Dynamic Manipulation With a One Joint Robot

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Abstract

We are interested in using low degree-of-freedom robots to perform complex manipulation tasks by not grasping. By not grasping, the robot can use rolling, slipping, and free flight to control more degrees-of-freedom of the part. To demonstrate this we study the controllability properties of planar dynamic nonprehensile manipulation. We show that almost any planar object is small-time locally controllable by point contact, and the controlling robot requires only two degrees-of-freedom (a point translating in the plane). We then focus on a one joint manipulator (with a two-dimensional state space) and show that even this simplest of robots, by using slipping and rolling, can control an object to a full-dimensional subset of its six-dimensional state space. We have developed a one joint robot to perform a variety of dynamic tasks, including snatching an object from a table, rolling an object on the surface of the arm, and throwing and catching.

1 Introduction

We are interested in using low degree-of-freedom robots to perform complex manipulation tasks by not grasping (*nonprehensile* manipulation). By not grasping, the robot can use gravitational, centrifugal, and Coriolis forces as virtual motors to control more degrees-of-freedom of the part. The extra motion freedoms of the part are exhibited as rolling, slipping, and free flight. An example is shooting a basketball—the ball is sent to the basket by using rolling and free flight (Figure 1).

One obvious advantage is that we may be able to build cheaper, simpler robots with fewer motors and joints. This comes at the expense of increased complexity in planning and control. Planning for pick and place manipulation requires only a kinematic model of the world; dynamic manipulation requires a dynamic model. In a previous paper (Lynch and Mason [15]) we addressed the planning problem for dynamic nonprehensile manipulation—how to choose manipulator trajectories to achieve the desired motion of the object via nonlinear coupling through the nonprehensile contact.

In this paper we study controllability properties of planar dynamic nonprehensile manipulation. The results parallel our previous results on the controllability of quasistatic pushing (Lynch and Mason [13]), but now we have second-order dynamics and there is no support friction resisting motion of the object. There is also a drift term corresponding to the object's motion when no control force is applied. The problems studied here can be thought of as dynamic pushing—imagine pushing or batting an object floating on an air table (which may be tilted to yield a gravitational acceleration in the plane of the table).

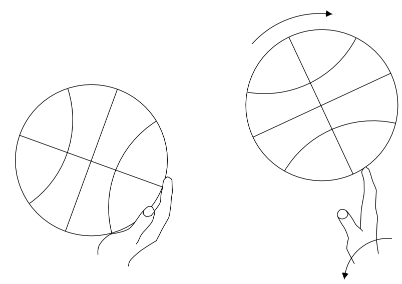

Figure 1: Shooting a basketball using rolling contact.

We begin by assuming no constraints on the motion of the robot, and we show that almost any planar object is small-time locally controllable by pushing with point contact. The controlling robot requires only two degrees-of-freedom (a point translating in the plane). We then focus on a one joint manipulator (with a two-dimensional state space) and show that even this simplest of robots, by using slipping and rolling, can control an object to a full-dimensional subset of its six-dimensional state space. We have developed a one joint robot to perform a variety of dynamic tasks, including snatching an object from a table, rolling an object on the surface of the arm, and throwing and catching.

This work pursues a minimalist approach to robotic manipulation. We are motivated by the academic interest to understand the simplest mechanisms capable of performing a given task and the economic motive to construct simpler, cheaper robots. Simple robots employing dynamic nonprehensile manipulation may be especially effective in industrial parts feeding or in space, where dynamic effects dominate.

Ths paper summarizes work presented in (Lynch [11]).

2 Related Work

Minimalist systems have received increasing attention in the robotics literature. Erdmann [7] studied minimal sensor design based on a task description. Bicchi and Sorrentino [4] demonstrated minimalist dextrous manipulation by rolling an object between two flat palms. Our previous work on 1JOC (1 Joint Over a Conveyor) (Akella *et al.* [1]) is closely related to the work described in this paper. 1JOC uses a sequence of pushes by a single joint robot to position and orient

[1] This research was conducted while the first author was at the Robotics Institute, Carnegie Mellon University.

parts coming down the conveyor in random configurations.

