, spring stiffness, and flapping frequency. The presented two-wing system is one of only three existing flapping wing platforms capable of vertical liftoff and controlling forces and torques, and the least mechanically complex.

The placement of the wing and motor pairs, or modules, is critical in the platform design and free flight performance. As the two-wing system does not well filter the wing produced wrench at the flapping frequency, a four-wing system was developed as a solution to slow system dynamics and to reduce the magnitude of the instantaneous wing forces and torques. The total of four wings also allows controlling torques to be produced by only changing wing flapping amplitude, more thoroughly decoupling wing control inputs and increasing control authority about the pitching axis.

Both the four-wing system and a damped two-wing system have demonstrated free flight without the 10 Hz oscillations influencing the motion of the original two-wing system and contributing to the error present in the linearized system model. Though each system is capable of flight, the four-wing platform holds the most promise for eventual well controlled maneuvers and hovering.
Chapter 5

Control of the Flapping Wing Platforms

5.1 Introduction

Over the past decade, there has been a number of theoretical approaches to controlled, flapping flight [42], [50], [51], [52]. Surprisingly, for such highly dynamic, nonlinear, and time-invariant systems, linear controllers have proven to be quite suitable and have been successfully demonstrated in experiment. In recent work a variation of the Harvard Fly has been controlled in free flight with a modified, model-free linear controller [53]. Linear controllers are attractive in their simplicity and ease of implementation for the highly resource constrained platforms. At the small scale, there are still a number of issues to solved to allow a system to be truly autonomous (ranging from platform power to lightweight sensing and feedback) and the ability to use a similarly ‘lightweight’ control strategy increase the feasibility eventual untethered flight.

Alternatively, more complex robust control strategies are better able to handle systems with either unknown or changing physical parameters as well as large disturbances. Air gusts, torn wings, and difficult to characterize wing aerodynamics are each practical problems in flapping flight systems. Recently, Chirarattananon et al has demonstrated performance improvements with an adaptive controller on the Harvard Fly [54].

The primary purpose of this chapter is to demonstrate piezo-driven and motor-driven platform
controllability through a series of restricted degree of freedom and free flight control tests. Both a robust controller based on Lyapunov stability and a linear controller are described and applied to each system respectively. Challenges with the two-wing motor-driven system are discussed with preliminary improvements demonstrated in the four-wing system.

5.2 Controller Design

5.2.1 Averaging Theory

In each of the previous approaches, control of the flapping-wing system relies upon the fact that the frequency of wing oscillations is faster than the system’s body dynamics. As mentioned in 4.3.1, the body acts can function as lowpass filter, with only the average wing forces and torques to affect body motion. Averaging theory is a well accepted method of simplifying such systems, and allows the time variant flapping wing system to be approximated as time invariant, easing analysis without significant change in overall system behavior. Consider the general nonlinear system [50]:

\[ \dot{x} = f(x, u), \quad (5.1) \]
\[ u = g(v, t) = g(v, t + nT), \text{ for } n = 0, 1, 2, \ldots, \quad (5.2) \]
\[ v = h(x), \quad (5.3) \]
\[ \hat{x} = \bar{f}(\bar{x}, \bar{u}), \quad (5.4) \]
\[ \bar{f}(\bar{x}, \bar{u}) = \frac{1}{T} \int_0^T f(x, g(v, t))dt, \quad (5.5) \]
\[ \bar{v} = h(\bar{x}), \quad (5.6) \]

where variable \( x \) is the system state, \( t \) is time, \( u \) is the open-loop control input, and \( v \) is the feedback control input. The open-loop control, \( u \), is periodic with the period \( T \). All functions
and their derivatives up to second order must be continuous. If the origin \( x = 0 \) of the averaged system, \( \bar{x} \), is an exponentially stable equilibrium point, there exists \( T^* \) and a constant \( k > 0 \), such that for any \( 0 < T < T^* \), the error between the instantaneous and averaged system is bounded as \( \|x(t) - \bar{x}\| < kT \), \( t \in [0, \infty) \). In addition, the system \( \dot{x} = f(x, u) \) has a unique, locally exponentially stable, \( T \)-periodic solution \( x_T(t) \) with \( \|x_T(t)\| \leq kT \) \([50], [127]\).

In flapping systems that are poorly approximated using this method there are two main issues that complicate control. The first is the oscillatory nature of the system itself. The passive wing rotation design prohibits the generation of arbitrary instantaneous wing forces; any body oscillations at the wing beat frequency cannot be reduced with even very careful controller design. The second is that any controller developed about the averaged system approximation may not be well suited for the actual system dynamics and lead to reduced performance or failure in free flight.

While \( T^* \) is not formally defined, it reasonably follows that a smaller operating \( T \) or wing beat period will generally lead to a better approximation and less oscillatory flight. However, wing beat frequency is closely tied to lift production and cannot be arbitrarily changed. As discussed in \([4.3.1]\), other alternatives are slowing the system dynamics and reducing the magnitude of the oscillation of the instantaneous forces and torques.

Both the piezo-driven systems, the two-wing motor-driven system with added damper, and four-wing motor-driven system are examples of platforms where the generated instantaneous wing forces and torques are well filtered and/or sufficiently reduced. The two-wing motor-driven system described in \([4.3.2]\) however, is an example of a platform with underdamped pitch oscillations.

### 5.2.2 Linear Controller Design

The two-wing and four-wing systems’ equations of motion have been described previously in Sections \([4.3.2]\) and \([4.3.3]\) and are relevant to both piezo-driven and motor-driven systems. Since
the platform is intended to operate about hover, their averaged equations of motion were linearized about this point. This time-invariant continuous system of equations can then be approximated with its discrete form

\[
\vec{x}(n + 1) = A\vec{x}(n) + B\vec{u}(n) + \delta_b(n) \\
\vec{y}(n) = \vec{x}(n) + \delta_s(n)
\] (5.7) (5.8)

where \(\vec{x}(n)\) is the vector of averaged system states, and \(\vec{y}(n)\) is the state position and orientation feedback. The time step \(n\) is based on the length of the wing beat period. The terms \(\delta_b(n)\) and \(\delta_s(n)\) represent the unmodeled body dynamics and added sensing noise respectively.

With the linearized system nicely decoupled longitudinal, lateral, heave, and yaw dynamics, separate linear controllers can be designed in the discrete domain using pole placement strategies. Though yaw could conceivably be controlled with an additional input modifying wing leading edge velocities, here it is left uncontrolled as hovering should be achievable without it. As with helicopters, a change in body pitch and roll about hover induces forwards and lateral motion respectively. In the controller used here, only pitch, roll, and altitude are controlled upon, though an additional higher level control scheme can induce desired translational motion.

Each individual controller is first designed based on the decoupled discrete system dynamics, and then further tuned when applied to the full nonlinear, time-variant system simulation. Two key features lie in the choice of system feedback and in the system calibration for individual prototypes. Later experimental control tests are performed with various sensor feedback, and depending upon the system simulated, the feedback model is updated to reflect the sensor used.

In addition, while the controller is based on the application of averaged lift force and pitch and roll torques, it is important to find an accurate mapping between these desired control inputs, the voltages necessary to produce them, and the actual mean and instantaneous wrench produced by the wing motion. In the case of the piezoelectric driven systems where wing produced forces and torques are low and only mean lift force is measured, this relationship is based on input volt-
Figure 5.1: Closed loop diagram of the system simulation. The desired mean lift, pitch, and roll torques are determined by the decouple linear controllers. This is translated into motor input voltages using the determined calibration matrix. Instead of using the developed wing, motor, and aerodynamic model to predict the resulting wing forces, the actual measured wing produced wrench is used which is first experimentally captured for a range of motor voltage inputs.

Average amplitude sweeps and observed wing sweep center of lift displacement. The instantaneous wing wrench applied to the body dynamics is taken simulated base on the determined aerodynamic model. In the motor-driven systems the wing wrench is fully characterized with a 6 DOF force/torque sensor, which is described later in this chapter. The wing wrench used in simulation is taken directly from the collected instantaneous wing forces and torques.