The problems studied in this paper are related to the controllability of a planar free-flying robot with gas jets, a forced planar rigid body (Lewis and Murray [10]), a hovercraft (Manikonda and Krishnaprasad [16]), or an unactuated link of a robot arm (Arai [2]). One distinguishing feature of nonprehensile manipulation is that contact forces are unilateral, while most controllability analyses assume bidirectional controls. An exception is the work by Goodwine and Burdick [8] based on Sussmann's [22] general theorem on local controllability.

Dynamic nonprehensile manipulation is also similar to the control of underactuated manipulators, except the unactuated freedoms are controlled through unilateral frictional contacts. The proof that a one joint robot can control an object to a full-dimensional subset of its state space is closely related to work by Oriolo and Nakamura [21] on the integrability of second-order constraints on underactuated manipulators. Building on previous work on the control of a passive joint, Arai and Khatib [3] demonstrated rolling of a cube on a paddle held by a PUMA, a kind of dynamic nonprehensile manipulation.

3 Definitions and Assumptions

All problems considered in this paper are planar. The planar object $\mathcal{O}$ can be contacted anywhere along its closed, piecewise smooth perimeter Γ. Coulomb friction acts between the robot and the object.

The configuration space of the object is $\mathcal{C} = SE(2) = \mathbf{R}^2 \times S^1$. An object frame $\mathcal{F}_\mathcal{O}$ is fixed to the center of mass of the object. Coordinates in this frame are $(x, y, \phi)^T$. The configuration of $\mathcal{F}_\mathcal{O}$ in the world frame $\mathcal{F}_\mathcal{W}$ is written $\mathbf{q} = (x_w, y_w, \phi_w)^T$. The state space of the object is the tangent bundle $T\mathcal{C} = SE(2) \times \mathbf{R}^3$, and the object's state is given by $(\mathbf{q}, \dot{\mathbf{q}})$. The tangent space at $(\mathbf{q}, \dot{\mathbf{q}})$ is written $T_{(\mathbf{q},\dot{\mathbf{q}})}T\mathcal{C}$.

Generalized forces $\mathbf{f} = (f_x, f_y, \tau)^T$ are written in the object frame $\mathcal{F}_\mathcal{O}$. A *pure force* is a force with a zero torque component ($\tau = 0$) and a *pure torque* is a force with a zero linear component ($f_x = f_y = 0$). A *force direction* $\hat{\mathbf{f}} = (\hat{f}_x, \hat{f}_y, \hat{\tau})^T$ is defined as $\mathbf{f}/|\mathbf{f}|$. The *force sphere* is the two-dimensional unit sphere S^2 of all force directions.

4 Accessibility

This section studies the accessible state space of a planar object $\mathcal{O}$ during dynamic nonprehensile manipulation. The object is *accessible from* $(\mathbf{q}, \dot{\mathbf{q}})$ if the set of states reachable from $(\mathbf{q}, \dot{\mathbf{q}})$ has nonempty interior in the state space. The object is *small-time accessible from* $(\mathbf{q}, \dot{\mathbf{q}})$ if, for any neighborhood U of $(\mathbf{q}, \dot{\mathbf{q}})$, the set of reachable states without leaving U has nonempty interior. The object is *controllable from* $(\mathbf{q}, \dot{\mathbf{q}})$ if, starting from $(\mathbf{q}, \dot{\mathbf{q}})$, the object can reach any point in the state space. The object is *small-time locally controllable from* $(\mathbf{q}, \dot{\mathbf{q}})$ if, for any neighborhood U of $(\mathbf{q}, \dot{\mathbf{q}})$, the set of reachable states without leaving U contains a neighborhood of $(\mathbf{q}, \dot{\mathbf{q}})$.