5.2.3 Robust Controller Design

For each controllable degree of freedom we can derive a control input such that the system is robustly stable within the control limits using the rules of Lyapunov stability and sliding mode control strategies. For each of $z$, $\theta_x$, and $\theta_y$, the nonlinear equations in the fixed body coordinate system can be written as

\[
\ddot{z} = f_z(x) + g_z(x, \bar{u})u_1, \tag{5.9}
\]

\[
\ddot{\theta}_y = f_{\theta_y}(x) + g_{\theta_y}(x, \bar{u})u_2, \tag{5.10}
\]

\[
\ddot{\theta}_x = f_{\theta_x}(x) + g_{\theta_x}(x, \bar{u})u_3 \tag{5.11}
\]
Following standard Lyapunov control techniques and with the choice of sliding manifold $s$ that satisfies $\dot{s} = -k_1 \text{sgn}(s) - k_2 s$, we find our inputs of the form

$$U(f(\vec{x}), g(\vec{x}, \vec{u}), C, E) = \frac{1}{g(\vec{x}, \vec{u})}[(1 - c_1^2 + \lambda_1)e_1 + c_1 e_2 - c_1 \lambda_1 X_1 (5.12)$$

$$+ k_1 \text{sat}(e_2/\varepsilon) + k_2 e_2 + \ddot{\theta}_d - f(\vec{x})], (5.13)$$

where

$$u_1 = \frac{m}{\cos(\theta_x) \cos(\theta_y)} U(f_z(\vec{x}), g_z(\vec{x}, \vec{u}), \vec{c}_z, \vec{e}_z), (5.14)$$

$$u_2 = U(f_{\theta_x}(\vec{x}), g_{\theta_x}(\vec{x}, \vec{u}), \vec{c}_{\theta_x}, \vec{e}_{\theta_x}), (5.15)$$

$$u_3 = U(f_{\theta_y}(\vec{x}), g_{\theta_y}(\vec{x}, \vec{u}), \vec{c}_{\theta_y}, \vec{e}_{\theta_y}). (5.16)$$

The system state is $\vec{x}$ and the three system control inputs are $\vec{u} = [u_1, u_2, u_3] = [\vec{F}_{L,z} + \vec{F}_{R,z}, \vec{F}_{L,z} - \vec{F}_{R,z}, \dot{\theta}]$. $C$ is a list of tunable positive constants $\{c_1, k_1, k_2, \lambda_1, \varepsilon\}$ and $E$ is a list of errors calculated from the desired set point $\{e_1, e_2, X_1\}$. Derivations and further parameter descriptions can be found in Appendix C.

As derived this allows control altitude, roll, and pitch body angles. Global translational $(x, y)$ motion, however, is dependent on both thrust, $u_1$, as well as system orientation. We can follow the same process and determine a desired force in both the $x$ and $y$ directions. The desired input force in the $x$ and $y$ direction, $u_x$ and $u_y$ are as follows:

$$u_x = m[(1 - c_{x1}^2 + \lambda_{x1}) e_{x1} - c_{x1} \lambda_{x1} X_x + k_1 \text{sat}(e_{x2}/\varepsilon) + k_2 e_{x2} + \ddot{x}_d - f_{\theta_x}(\vec{x})] (5.17)$$

$$u_y = m[(1 - c_{y1}^2 + \lambda_{y1}) e_{y1} - c_{y1} \lambda_{y1} X_y + k_1 \text{sat}(e_{y2}/\varepsilon) + k_2 e_{y2} + \ddot{y}_d - f_{\theta_y}(\vec{x})], (5.18)$$

For a given thrust $u_1$ and system orientation, the translational force applied to the system can be
written as follows with the appropriate rotation angle transformation:

\[
\begin{align*}
    u_x &= \cos(\tilde{\theta}_x) \sin(\tilde{\theta}_y) \cos(\tilde{\theta}_z) + \sin(\tilde{\theta}_x) \sin(\tilde{\theta}_z)u_1, \\
    u_y &= \cos(\tilde{\theta}_x) \sin(\tilde{\theta}_y) \sin(\tilde{\theta}_z) + \sin(\tilde{\theta}_x) \cos(\tilde{\theta}_z)u_1.
\end{align*}
\]  

(5.19)  
(5.20)

The desired roll and pitch angle, \( \theta_{xd} \) and \( \theta_{yd} \), can be solved for with a given \( u_1 \) and current vehicle \( \theta_z \), and which is then used in the calculation of \( u_2 \) and \( u_3 \).

As with the linear controller, the developed robust controller can be applied to the simulated system as in Fig. 5.1. The decoupled linear controllers are replaced with the control laws determined as above.

## 5.3 Piezo-Driven Platform Simulations and Experiments

### 5.3.1 Platform Characterization

As depicted in Fig. 4.14 roll torques can be created with differential actuator voltage amplitudes for the left and right wings, resulting in a difference in wing flapping amplitudes and wing lift forces. This, along with the relatively large moment arm, \( R \), results in roll torques being larger and easier to produce than torques in either the pitch and yaw directions.

Pitching torques can be accomplished by varying the actuator voltage bias, \( V_b \) as depicted in Fig. 3.4. The voltage bias shifts the location of the wing sweep lift center, moving it behind or in front of the system center of gravity. Though this distance is limited by possible actuator depoling and cracking, a small lever arm can be produced, creating a pitching torque. This pitching lever arm is examined in Prototype 2 and shown in Figure 5.4. An example of a normal wing sweep as well as with an added positive and negative voltage bias is shown. The resulting lift center displacement was \( \pm 1.5 \text{ mm} \).

To characterize the relation between amplitude of the sinusoidal input \( V_a \) and the value of \( V_b \),
Figure 5.2: Images of both a large and small scale piezo-driven system with coordinate system defined.

Figure 5.3: Experimental mean lift as a function of actuator input voltage amplitude for both right and left wing at 30 Hz. Each point averages approximately one second of wing operation, or 1000 samples.
Figure 5.4: Top view photograph wing sweep changes with modified actuator bias. Each pair of colored lines (red, black, and green) denote wing sweep bounds for +40 V, 0 V, and -40 V actuator bias, respectively. The system was run at 100 V peak-to-peak amplitude and 35 Hz (right wing), 29 Hz (left wing).

a series of mean lift experiments are performed on the two-wing piezoelectric actuator system. Using the setup depicted in Fig. 3.11, input voltage amplitude sweeps are performed for each wing. Similarly the displacement of the wing sweep center is measured with an overhead camera for varying values of $V_b$. Where it is applicable, linear or quadratic functions are fitted to the gathered data, resulting in the relations

$$
\bar{F}_{z,L} = K_1 V_{a,L}^2 + K_2 V_{a,L} + K_3, \quad \bar{\tau}_{y,L} = K_4 V_{b,L} \bar{F}_{z,L},
$$

$$
\bar{F}_{z,R} = K_5 V_{a,R}^2 + K_6, \quad \bar{\tau}_{y,R} = K_7 V_{b,R} \bar{F}_{z,R}
$$

for both the right and left wings where $K_i$ are the fitted constants.

### 5.3.2 Single DOF Control Experiments

As a sufficiently sensitive sensor was not available to directly measure generated roll and pitch torques, roll and pitch control was tested experientially on a restricted degree of freedom rig. A custom single rotational DOF rig was manufactured from carbon fiber rods and 3D printed plastic components to allow rotation of the platform only around the interested axis. To minimize the effect of friction force, the rig is suspended on a set of jewel bearings. To allow conformation to manufacturing imperfections of the body and imperfect body to rig mounting, the rig was
designed such as to allow offset of the rotation axis, illustrated in Fig. 5.5, via sliding of the robot body along designated channels, as well as allowing offset of the system CG, via orthogonally mounted bolts. The system requires careful calibration, by positioning the rotation axis to pass through the wings’ centers of lift such that any biased deviation of the flapping stroke causes a pitching torque. A scanning sheet laser micrometer (Keyence LS-3100) provides feedback on the angular displacement of the body for control. Prototype 2, from Table 3.2, was used in the rig control experiments. Lift for inputted voltage amplitude can be seen in Fig. 5.3.

After mounting the flapping wing platform on the rig, the entire assembly was fully characterized. In particular, the following effects were taken into account:

1. Wire Stiffness: Power delivery to the robot is done via 45 AWG wire, channeled and wound such that it passes close to the rotation axis. Even so, the connections act as a spring, creating a restoring force given system perturbations.

2. Rig Drag: The connection points, both wire and rotation joint, contribute to non-negligible system damping.

3. Center of Gravity Location: Ideally the CG of the platform would have to be located exactly at the rotation axis, since this is the point about which the body will rotate in free flight. The CG of the body and rig can be shifted by adjusting the two added bolts in the
rig. Given that the exact location of the CG cannot be ensured, the effect of its offset is considered when learning rig parameters from the impulse response of the system.

The rig was modeled as a pendulum with a torsional spring of stiffness $k$, damping with coefficient $d$, and length $L$. The mass of the system is measured experimentally and the inertia of the aligned system is estimated from a Solidworks model of the combined rig components and body. The model is then fitted to an experimentally measured rig impulse response.

A reasonable set of hand-tuned PID gains for vehicle roll was first found in simulation for the un-rig encumbered full scale prototype. This controller was then applied to the roll mounted prototype, with compensation as described previously. The PID controller in roll is differential, increasing and decreasing the left and right wing amplitudes respectively from a nominal operating amplitude. The nominal amplitude was set to produce a low, but not nonzero force, 110 V for the left wing, 100 V for the right wing. Sensor feedback was read at 400 Hz, as limited by the laser micrometer used for body angle measurement. System response to a step of 10 and 20 deg can be seen in Fig. 5.6.

While the pitch control experiment demonstrated that the prototype was able to generate mean torque in both directions around the CG, overall system response was quite poor. Because the prototype is able to produce comparatively little torque, the additional force necessary to fully compensate for the rig cannot be produced, leaving the controller to threshold system inputs when attempting to achieve and maintain a nonzero body angle. For this reason, the simulation of the unencumbered prototype pinned to allow only pitch motion is not included here.

The larger magnitude body vibrations are in part due to excitation of the resonance of the rig, as seen by the low frequency oscillations. In general, however, pitch oscillations are expected to be larger than roll due to the additional contribution of uncontrolled pitch torques. While shifting the wing center of lift produces a mean controlled torque around the body, the wings also continually produce drag force and any unbalance between wing strokes causes net torques, acting to perturb the system. In addition, there is also a purely inertial reaction force from wing
Figure 5.6: (a) System roll trajectory tracking with a PID controller, experimentally measured and simulated. Controller gains: $P=400$, $I=400$, $D=25$. (b) System pitch trajectory tracking with a PID controller, experimentally measured. Controller gains: $P=800$, $I=0$, $D=10$. 
rotation, though this should be minimal due to low wing mass in comparison to total system weight.

5.3.3 Robust Controller Simulations and Experiments

The designed robust controller is implemented in simulation on a theoretical $\frac{1}{2}$ scale system model to allow free flight control testing, parameters of which are listed in Table C.1. Control inputs are limited as in the actual system: actuator input voltage amplitude and bias are limited by potential depoling of the actuator and the maximum allowable stress of PZT-5H. The maximum shifted distance of wing center of lift, $u_3$, is obtained from a kinematic model of the vehicle transmission for a given input voltage bias. The controller is limited to commands once per wing stroke to ensure that the expected wing motion and corresponding lift would be achieved. The system is assumed to be fully observable and controller parameters were optimized using the Nelder-Mead optimization routine of Matlab.

A series of simulation runs were made with the theoretical system, including several possible controller error types and additional feedback noise. Table C.4 contains the root mean squared (RMS) translational error, for comparison purposes, of a step response in the x, y, and z directions. Each column contains results for a different increasing Gaussian noise level added to the system feedback, with standard deviations of $\{0, 0.1, 0.5\}$ (rad and m). Controller errors tested include errors in the approximation of system parameters (mass and inertia), as well as an offset and a multiplicative error in the function relating input voltage amplitude to produced lift force. Figure 5.7 shows an example system response with no added error. Simulation results show a reasonable robustness to both controller error and added noise, though some sensitivity to offset errors in the expected lift function is observed.

As the current prototype cannot produce enough lift for liftoff, a passive weight compensating rig is constructed to conduct control experiments. As seen in Fig. 5.8, the rig is composed of a gimbal base with the prototype mounted to an extension rod. As the system is capable of
Figure 5.7: Free flight step response simulation of the scaled theoretical system with no added errors including the system translational position (a), roll and pitch angles (b), and control inputs (c), where $F_R$ and $F_L$ are the right and left wing forces in mN and $U_3$ is the distance of wing center of lift shift from nominal in millimeters. The black dotted line denotes the desired translational position.
producing much greater roll than pitch torques, the roll degree of freedom was chosen as a focus in rig development to ensure the system could handle the additional mass and inertia of the rig. In total, the rig has 3 DOFs within a restricted working area, allowing system roll as well as vertical and horizontal translational motion on the surface of a sphere. With an extension rod of 22 cm, an angle change of 5 deg translates to approximately 2 cm of translational motion.

The rig is constructed from laser machined Delrin and carbon fiber to reduce added inertia and jewel bearings are used for the rotational joints to reduce frictional effects. In addition, magnetic encoders (AM256, RLS, Ljubljana, Slovenia) are used on each axis to provide non-contact position feedback. Two orthogonal screws and a larger rear counterbalancing nut are used to shift the system center of gravity. The moment of inertia about each rig axis is \( \mathbf{J}_{\text{rig}} = \begin{bmatrix} J_\theta, J_\phi, J_\psi \end{bmatrix} = [1370, 5640, 5700] \text{ g.mm}^2 \), as predicted by the rig Solidworks model. Effective lift-to-weight ratio of the vehicle on the rig was approximately 2 in the nominal position, though this value varies depending upon the robot’s instantaneous z position due to constrained operation on a sphere and the rigid connection between the vehicle and extension spar.

Although within a certain operating area the prototype could be viewed as a constrained planar version of the free-flight system, the rig adds additional effects that preclude experimental runs to be directly mapped to free flight performance. As both power and computation is done offboard, power wires to the system and connections to the magnetic encoders must be routed, which contribute rotary stiffness and damping to each axis. The counter weight necessary to balance the system increases inertia significantly, slowing the system response. Placement of the magnetic encoders to accommodate the jewel bearing joint resulted in a noise range of \( \pm 2.5 \) deg with standard deviation of \( \pm 1.5 \) deg, requiring the feedback signal to be filtered. A simple moving average filter was used.

A series of experimental runs were performed on the 3 DOF rig system including a trajectory following task, position keeping under an air disturbance, and step horizontal input commands with various added errors similar to the simulation trials. The controller used was of the
Figure 5.8: Photo of the 3 DOF passive weight compensating rig used in control experiments: a) full system in the nominal zero position, b) close-up view of rig base with rotational axes marked.
Figure 5.9: Snapshots (A-H) in time of the flying robot prototype following figure eight trajectory. Prototype locations are labeled in Fig. 5.10(a) while time, t, corresponds the rig angles depicted in Fig. 5.10(b)-(d).

Table 5.1: Experimental Rig Test Results with Added Error

<table>
<thead>
<tr>
<th>Added Error Type</th>
<th>RMS Tracking Error (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\psi_{\text{rig}}$</td>
</tr>
<tr>
<td>No Added Error</td>
<td>6.6 ± 9.8</td>
</tr>
<tr>
<td>Parameters $x1.5$</td>
<td>6.5 ± 9.2</td>
</tr>
<tr>
<td>Parameters $x2$</td>
<td>7.6 ± 11.0</td>
</tr>
<tr>
<td>Parameters $x0.5$</td>
<td>7.0 ± 9.1</td>
</tr>
<tr>
<td>Right Wing Force +0.1mN</td>
<td>6.9 ± 9.0</td>
</tr>
<tr>
<td>Right Wing Force -0.1mN</td>
<td>9.4 ± 11.5</td>
</tr>
<tr>
<td>Right Wing Force 90%</td>
<td>7.2 ± 10.2</td>
</tr>
<tr>
<td>Right Wing Force 110%</td>
<td>6.8 ± 10.0</td>
</tr>
<tr>
<td>Right Wing Force 50%</td>
<td>12.3 ± 10.2</td>
</tr>
</tbody>
</table>

same form as derived previously but modified to reflect the changes in the physical system and restricted DOFs. Vertical motion and roll angle are directly controlled while horizontal translational motion is controlled similarly to the free flight $(x, y)$ translational motion described in equations (5.19) and (5.20). The calculated force to be applied to the system is transformed into actuator input voltage amplitude based on the system calibration and is allowed to change only once per wing stroke. Controller parameters were first optimized using a model of the rig system and then further hand-tuned on the experimental system. Roll commands by the controller are capped at ±40 deg due to the physical limitations of axis $\theta$ in the rig.
Figure 5.10 depicts the system following two cycles of a figure eight trajectory to be completed in 30 sec. Snapshots of the prototype during this trajectory following are shown in Fig. 5.9. As the desired roll angle is calculated using measured noisy system position rather than set directly, here and in other experiments, the desired roll angle is noisy as well.

Controller errors, including incorrect physical parameter estimation, offset error in the expected lift curve, and multiplicative error in the expected lift curve were added to the system on successive trials with step input commands. The system was asked to maintain vertical height while performing a step horizontal motion. An example step response with an error in estimation of produced lift can be seen in Fig. 5.11. Table 5.1 contains the RMS error over each axis for comparison purposes between the experimental runs. Each value is the average of at least 4 trials.

As can be seen in Table 5.1, the performance between the nominal case, where no controller error is added, and the trials with various added error (except 50% expected lift error) is comparable. The constant noise present in the system serves to mask the differences between the trial runs. The controller applied to the rig system is able to handle reasonable system approximation errors and feedback noise. However, the rig in general is less sensitive than the free flight system due to the added mass and inertia of the counter weight. Failure, when the system reaches the bounds of the rig, would occur at what would be considered unreasonably large errors, approximations that would, if true, render the system physically incapable of flight. A significant performance drop only occurred here when the expected right wing lift was 50% of its actual value.