We begin by examining the case of no constraints on the motion of the manipulator $\mathcal{M}$ which can contact any point on the object's perimeter Γ. With this assumption, we demonstrate necessary and sufficient conditions for the controllability of the object by pushing and batting.

Because of the difficulty of breaking contact and recontacting a moving object, we then include manipulator motion constraints in the analysis. We study the simplest possible case: a single-degree-of-freedom robot which maintains point contact with the object as it moves. We show that a one-degree-of-freedom revolute robot, with just a two-dimensional state space, can take a planar object to a six-dimensional subset of its six-dimensional state space. The equality constraints on the motion of the manipulator (the pivot remains fixed) usually do not translate to equality constraints on the motion of the object.

4.1 No Manipulator Constraints

To visualize the control system with no manipulator constraints, imagine an object floating on an air table which may be tilted to yield a gravitational acceleration in the plane of the table. The object is pushed or batted by a manipulator. Alternatively, the object can be considered to be a planar free-flying rigid body with gas jets attached to its perimeter. The angle the gas jets can take with respect to the normal of the perimeter is determined by the friction coefficient μ.

The control system is written

$$(\dot{\mathbf{q}}, \ddot{\mathbf{q}}) = X_0(\mathbf{q}, \dot{\mathbf{q}}) + uX_i(\mathbf{q}, \dot{\mathbf{q}}), \;\; i \in \{1, \ldots, n\},$$

where X_0 is a drift vector field, $u \in [0, \infty)$ is a nonnegative scalar control, and X_i is the control vector field, where i chooses which control vector field is used. (Note that only one control force is applied at a time, and u must be nonnegative due to unilateral contact.) The vector field X_i, $i \in \{1, \ldots, n\}$, corresponds to a force direction $\hat{\mathbf{f}}_i$ (and resulting acceleration direction $\hat{\mathbf{a}}_i$) fixed in the object frame $\mathcal{F}_\mathcal{O}$. For simplicity, the mass and radius of gyration of $\mathcal{O}$ are assumed to be unit, yielding $\hat{\mathbf{f}}_i = \hat{\mathbf{a}}_i$. The set of force directions $\cup_i \hat{\mathbf{f}}_i$ is denoted $\hat{\mathcal{F}}$. Forces arise from point contact on the perimeter Γ of the object.

The state of $\mathcal{O}$ is $(\mathbf{q}, \dot{\mathbf{q}}) = (x_w, y_w, \phi_w, \dot{x}_w, \dot{y}_w, \dot{\phi}_w)^T$, and the tangent vector is $(\dot{\mathbf{q}}, \ddot{\mathbf{q}}) = (\dot{x}_w, \dot{y}_w, \dot{\phi}_w, \ddot{x}_w, \ddot{y}_w, \ddot{\phi}_w)^T$. The drift field X_0 is written $(\dot{x}_w, \dot{y}_w, \dot{\phi}_w, 0, 0, 0)^T + \mathbf{g}$, where $\mathbf{g}$ is the gravitational acceleration vector $(0, 0, 0, 0, g, 0)^T$. The gravitational acceleration g may be either 1 or 0.

We first consider a single control, $n = 1$. To determine accessibility, we can examine the Lie algebra generated by the vector fields X_0 and X_1. (Recall that a system is small-time accessible at $\mathbf{p}$ if it satisfies the *Lie Algebra Rank Condition*—the vector fields and their Lie brackets evaluated at $\mathbf{p}$ span the tangent space at $\mathbf{p}$ [20].) Without loss of generality, assume the control force applied to the object is $(0, f_y, \tau)^T$ in the object frame $\mathcal{F}_\mathcal{O}$, so the control vector field X_1 is written $(0, 0, 0, -f_y \sin\phi_w, f_y \cos\phi_w, \tau)^T$. We define the Lie bracket vector fields $X_2 = [X_0, X_1]$, $X_3 = [X_1, [X_0, X_1]]$, $X_4 = [X_1, [X_0, [X_0, X_1]]]$, $X_5 = [X_1, [X_1, [X_0, [X_0, X_1]]]]$, $X_6 = [X_0, [X_1, [X_1, [X_0, [X_0, X_1]]]]]$. We find that