Controller gains for both simulation and experimental tests are given in Table IV and V, respectively. In both cases, gains were optimized using Matlab’s Nelder-Mead optimization routine. Gains used in the experimental rigs tests were then further tuned by hand.
Figure 5.10: Experimental figure eight trajectory on the 3 DOF rig is shown in (a). As the rig arm extension is 22 cm this motion corresponds to approximately 15 cm horizontal and 5 cm vertical displacement. In (b)-(c), motion over individual axes is shown over time. The dashed black line is the desired position while the solid blue line is the actual system position after filtering. Desired horizontal and vertical position ($\psi$ and $\phi$) are entered while the desired roll angle is determined by the controller dependent on actual system position and desired trajectory.
Figure 5.11: Example experimental system response to a step input with right wing lift force 90% as expected. (a)-(c) depict motion about each of the rig axes. Blue denotes actual system position and the dotted black line is the desired system position.
5.4 Motor-driven Platform Simulations and Experiments

As with the piezoelectric actuator-driven system, the two-wing motor-driven system is able to create roll torques through varying wing flapping amplitudes and changing the wing stroke center of lift position. In this section we characterize and experimentally test the generated control inputs using a 6 DOF force/torque sensor. The linear controller is applied to both the two and four-wing system in simulation and a series of free flight experimental runs.

5.4.1 Generation of Controlling Forces and Torques

Platform calibration

To characterize the relationship between our desired mean forces and torques and the necessary motor voltage inputs, the instantaneous wing wrench is measured over the range of possible input voltage amplitudes and offsets. Specifically we would like to find a mapping between the body force $\bar{F}_b^z$ and torques $\bar{\tau}_b^x, \bar{\tau}_b^y$, which here will be noted with the superscript $b$, and the input voltage sinusoidal parameters $V_{amp,i}$ and $V_{b,i}$ for each motor $i$.

A fixture is used to mount the system to an ATI Nano-TI transducer sensor plate, as shown in Figs. 5.13 and 5.14, which is capable of measuring forces and torques in six degrees of freedom. In order to find the wing wrench about the system center of mass, we need to compensate for rotational and translational difference between the sensor and system CG-fixed coordinate frame.

![Figure 5.12: Two-wing motor-driven system coordinate system definition](image)

Figure 5.12: Two-wing motor-driven system coordinate system definition
Figure 5.13: Image of two-wing motor-driven system mounted to ATI Nano17-TI.

Figure 5.14: Diagram of ATI Nano17-TI sensor mount.
For the fixture used, the sensor and system frame differs by \( \theta_b = 120 \) deg about the z axis. In addition, as the position of the system’s CG is displaced from the origin of the sensing reference frame, the sensed torque is a function of both the body forces and torques. The forces and torques in the \( \vec{E}_b' \) frame are

\[
\mathbf{F}^{b'} = \mathbf{R}(\theta_b) \mathbf{F}^m \quad (5.21)
\]
\[
\mathbf{\tau}^{b'} = \mathbf{R}(\theta_b) \mathbf{\tau}^m \quad (5.22)
\]

where \( \mathbf{F}^m = [F_{x}^m, F_{y}^m, F_{z}^m]^{\top} \) and \( \mathbf{\tau}^m = [\tau_{x}^m, \tau_{y}^m, \tau_{z}^m]^{\top} \) are the measured forces and torques and \( \mathbf{R}(\theta) \) is the standard rotation matrix about a single axis.

While the forces in the \( \vec{E}_b' \) frame are equal to our desired body forces, the mount displacement must be accounted for when calculating torque. For example, about the \( \vec{E}_{xb}' \) axis depicted in Fig: 5.13, \( \mathbf{\tau}^{xb'} \) is

\[
\tau_{xb}' = \delta_y F_{z}^b - \delta_z F_{y}^b + \tau_{xb}^b, \quad (5.23)
\]

where \( \delta_y \) and \( \delta_z \) are the distances in the y and z direction respectively between the system mounting point and the sensor origin.

To find \( \delta_x, \delta_y, \) and \( \delta_z \) as well as our body torques \( \tau_{xb}, \tau_{yb}, \) and \( \tau_{zb}, \) a linear least squares problem can be formed and solved using singular value decomposition (SVD). For data gathered
in the time series \( t = \{ t_1, \ldots, t_n \} \),

\[
\begin{bmatrix}
\tau^b(t_1) \\
\vdots \\
\tau^b(t_n) \\
\delta_x \\
\delta_y \\
\delta_z
\end{bmatrix} = \mathbf{A}^+ \begin{bmatrix}
\tau^m(t_1) \\
\vdots \\
\tau^m(t_n)
\end{bmatrix}
\]

(5.24)

where \( \mathbf{A} \) is a \( 3n \times 3n + 3 \) matrix composed of our measured force values and \( \mathbf{A}^+ \) is the pseudo inverse found using SVD. After an initial calibration run determining \( \delta_x, \delta_y, \) and \( \delta_z \), the body torques at time \( t \) can be calculated as

\[
\tau^b(t) = \tau^m(t) - \begin{bmatrix}
0 & F^b_x(t) & -F^b_y(t) \\
-F^b_x(t) & 0 & F^b_z(t) \\
F^b_y(t) & -F^b_z(t) & 0
\end{bmatrix} \begin{bmatrix}
\delta_x \\
\delta_y \\
\delta_z
\end{bmatrix} \]

(5.25)

For the sensor and mount described above, \((\delta_x, \delta_y, \delta_z) = (11.2, 4.1, 37.3) \) mm.

A series of input voltage amplitude sweeps is performed on each wing in order to characterize the relation between the input voltage parameters and the resulting mean body forces and torques. For each wing \( i \), a multivariable quadratic function of the input parameters is then fitted in order to represent this space. This relation can be written as

\[
\begin{bmatrix}
\bar{F}^b \\
\bar{T}^b
\end{bmatrix} = \mathbf{C}_{VFT} \begin{bmatrix}
V^2_{amp} & V_{amp} & V^2_b & V_b & 1
\end{bmatrix}^\top
\]

(5.26)

where \( \mathbf{C}_{VFT} \) is the matrix of function coefficients, which is solved for using SVD. An example of a typical fitted map for a single wing can be seen in Fig. \ref{fig:5.15} However, for purposes of system control calibration, we desire a function that relates total desired average lift force \( \bar{F}^b_z \),
roll and pitch torque $\vec{\tau}_x^b$ and $\vec{\tau}_y^b$ to the input voltage parameters for each wing 1 to $n$. Similarly, we can write

$$\begin{bmatrix} V_{amp,1} & V_{b,1} & \cdots & V_{amp,n} & V_{b,n} \end{bmatrix}^\top = C_{FTV}x_{FT}$$

(5.27)

where $x_{FT} = [(\vec{F}_z^b)^2, (\vec{\tau}_x^b)^2, (\vec{\tau}_y^b)^2, \vec{F}_z x, \vec{F}_z y, \vec{\tau}_x x, \vec{\tau}_x y, \vec{\tau}_y]$. 

Testing of Generated Control Inputs

The accuracy of the calibration matrix $C_{FTV}$ is first tested on the collected mean force and torque data for steady state wing flapping. Five hundred randomly selected combinations of $\vec{F}_z$, $\vec{\tau}_x$, and $\vec{\tau}_y$ are compared to the value expected from the model, and plotted in Fig. 5.16. However, in practice system commands are allowed to change once per wing stroke and are subject to wing transients. A more accurate representation of the determined mapping was found through a trial of 21 experimental runs where command inputs $V_{amp,i}$ and $V_{off,i}$ were varied for each wing according to the determined calibration matrix $C_{FTV}$ for the two-wing system. Expected versus achieved $\vec{F}_z$, $\vec{\tau}_x$, and $\vec{\tau}_y$ are plotted in Fig. 5.17. The commanded inputs in the experimental trials were not allowed to vary more than 2 mN or 50 mN.mm per change. While error does increase, the system is still able to produce independent control forces and torques over the same range.
Figure 5.15: Mean wing lift and torque for varied control inputs. The red points denotes experimentally measured data and the surface is the fitted function as noted in (5.26).
Figure 5.16: Expected versus measured mean force and torque for steady state wing flapping. \( \bar{F}_z \), \( \bar{\tau}_x \), and \( \bar{\tau}_y \) were first measured from a series of input voltage amplitude and offset sweeps for each wing. Five hundred force and torque values were then randomly selected and compared to the forces and torques expected using the calibration matrix \( C_{FTV} \). The spread about the dotted diagonal lines denotes the combined calibration matrix modeling and sensor error.
Figure 5.17: Expected versus measured mean force and torque for commanded control inputs. The mean and standard deviation of $F_z$, $\tau_x$, and $\tau_y$ were calculated from a total of 21 experimental runs where the command inputs $V_{amp}$ and $V_b$ were varied for each wing according to the determined calibration matrix $C_{FTV}$ for the two-wing system. The command inputs were applied at 10 Hz and were varied no more than 2 mN or 50 mN.mm per change. The uncontrolled forces and torques for this system ($F_x$, $F_y$, and $\tau_z$) had a mean and standard deviation of -3±0.7 mN, 3±0.5 mN, and -300±30 mN.mm respectively.
5.4.2 Experimental Testing Setup

As the motor-driven system is capable of liftoff, tethered free flight trials can be performed. Experimental free flight runs were performed in an arena surrounded with eight Bonita Vicon cameras to provide feedback on system position and orientation, which is depicted in Fig. 5.18. Markers are added to the flapping system body to allow tracking. The Vicon system operates at 240 Hz and provides information on marker position at sub-millimeter accuracy. The system is always tethered to the take-off platform with additional wires providing off board motor power. Though the length of the tether varies between trials, the system has an operating space of an approximate sphere with a radius of one meter. Free flight control runs were performed with the linear controller; Fig. 5.19 depicts the experimental closed loop system with each physical component noted appropriately.

5.4.3 Two-wing System

Control of the motor-driven two-wing system is tested both in simulation and with free-flight experiments. A linear controller, as described in Section 5.2.2 is used in both cases.

Figure 5.20 depicts the simulation results for linear, decoupled control of altitude, roll, and pitch. The system is asked to maintain hover while perturbed by the instantaneous wing generated wrench. The transfer functions of discrete-time controllers used are

\[ G_z = \frac{0.89(z^2 - 1.23z + 0.44)}{z^2 - 0.29z - 0.72} \]

\[ G_{\theta_x} = \frac{5.56e^{-3}(z^2 - 1.06z + 0.34)}{(z^2 - 0.31z - 0.7)} \]

\[ G_{\theta_y} = \frac{1.62e^{-4}(z^2 - 1.13z + 0.33)}{(z^2 - 0.53z - 0.47)} \]

for error in \( z \), \( \theta_x \), and \( \theta_y \) respectively. Units are metric standard m, rad, and N. To avoid confusion, \( z \) is used as the discrete time variable with sampling time \( \frac{1}{f} \), or 0.1 sec in this case. Performance is quite poor which is contributed to by the large 10 Hz pitch oscillations previously described. Indeed, even ignoring the input bounds defined by the experimental system, after two seconds the system effectively crashes by completing a complete revolution midair.

The linear controllers found in simulation are then applied to the system in free flight and further tuned. Each controller can be tested individually to examine performance about the rele-
Figure 5.18: Diagram depicting the experimental free flight setup. Fig. 5.19 provides a more detailed description of the components between the camera array and generated motor inputs.

Figure 5.19: Closed loop diagram of experimental free flight system. Feedback is measured using a Vicon motion capture system.
Figure 5.20: Closed loop control of the two-wing motor-driven system in simulation. Altitude, body angles, and control inputs are shown for a station keeping attempt. Plotted here are $\hat{u}_1$, $\hat{u}_2$, $\hat{u}_3$ which are the stroke averaged lift force minus trim, and stroke averaged roll and pitch torque respectively. Though inputs are allowed to be unbounded in this case, the system quickly destabilizes.
vart controlled axis. Figure 5.21 depicts several trajectories; the first is typical open loop flight, the second subplot depicts altitude and body angles of a system with a running altitude controller and roll controller. Figure 5.22 provides a closer look at the control inputs applied and the flight trajectory in free space.

The altitude controller performs as expected, reaching and maintaining the desired altitude with some error. The roll controller, as examined over a single prototype’s five open loop trials and four closed loop runs, provides a relatively small improvement on roll RMS error. The pitch controller, however, fails, which is not entirely unexpected given the flapping oscillations. Applied, the pitch controller consistently shortens the system flight time.
Figure 5.21: Open loop altitude and body angles (a) and two runs of the system with the roll and altitude controller active (b). Desired height was 600 mm.
Figure 5.22: Altitude, body angles, and control inputs for the implemented roll and altitude controller in a hand released system are shown in (a). Desired height was 600 mm. In (b) snapshots at 0.1 sec are overlaid of the system flight. An open loop trial for the same system is shown in (c).
5.4.4 Four-wing System

As with the two-wing system, the linear controller is applied and tested on the four-wing system. Figure 5.23 depicts altitude and body angles of the simulated system asked to hover given an initial error. The discrete-time controller transfer functions used are

\[
G_z = \frac{1.60e^{-3}(z^2 - 1.55z + 0.61)}{z^2 - 0.12z - 0.88},
\]

\[
G_{\theta_x} = \frac{1.62e^{-3}(z^2 - 1.24z + 0.47)}{(z^2 - 0.27z - 0.74)},
\]

\[
G_{\theta_y} = \frac{1.56e^{-3}(z^2 - 1.24z + 0.47)}{(z^2 - 0.27z - 0.74)}
\]

for error in \( z \), \( \theta_x \), and \( \theta_y \) respectively with metric standard units of m, rad, and N. Control inputs are bounded to what is achievable in the experimental platform. The decreased control input coupling and flapping oscillations significantly improves control of the system in simulation. Though there remains oscillations in the controlled body angle, the magnitude is comparably small and the system stabilizes.

In order to verify torque generation in free flight, a series of open loop runs are perform with varied set control input biases. In Fig. 5.24 and 5.25 the four-wing system is released from the testing platform with modified motor input voltages that are held constant during the flight. Instead of the typical trim inputs, a bias about either the pitch or roll axis is applied. The system is started at the same body orientation at release. As can be seen in Fig. 5.24, open loop control input change allows the system to move translationally in any direction.

Large oscillations at the flapping frequency were debilitating for the two-wing motor-driven system. Given that open loop flights were possible, it is possible that a control method utilizing predictive modeling could compensate, but at the cost of complicating control. The robust control method discussed in this work may show an improvement, but as with the linear controller, it is designed and operates on the average system state. The significant improvement, however, of the four-wing over the two-wing system in simulation suggests that the linear controller, with some experimental gain tuning, will be adequate to control the four-wing platform in experimental free flight.
Figure 5.23: Closed loop control of the four-wing motor-driven system in simulation. Altitude, body angles, and control inputs are shown for desired set point of \( \{ z, \theta_x, \theta_y \} = \{ 0, 0, 0 \} \) given an initial error. Plotted here are \( \hat{u}_1, \hat{u}_2, \hat{u}_3 \) which are the stroke averaged lift force minus trim, and stroke averaged roll and pitch torque respectively. Inputs are bound to limits of what has been shown to be achievable in experiment.
Figure 5.24: Overlaid snapshots taken at intervals of 0.1 sec of three open loop flights of the tethered 4 wing prototype. With differing constant input driving signals, the system travels to the left (a), forward (b), and right (c).
Figure 5.25: Trajectories of sixteen open loop flights of the 4 wing system. Displacement in the $x$ and $y$ directions is shown in (a). Each run begins with the system in the same position and orientation. Between runs the constant wing input signal is varied, resulting in varied travel directions. In (b) the vertical displacement of each trajectory is plotted. The sharp decrease in elevation seen in many of the runs is due to the system hitting its tether.
5.5 Yaw Torque Generation

While yaw is intended to be left uncontrolled in experimental trials, in this section a possible means of generating yaw torques is examined. In a fully controlled wing, yaw torques can be created by asymmetrically delaying the motion of the trailing edge of the wing during each stroke. Given the underactuated wing design presented here, we can instead force asymmetric acceleration of the leading edge; larger accelerations result in overall higher wing velocities generating large drag forces with the converse true for slow accelerations. An example complete wing stroke can be seen in Figure 5.26. Driving the wing with an asymmetrical waveform results in a net drag force over the entire stroke, thus creating yaw torques.

During normal operation, the wing is driven with a simple symmetric sinusoid. To generate the required waveforms for yaw torque generation, the peak voltage point of the wave is advanced and retarded during the half cycle, as portrayed in Figure 5.27. The peak position parameter is defined as the ratio between the location of the positive wave peak $\zeta$ and the half stroke length $\xi$, and is in the range [0, 1]. A peak location in the half wing cycle of 0.5 denotes the normal sinusoid; shifting the peak position sooner (such as to 0.1) or later (such as to 0.9) will create an overall greater drag force in the clockwise or counterclockwise direction. Note that, because the drive signals are constructed from the sum of four sinusoids, there are oscillations in the waveforms; in practice, however, these higher frequency, low amplitude vibrations are damped out by wing aerodynamics during operation. Whenever needed, additional terms can be added to smooth the drive signal. Figure 5.27a depicts example sinusoids over one wing stroke.

5.5.1 Piezo-driven Platform Yaw Simulation

To investigate the effect of the created driving waveforms, a wave peak position response plot was simulated using the piezoelectric actuator-driven system model described in Chapter 2. Wing kinematics along with mean lift and mean drag over a wing stroke are plotted in Fig. 5.27b. As might be expected, varying the wing driving waveform from a symmetric sinusoid generally
Figure 5.26: Time spent on forward and reverse motion during one wing stroke for generation of a yaw torque, accompanied with example wing rotation angles. In this case the forward stroke is completed much faster than the reverse stroke, resulting in larger wing rotation angles but overall greater wing speed and greater drag force.

results in the loss of flapping amplitudes, and, therefore, mean lift force. The wave peak position of 0.5 does not yield maximum flapping amplitude, occurring instead at 0.58. This is due to the asymmetric torque transmission curve of the spherical 4-bar, depicted in Figures 3.10a and 3.15a. This intrinsic asymmetry requires slightly different accelerations during each half stroke of the wing for optimum wing trajectory.

As seen in Fig. 5.27b, the net drag force changes sign, demonstrating the capability for bi-directional yaw torques. Though the total mean drag force that can be produced is small compared to the total lift force, it shows at least the possibility of compensating for the asymmetric torque transmission curve or reasonable manufacturing irregularities between wings, if not for creating large controlling yaw torques.