$$\det(X_1\ X_2\ X_3\ X_4\ X_5\ X_6) = -16 f_y^4 \tau^8,$$

indicating that these six vector fields span the tangent space $T_{(\mathbf{q},\dot{\mathbf{q}})}T\mathcal{C}$ at any state $(\mathbf{q},\dot{\mathbf{q}})$, provided $f_y \neq 0$ (the control must not be a pure torque) and $\tau \neq 0$ (the control must not be a pure force through the center of mass). Note that a pure torque is not possible by frictional contact with the perimeter of a bounded object.

The tangent vectors $X_1(\mathbf{q},\dot{\mathbf{q}})$, $X_3(\mathbf{q},\dot{\mathbf{q}})$, and $X_5(\mathbf{q},\dot{\mathbf{q}})$ span the acceleration space at $(\mathbf{q},\dot{\mathbf{q}})$, and $X_2(\mathbf{q},\dot{\mathbf{q}})$, $X_4(\mathbf{q},\dot{\mathbf{q}})$, and $X_6(\mathbf{q},\dot{\mathbf{q}})$ span the velocity space.

Proposition 1 *The state of the planar object $\mathcal{O}$ is small-time accessible if and only if $\hat{\mathcal{F}}$ contains a force direction which is neither a pure force nor a pure torque.*

Proposition 1 implies that all planar objects are small-time accessible by point contact, except for a frictionless disk centered at its center of mass. For such an object, all contacts with its perimeter result in a pure force.

Clearly $n = 1$ is never sufficient for controllability; the angular velocity of the object can only change in one direction. It can be shown that $n = 2$ is sufficient for controllability, provided the signs of $\hat{\tau}_1$ and $\hat{\tau}_2$ are opposite.

Proposition 2 *The state of the planar object $\mathcal{O}$ is controllable, with or without gravity, if and only if $\hat{\mathcal{F}}$ contains force directions $\hat{\mathbf{f}}_1$ and $\hat{\mathbf{f}}_2$ such that $\hat{\tau}_1 > 0$, $\hat{\tau}_2 < 0$, and they are not both pure torques.*

Proof: See (Lynch and Mason [14]).

Manikonda and Krishnaprasad [16] proved a similar result for the case $\hat{\mathbf{f}}_1 = -\hat{\mathbf{f}}_2$ (not pure forces or torques).

Proposition 2 implies that any object is controllable by point contact with its perimeter Γ except for a frictionless disk centered at its center of mass. In fact, if friction is nonzero, Proposition 2 implies that the object is controllable from a single point of contact.

Theorem 1 *Any planar object $\mathcal{O}$ with a closed, piecewise smooth curve Γ of available contact points is controllable by pushing at a single point of Γ if the friction coefficient is nonzero.*

Proof: The radius function $r : \Gamma \rightarrow \mathbf{R}$ measures the distance from the center of mass to points γ on the object's perimeter Γ. At each point γ where $dr(\gamma)/d\Gamma = 0$, the contact normal of Γ passes through the center of mass. There are at least two such points because Γ is closed. If $r(\gamma) \neq 0$ and $dr(\gamma)/d\Gamma = 0$, then if the friction coefficient is nonzero, positive and negative torques can be applied from γ. The object is controllable by Proposition 2. □

We now consider conditions for small-time local controllability about a state $(\mathbf{q},\mathbf{0})$. The object $\mathcal{O}$ is small-time locally controllable about $(\mathbf{q},\mathbf{0})$ if, given any neighborhood U of $(\mathbf{q},\mathbf{0})$, $(\mathbf{q},\mathbf{0})$ is interior to the set of states reachable by trajectories remaining in U. This is a stronger condition than global controllability.

Proposition 3 *In the presence of gravity (or other disturbance forces), the planar object $\mathcal{O}$ is small-time locally controllable at all states $(\mathbf{q},\mathbf{0})$ if and only if the set of force directions $\hat{\mathcal{F}}$ positively spans the force sphere. A minimum of four controls ($n \geq 4$) is necessary.*

The proof of Proposition 3 is straightforward and can be found in (Lynch [11]). This is the familiar condition for a "force closure" grasp of a planar object; the difference is that for a grasp, all contacts are simultaneously active. We can directly apply various theorems regarding the existence of positive grasps.

Theorem 2 *Any planar object $\mathcal{O}$ with a closed, piecewise smooth curve Γ of available contact points is small-time locally controllable at all states $(\mathbf{q},\mathbf{0})$, with or without gravity, unless the contact is frictionless and Γ is a circle.*

Proof: See, e.g., (Mishra *et al.* [19]; Markenscoff *et al.* [17]).

We can also apply results concerning the number of fingers (respectively controls) sufficient for force closure (respectively small-time local controllability), with and without friction at the contacts, but they are not listed here because their application is direct. It should be noted, however, that sufficient conditions for a force closure grasp of a spatial object are also sufficient conditions for small-time local controllability of the object.

Tighter sufficient conditions on the set of control forces $\hat{\mathcal{F}}$ can be found for small-time local controllability in zero gravity and for subsets of the zero velocity space in the presence of gravity.

Proposition 4 *The planar object $\mathcal{O}$ is small-time locally controllable about a state $(\mathbf{q},\mathbf{0})$ if the negated gravitational force direction, expressed in the object's frame $\mathcal{F}_{\mathcal{O}}$, is in the interior of $CH_{S^2}(\hat{\mathcal{F}})$, the convex hull of the force directions $\hat{\mathcal{F}}$ on the force sphere.*

If the condition of Proposition 4 is satisfied, $n \geq 3$ and the object is small-time locally controllable on the simply connected three-dimensional subset of its configuration space $\{\mathbf{q} \in \mathcal{C} \mid \phi_{min} < \phi_w < \phi_{max}\}$, for a suitably defined world frame $\mathcal{F}_{\mathcal{W}}$. This is just the angle range for stable equilibrium if all contacts were acting simultaneously.

Proposition 5 addresses small-time local controllability in zero gravity, and is relevant to controlling the position and attitude of a free-flying planar robot with gas jets, a hovercraft with a single rotating thruster (Manikonda and Krishnaprasad [16]), or an unactuated joint of an underactuated manipulator (Arai [2]).

Proposition 5 *In the absence of gravity, the object $\mathcal{O}$ is small-time locally controllable about any state $(\mathbf{q},\mathbf{0})$ if the set of force directions $\hat{\mathcal{F}}$ positively spans a great circle of the force sphere that does not lie in the $\tau = 0$ plane.*

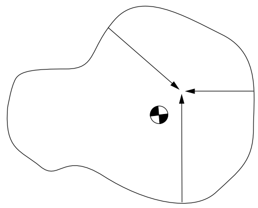

Figure 2: Without gravity, the object is small-time locally controllable at any state $(\mathbf{q}, \mathbf{0})$ by the three force directions shown.

Remark: The condition of Proposition 5 is satisfied by any set of three or more force lines that intersect at a single point, provided the force lines positively span the plane and the intersection point is not at the object's center of mass (see Figure 2).

Proof: We consider the system (Lewis and Murray [10])

$$(\mathbf{q}, \dot{\mathbf{q}}) = X_0(\mathbf{q}, \dot{\mathbf{q}}) + u_1 X_1(\mathbf{q}, \dot{\mathbf{q}}) + u_2 X_2(\mathbf{q}, \dot{\mathbf{q}}),$$

where $u_1, u_2 \in [-1, 1]$, and the bracket terms $X_3 = [X_0, X_1], X_4 = [X_0, X_2], X_5 = [X_1, [X_0, X_2]], X_6 = [X_0, [X_1, [X_0, X_2]]]$.