5.5.2 Motor-driven Platform Yaw Experiment

While the low drag forces of the piezo-driven system precludes measurement with the available setup, a similar experiment can be run on the larger, two-wing motor-driven system. A prototype was mounted to a 6 DOF force/torque sensor as described in Section 5.4.1. Seven trials were run where the input wave peak position, defined by $\zeta / \xi$, was stepped from 0.2 to 0.8. As the
Figure 5.27: (a) Wing drive waveforms for generation of body yaw torques. The signals are constructed by fitting a sum of 4 sinusoid signals to 9 control points (blue circles) defining the wave. (b) Sensitivity of system dynamics to drive wave peak position parameter and constant actuator voltage amplitude.
two-wing system was tested, the shift direction of the input sinusoid was reversed between each wing; in order to prevent the produced wing drag forces from canceling instantaneously, while wing one had an input wave peak position of 0.2, wing two was set to 0.8. Each varied input waveform step was held for five complete wing strokes, or 0.5 sec. Input flapping frequency and peak input voltage magnitude was held constant at 10 Hz and 12.5 $V_{pp}$ respectively. Figure 5.28 shows the average and standard deviation of mean lift and yaw torque produced over all the trials.

Due to a bias in wing rotation angle, the maximum negative yaw torque is significantly larger than the maximum positive yaw torque produced. As in the piezo-driven system simulation, mean lift generally falls as the input sinusoid becomes more skewed, though this could be com-
pensated for to a certain extend by increasing peak input voltage magnitude appropriately. Never-
theless, with a shift input waveform peak, the system is capable of producing small, but varying
yaw torques as predicted.

5.6 Summary

In this chapter we attempt to demonstrate both piezoelectric actuator-driven and motor-driven
system controllability. Both a robust and linear controller are described and tested in simulation
on each system respectively. Due to lack of sufficient lift, control of the piezo-driven system is
performed on a restricted degree of freedom rig. Lift force and roll and torque generation is tested
in the two-wing motor-driven platform using a 6 DOF force/torque sensor capable of capturing
the instantaneous wing wrench. Linear control of the two-wing motor-driven system in exper-
imental free flight is not successful in part due to the large pitch oscillations and assumptions
made based on averaged system performance and operation about hover. The four-wing system
proves to perform better in simulation, and will likely be controlled in experiment in future work.
Chapter 6

Conclusions and Future Work

This work focuses on the ongoing challenge of balancing mechanical complexity, system controllability, and weight for flapping flight systems, with the goal of creating a new system capable of liftoff and of controlling torques. As of yet, the acrobatic agility of the fly or hummingbird is unmatched outside of the natural world. Compared to their fixed and rotary winged counterparts, insect-inspired flapping wing vehicles would present a large leap forward in maneuverability. While there are a number of practical problems remaining in order to make agile artificial fliers, this work advances efforts by developing new systems at two size scales, and in the end, demonstrating the liftoff of a simple flapping system capable of producing controlling torques.

6.1 Contributions and Concluding Remarks

Some of the tasks accomplished in this work include: 1) development of a modular flapping system considering both controllability and weight minimization, 2) construction of prototypes at several size scales, 3) achieving liftoff in a flapping system capable of producing controlling torques, 4) conducting control experiments on the developed platforms, and 5) developing a new lightweight control actuator that can be incorporated into flexural mechanisms. Below is a brief overview of the progress made towards the development of a flapping wing micro aerial vehicle:
• **Piezoelectric actuator-driven platform design:** The piezo-driven platform design considers system controllability, with one actuator per wing, along with the requirement that the body structure be lightweight and rigid. The transmission, a spherical 4-bar, centralizes system mass below the center of lift while increasing output torque. Using the system dynamic model, the system is optimized and platforms at two size scales are constructed using the Smart Composite Microstructures (SCM) manufacturing techniques [100]. The small 160 g platform demonstrated a maximum lift-to-weight ratio of 3/8.

• **Motor-driven platform design:** In order to take full advantage of the benefits of resonance, a motor-driven flapping system is designed with a linear transmission and elastic element. Independence matching was used to chose a wing size, flapping frequency, and motor. Though gearbox limitations restricted the stiffness of spring used, a lift to weight ratio of greater than one was achieved. With a single actuator per wing, both controlling roll and pitch torques can be created. While the original two-wing system exhibits large oscillations about the pitching axis, a modified four-wing system resolves this issue.

• **Platform control tests:** Two controllers are developed and used to demonstrate both piezo and motor-driven controllability. A robust controller based on Lyapunov stability and sliding-mode techniques is derived and tested on the piezo-driven system mounted on a restricted three DOF rig. Roll and pitch torque generation are tested individually on a single DOF rig. The motor-driven system, with a higher generated wing wrench, is characterized on a six DOF sensor and control inputs tested. Free flight linear control trials with the two-wing system resulted in poor performance, but hold promise with the modified four-wing platform.

• **Theoretical motor-driven system model:** Arabagi et. al developed a dynamic model of a passively rotating wing driven by a piezoelectric actuator in [84], which was both used here to numerically optimize the piezo-driven platform and adapted for use with the motor-driven system. The aerodynamic model, originally intended for lower Reynolds number
flapping, was verified for the larger flapping-wing platform.

- **Lightweight control actuator for flexural mechanisms:** Using a shape memory polymer coated flexure, lift control is demonstrated on the piezo-driven system. When placed in the transmission, applying heat to the flexure changes the mechanism resonance frequency, allowing a controlled decrease in lift. The power requirement remains relatively high, but was significantly decreased when using an embedded heating element.

### 6.2 Future Directions

This work focuses on only a few aspects of a functional flapping wing robot. On board power, sensing, and computation is left as future work for an untethered system. There are a few open areas that can be considered immediate extensions of this work, however, that may lead to interesting results:

- **Free flight control of motor-driven platform:** The modular platform design allows almost arbitrary placement of wings. While the nominal two-wing motor-driven design does not well filter the wing generated torques about the pitching axis, a modified four-wing design increases body damping and decreases the magnitude of the instantaneously generated wing wrench. In free flight it is considerably less oscillatory and better matches the assumptions made based on averaging theory. Implementing a model predictive controller to better handle the two-wing system motion and continued linear or robust controller testing with the four-wing system are reasonable next steps to take.

- **Lift improvements in motor-driven platform:** There are several improvements that can be made to increase lift in the motor-driven platform. The current GM15 gearbox fails with large elastic elements, lowering the maximum system resonance frequency. Currently the system operates above system resonance to maximize lift. Additional power can be saved and expended instead on lift if a higher elastic element stiffness could be used. Design-
ing a new gearbox or motor specific to this application could have considerable benefits. Using brushless instead of brushed DC motors could also allow an increase in system lift. Brushless motors are typically more powerful, robust, and efficient, and if implemented with motor position feedback, could allow closed-loop control of wing motion for better and more consistent lift and torque generation.

As a whole, the commercially available motors are not designed for the relatively low speeds, high torque, and oscillatory motion needed for the flapping wing system. Optimizing and constructing a motor specifically for this application may also lead to increased lift production per unit mass.

- **Wing optimization:** The wing is one of the more important features influencing system lift on both the small and large scale. A rigid wing shape is assumed for this work, with rotation occurring only about the incorporated wing flexure. Wing deformable chamber, wing twist, and wing surface texture are all elements observed in nature, and may be worth continued study as applied to an artificial system. Computational fluid dynamics along with tests of manufactured wings could be performed to capture all nonlinear aerodynamic effects.

- **Characterizing passive wing damping:** The wings and the wings’ motion is the highest contributor to body damping and therefore has a significant effect on the system dynamics. Cheng *et al.* has well characterized the influence of changing wing kinematics on the system damping coefficient [49], but initial tests suggest that using a passive wing may further influence the result. Better understanding this effect will help better characterize the existing system and may shed light on the effect on deformable wings in nature.

- **Mechanism control with shape memory polymer flexures:** While the embedded shape memory polymer flexure exhibited a relatively slow transition speed in regards to flapping platform control, this same technique could be applied to slower crawling robots for steering or in rig mounted flexural mechanisms. Being able to tune a mechanism reso-
nance frequency not only allows control of the outputted motion, but also allows greater flexibility of load and application for devices used for measurement.
Nomenclature

\( \beta_{CG} \)  vertical distance of wing CG from wing rotational axis

\( \eta_g \)  gearbox efficiency

\( \nu \)  velocity

\( \omega \)  angular velocity

\( \phi \)  wing rotation angle

\( \rho \)  air density (SATP 1.18 kg.m\(^{-3}\))

\( R \)  rotation matrix

\( \theta \)  wing flapping angle

\( \theta_x, \theta_y, \theta_z \)  body angle about coordinate axes

\( \vec{\beta}_t, \vec{\beta}_{rot} \)  position of translational and rotational lift centers

\( b_m, b_0 \)  motor internal damping, motor internal damping as seen at gearbox output shaft

\( b_w \)  wing rotational flexure damping

\( c \)  wing chord length

\( C_l, C_d, C_{rot} \)  coefficient of wing translational lift, drag, and wing rotation

\( d_w \)  wing offset, radial distance from wing base to flapping axis

\( f \)  flapping frequency

\( F, M, \tau \)  force, moment, torque
$g$ acceleration due to gravity (9.81 m/s$^2$)

$h_{yoff}, h_{yoff,m}$ horizontal offset between wing center of lift and system CG, and between wing center of lift and individual module CG

$h_{zoff}, h_{zoff,m}$ vertical offset between wing center of lift and system CG, and between wing center of lift and individual module CG

$J, \mathbf{J}$ inertia component, inertial tensor

$k_{act}$ stiffness of piezoelectric actuator

$k_{rot}$ wing rotational flexure stiffness

$k_s$ elastic element stiffness in motor-driven system

$k_t$ motor constant

$L$ dynamical system Lagrangian

$m$ mass

$N_g$ gearbox gear ratio

$R$ wing length measured from base of wing to wing tip

$r$ radial distance from the wing flapping axis

$R_0$ motor armature electrical resistance

$R_{CG}$ radial distance of wing CG from wing base

$Re$ Reynolds number

$T_{energy}, T_{wing}, T_{act}$ kinetic energy, of wing and actuator respectively

$U$ wing velocity in the direction of wing flapping motion

$V_a, V_b$ input voltage sinusoid amplitude, input voltage sinusoid bias from zero

$V_{energy}, V_{wing}, V_{act}$ potential energy, of wing and actuator respectively

$V_{in}, V_{pp}$ input voltage, peak-to-peak magnitude of input voltage sinusoid
<table>
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<th>Acronym</th>
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<td>CG</td>
<td>center of gravity</td>
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<tr>
<td>DAQ</td>
<td>data acquisition</td>
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<td>FMAV</td>
<td>flapping wing micro aerial vehicle</td>
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<tr>
<td>NiCr</td>
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<td>PZT</td>
<td>piezoelectric ceramic lead zirconate titanate</td>
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<tr>
<td>SATP</td>
<td>standard ambient temperature and pressure (25°C, 14.5 psi)</td>
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<td>SMP</td>
<td>shape memory polymer</td>
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Appendix A

SMP Flexure Prototypes and Additional Results

A.1 SMP Flexure Prototypes

The dimensions and experimental angular bending stiffnesses and damping of the manufactured SMP flexures is included here in Table A.1.

A.2 Experimental Lift Change for Flexures 11 and 12

Additional figures detailing the lift change induced by SMP flexures 11 and 12, as defined in Table A.1.
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Table A.1: Manufactured Flexure Measured Stiffness and Damping
Figure A.1: Mean lift over time at input frequency of 46 Hz (a) and 44 Hz (b). A voltage amplitude of 200 V is held constant while heat is applied to the flexure at the times indicated with the red shaded bands. Flexure 12 was used here and was warmed using an embedded NiCr heating element. In (a) lift decreases when the flexure is warmed. In (b), due to the reduced operating frequency and the difference in system resonance frequency between the flexure warm and cool states, lift instead increases when heat is applied.
Figure A.2: Mean lift over time at an input frequency of 46 Hz. A voltage amplitude of 200 V is held constant. Flexure 11 was used here and was warmed using an embedded NiCr heating element. Heat is applied to the flexure at the times indicated with the red shaded bands. (b) provides a closer look at the change in mean lift values.
Appendix B

Motor Parameters

The physical parameters of over 60 commercially available motors less than 10 grams can be found here in Tables B.1, B.2, and B.3.
Table B.1: Commercially Available Motor Physical Parameters, Part 1

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Company Abbreviations: Precision Microdrives (PM), Vigor Precision (VP)
Table B.2: Commercially Available Motor Physical Parameters, Part 2

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Note: The table continues with similar entries for other companies and models, providing detailed specifications such as voltage, torque, current, and efficiency values.
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Company Abbreviations: Precision Microdrives (PM), Vigor Precision (VP)
Appendix C

Robust Controller Derivation and Simulation

C.1 Robust Controller Derivation

Here the derivation of a robust controller for a single degree of freedom is described. As an example, we can examine the averaged body roll angle $\bar{\theta}_x$, which from 4.25-4.31 has the nonlinear equation of motion

$$\ddot{\bar{\theta}}_x = \frac{1}{J_1}(\bar{\omega}_y \bar{\omega}_z + \bar{L}_y(\nu_y - \bar{\omega}_x h_{yoff}) - \bar{L}_x + h_{yoff} \cos(\bar{\theta})(\bar{F}_{L,z} - \bar{F}_{R,z}))$$  \hspace{1cm}(C.1)

where the three system control inputs are $\bar{u} = [u_1, u_2, u_3] = [\bar{F}_{L,z} + \bar{F}_{R,z}, \bar{F}_{L,z} - \bar{F}_{R,z}, \bar{\theta}]$.

This can be written in the general form:

$$\ddot{\bar{\theta}}_x = f_{\theta_x}(\bar{x}) + g_{\theta_x}(\bar{x}, \bar{u})u_2,$$  \hspace{1cm}(C.2)
where $\bar{x}$ is the system state. We define the roll angle error, $e_1$, as follows:

$$e_1 = \theta_{xd} - \bar{\theta}_x,$$

(C.3)

$$\dot{e}_1 = \dot{\theta}_{xd} - \dot{\bar{\theta}}_x,$$

(C.4)

with $\theta_{xd}$ being the desired roll angle. Instead of the standard angular velocity error we use a virtual input, $v_{xd}$, and define our second state error, $e_2$ as:

$$e_2 = \dot{v}_{xd} - \dot{\bar{\theta}}_x,$$

(C.5)

$$\dot{v}_{xd} = c_1 e_1 + \dot{\theta}_{xd} + \lambda_1 X_1,$$

(C.6)

$$X_1 = \int_0^t e_1(\tau) d\tau,$$

(C.7)

where $c_1$ and $\lambda_1$ are positive constants and $X_1$ is the position error integral term. The derivative of $e_2$ is therefore:

$$\dot{e}_2 = c_1 \dot{e}_1 + \ddot{\theta}_{xd} + \lambda_1 e_1 - \dot{\bar{\theta}}_x.$$

(C.8)

We can formulate a Lyapunov function for the system and choose our control input $u_2$ so the conditions for Lyapunov stability hold. Considering the Lyapunov function:

$$L = \frac{\lambda_1}{2} X_1^2 + \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2,$$

(C.9)

$$\dot{L} = \lambda_1 X_1 \dot{X}_1 + e_1 \dot{e}_1 + e_2 \dot{e}_2.$$

(C.10)

Simplifying the derivative of both $e_1$ and $e_2$ and substituting in both the derivative of the virtual
input $v_{xd}$ of (C.6) and the $\ddot{\theta}_x$ of (C.2) results in:

\begin{align*}
\dot{e}_1 &= e_2 - c_1 e_1 - \lambda_1 X_1, \\
\dot{e}_2 &= c_1 \dot{e}_1 + \ddot{\theta}_{xd} + \lambda_1 e_1 - [f_{\theta_x}(\bar{x}) + g_{\theta_x}(\bar{x}, \bar{u})]u_2. 
\end{align*}

(C.11) \quad (C.12)

The variables $\dot{e}_1$ and $\dot{e}_2$ can be substituted back into the Lyapunov function derivative. For convenience a new function $\kappa(\bar{x}, \bar{u})$ is defined, simplifying (C.10) as follows:

\begin{align*}
\dot{L} &= -c_1 e_1^2 + e_2((1 - c_1^2 + \lambda_1)e_1 + c_1 e_2 - c_1 \lambda_1 X_1 + \ddot{\theta}_{xd} - (f_{\theta_x}(\bar{x}) + g_{\theta_x}(\bar{x}, \bar{u}))u_2), \\
\dot{L} &= -c_1 e_1^2 + e_2 \kappa(\bar{x}, \bar{u}).
\end{align*}

(C.13) \quad (C.14)

To ensure Lyapunov stability $L \geq 0$ and $\dot{L} \leq 0$ must hold true. As it is defined, $L$ will always be greater than zero. Depending upon the value of $\kappa(\bar{x}, \bar{u})$, however, the latter condition may or may not hold. Since we would like the controller to be robust against both modeling errors and feedback noise we choose a sliding mode control law. Let $s$ be a manifold that satisfies

\begin{align*}
\dot{s} &= -k_1 \text{sgn}(s) - k_2 s, 
\end{align*}

(C.15)

a constant plus proportional rate reaching law where $k_1$ and $k_2$ are positive constants. To minimize chattering the sign function can be replaced with a saturation function, defined for a variable $y$ below:

\begin{align*}
\text{sat}(y) = \begin{cases} 
  y, & \text{if } |y| \leq 1 \\
  \text{sgn}(y), & \text{if } |y| > 1
\end{cases}
\end{align*}

(C.16)

For a positive constant $\varepsilon$, sat($s/\varepsilon$) will result in a linear controller near desired state.
Using our chosen manifold, we can solve for our control input such that:

\[
\kappa(\vec{x}, \vec{u}) = -k_1 \text{sat}(e_2/\varepsilon) - k_2 e_2, \quad (C.17)
\]

\[
e_2 \kappa(\vec{x}, \vec{u}) \leq 0. \quad (C.18)
\]

Using (C.17) and solving for \(u_2\) results in a roll control input of:

\[
u_2 = \frac{1}{g_{\theta_x}(\vec{x}, \vec{u})} \left[ (1 - c_1^2 + \lambda_1) e_1 + c_1 e_2 - c_1 \lambda_1 X_1 
+ k_1 \text{sat}(e_2/\varepsilon) + k_2 e_2 + \dot{\theta}_{zd} - f_{\theta_x}(\vec{x}) \right]. \quad (C.19)
\]

In the ideal case wing motion would be symmetric and total mean drag force over a wing stroke would be zero. With this assumption and considering operation about the nominal hovering position, yaw velocity will be driven to zero due to damping about this axis. In actuality, platform maneuvers as well as small manufacturing irregularities can result in a small yaw acceleration being produced. While it may be possible to modify the driving waveform to produce small compensating yaw torques, the Lyapunov function \(L_{\theta_x}\) can also be modified to ensure stability about this axis. Let us define \(\hat{e}_1\) and \(\hat{e}_1\):

\[
\hat{e}_1 = \theta_{zd} - \hat{\theta}_x + \theta_z - \hat{\theta}_z, \quad (C.21)
\]

\[
\hat{e}_2 = \ddot{\theta}_{zd} - \dot{\theta}_x, \quad (C.22)
\]

\[
\dot{\theta}_{zd} = c_1 \hat{e}_1 + \hat{\theta}_{zd} + \lambda_1 X_1 + \ddot{\theta}_{zd} - \hat{\theta}_z. \quad (C.23)
\]

Yaw acceleration is defined as \(\dot{\theta}_z = f_{\theta_z}(\vec{x})\). A minimum, we would like to drive yaw velocity to zero and not concern ourselves with actual yaw angle. Simplifying and setting \(\theta_{zd} = \hat{\theta}_z\), \(\hat{e}_1\) and \(\hat{e}_1\) are
\[
\dot{e}_1 = \dot{\theta}_{xd} - \ddot{\theta}_x - \ddot{\theta}_z,
\]
\[
= \dot{e}_2 - c_1\dot{e}_1 - \lambda_1 X_1
\] (C.25)
\[
\dot{e}_2 = c_1\dot{e}_1 + \lambda_1\dot{e}_1 + \ddot{\theta}_{xd} - \ddot{\theta}_x - \ddot{\theta}_z.
\] (C.26)

This results in our input \( U_2 \) being

\[
\ddot{u}_2 = \frac{1}{g_{\theta_x}(\vec{x}, \vec{u})}[(1 - c_1^2 + \lambda_1)e_1 + c_1e_2 - c_1\lambda_1 X_1
\]
\[+ k_1 \text{sat}(e_2/\varepsilon) + k_2 e_2 + \ddot{\theta}_{xd} - f_{\theta_x}(\vec{x}) - f_{\theta_z}(\vec{x})].
\] (C.27)

\[
+ k_1 \text{sat}(e_2/\varepsilon) + k_2 e_2 + \ddot{\theta}_{xd} - f_{\theta_x}(\vec{x}) - f_{\theta_z}(\vec{x})].
\] (C.28)

C.2 Robust Controller Gains, System Parameters, and Simulation Results

In this section, additional details regarding the robust controller tests with the piezo-driven system are included.

Table C.1 lists system parameters for the constructed piezoelectric actuator system used in the three DOF rig experiments as well as for a theoretical \( \frac{1}{2} \) scale system capable of liftoff. In Tables C.2 and C.3, controller gains used in both simulation and experiment are shown. In Table C.4, the simulation results for the robust controller applied to the half scale piezoelectric system are shown.
Table C.1: MAV Physical Parameters and Operating Information Used in the Robust Controller Simulation and Experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Body Scale</th>
<th>x1</th>
<th>x0.5†</th>
</tr>
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<tbody>
<tr>
<td>Body Mass (mg)</td>
<td></td>
<td>690</td>
<td>140</td>
</tr>
<tr>
<td>Inertia (g.mm²)</td>
<td>[62.0, 33.7, 28.8]*</td>
<td>[4.1, 3.6, 1.7]*</td>
<td></td>
</tr>
<tr>
<td>Horz. Wing Center of Lift to CG, h_{yoff} (mm)</td>
<td>23</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Vert. Wing Center of Lift to CG, h_{zoff} (mm)</td>
<td>26</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Viscous Drag Coefficient, b (kg/s)</td>
<td></td>
<td>0.0114</td>
<td>0.0029</td>
</tr>
<tr>
<td>Actuator Length (mm)</td>
<td></td>
<td>18.5 (7.5 ext.)</td>
<td>11 (4 ext.)</td>
</tr>
<tr>
<td>Actuator Width (mm)</td>
<td></td>
<td>5 (taper to 4)</td>
<td>3 (taper to 1.5)</td>
</tr>
<tr>
<td>Actuator Mass (mg)</td>
<td></td>
<td>130</td>
<td>32</td>
</tr>
<tr>
<td>Wing Length (mm)</td>
<td></td>
<td>21</td>
<td>13</td>
</tr>
<tr>
<td>Maximum Actuator Drive Voltage (Vpp)</td>
<td></td>
<td>250</td>
<td>320</td>
</tr>
<tr>
<td>Resonant Frequency (Hz)</td>
<td></td>
<td>30</td>
<td>95</td>
</tr>
<tr>
<td>Max Wing Flapping Angle (deg)</td>
<td></td>
<td>90</td>
<td>50</td>
</tr>
<tr>
<td>Lift-to-Weight Ratio</td>
<td></td>
<td>0.15</td>
<td>2.1</td>
</tr>
</tbody>
</table>

*Predicted from Solidworks model
†Values for half-scale system are predicted by simulation

Table C.2: Free Flight Simulation Controller Gains

<table>
<thead>
<tr>
<th>c</th>
<th>k₁</th>
<th>k₂</th>
<th>λ₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>1</td>
<td>30</td>
<td>0.1</td>
</tr>
<tr>
<td>y</td>
<td>1</td>
<td>30</td>
<td>0.1</td>
</tr>
<tr>
<td>z</td>
<td>10.7</td>
<td>1.9</td>
<td>1.9</td>
</tr>
<tr>
<td>θₓ</td>
<td>15.8</td>
<td>50</td>
<td>0.3</td>
</tr>
<tr>
<td>θᵧ</td>
<td>15.8</td>
<td>25</td>
<td>3</td>
</tr>
</tbody>
</table>

Table C.3: Experimental Controller Gains

<table>
<thead>
<tr>
<th>c₁</th>
<th>k₁</th>
<th>k₂</th>
<th>λ₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ_{rig}</td>
<td>3.3</td>
<td>0.6</td>
<td>0.1</td>
</tr>
<tr>
<td>ψ_{rig}</td>
<td>200</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>φ_{rig}</td>
<td>40</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>
| Added Error Type | x axis | y axis | z axis | Noise with 0.5 st. dev. | No Noise | RMS Tracking Error (cm) with added Gaussian feedback noise | Added Error (cm) | DIV
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No Added Error</td>
<td>1.7 ± 1.2</td>
<td>2.0 ± 1.6</td>
<td>0.9 ± 1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0.1 mN</td>
<td>1.6 ± 1.5</td>
<td>2.0 ± 1.7</td>
<td>0.9 ± 1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.1 mN</td>
<td>1.7 ± 1.5</td>
<td>1.9 ± 1.7</td>
<td>1.0 ± 1.4</td>
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<td></td>
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<tr>
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<td>1.9 ± 1.6</td>
<td>1.0 ± 1.4</td>
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<tr>
<td>-0.07 mN</td>
<td>1.7 ± 1.4</td>
<td>1.9 ± 1.6</td>
<td>1.0 ± 1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>+0.03 mN</td>
<td>1.6 ± 1.4</td>
<td>1.9 ± 1.6</td>
<td>1.0 ± 1.4</td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>-0.03 mN</td>
<td>1.7 ± 1.4</td>
<td>1.9 ± 1.6</td>
<td>1.0 ± 1.4</td>
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Table C.4: Simulation Results with Added Error

<table>
<thead>
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<th>DIV</th>
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<tbody>
<tr>
<td>0 - 2 cm</td>
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<tr>
<td>2 - 4 cm</td>
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</tr>
<tr>
<td>4 - 10 cm</td>
<td></td>
</tr>
<tr>
<td>&gt;10 cm</td>
<td></td>
</tr>
</tbody>
</table>

Performance levels:
- Good performance
- Acceptable performance
- Poor performance
- Unacceptable/divergent
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