Now we give some definitions necessary to apply Sussmann's [22] sufficient condition for small-time local controllability. For a bracket term B, we define $\delta_i(B)$ as the number of times X_i appears in B, and the degree of B is $\sum_{i=0}^{n} \delta_i(B)$. B is called a "bad" bracket if $\delta_0(B)$ is odd and $\delta_i(B)$ is even for all $i \in \{1, \ldots, n\}$, and B is a "good" bracket otherwise. A "bad" bracket B is "neutralized" at a state **p** if B, evaluated at **p**, is the linear combination of "good" brackets of lower degree evaluated at **p**. Sussmann proved that if a system satisfies the Lie Algebra Rank Condition at **p** and all "bad" brackets evaluated at **p** are neutralized, then the system is small-time locally controllable at **p**.

Consider the control vector fields $X_1 = (0, 0, 0, \cos\phi_w, \sin\phi_w, 0)^T$ and $X_2 = (0, 0, 0, 0, 0, 1)^T$. The force $\mathbf{f}_1$ acts through the center of mass in the x direction of the object frame $\mathcal{F}_\mathcal{O}$ and $\mathbf{f}_2$ is a pure torque. Calculating the brackets defined above, we find that $\det(X_1\ X_2\ X_3\ X_4\ X_5\ X_6) = 1$; the Lie Algebra Rank Condition is satisfied. Because we only use brackets up to degree four, the only "bad" brackets to be neutralized are the drift field (which vanishes at $\dot{\mathbf{q}} = \mathbf{0}$) and the "bad" brackets of degree three $[X_1, [X_0, X_1]]$ and $[X_2, [X_0, X_2]]$. Here

$$[X_1, [X_0, X_1]] = [X_2, [X_0, X_2]] = (0, 0, 0, 0, 0, 0)^T$$

and the system is small-time locally controllable at states $(\mathbf{q}, \mathbf{0})$. Considering X_1, $-X_1$, X_2, and $-X_2$ as four separate control vector fields with nonnegative controls, the corresponding force directions $\hat{\mathbf{f}}_1$, $-\hat{\mathbf{f}}_1$, $\hat{\mathbf{f}}_2$, and $-\hat{\mathbf{f}}_2$ positively span a great circle of the force sphere orthogonal to the $\tau = 0$ plane. Applying Sussmann's Proposition 2.3 [22] we see that any set of control forces which positively spans the same great circle is sufficient for small-time local controllability.

Now consider a force $\mathbf{f}_2$ in the y direction of the object frame $\mathcal{F}_\mathcal{O}$ with some torque about the center of mass, and its corresponding vector field $X_2 = (0, 0, 0, -\sin\phi_w, \cos\phi_w, \tau)^T$. Then X_i, $i = 1, \ldots, 6$, satisfies the Lie Algebra Rank Condition provided τ is not zero. The "bad" brackets

$$[X_1, [X_0, X_1]] = (0, 0, 0, 0, 0, 0)^T$$
$$[X_2, [X_0, X_2]] = (0, 0, 0, -2\tau\cos\phi_w, -2\tau\sin\phi_w, 0)^T$$

are clearly neutralized (the latter being a multiple of X_1), and the system is small-time locally controllable. The two forces $\mathbf{f}_1$ and $\mathbf{f}_2$ can span any plane which is neither the $\tau = 0$ plane nor orthogonal to the $\tau = 0$ plane. As above, any set of forces which positively spans the same plane also yields small-time local controllability.

Taking the two cases together, we see that small-time local controllability holds provided the set of force directions $\hat{\mathcal{F}}$ positively spans a great circle of the force sphere that does not lie in the $\tau = 0$ plane. □

Proposition 5 allows us to strengthen Theorem 2 for the case of zero gravity.

Theorem 3 *In the absence of gravity, any object* $\mathcal{O}$ *with a closed, piecewise smooth curve* Γ *of available contact points is small-time locally controllable at all states* $(\mathbf{q}, \mathbf{0})$*, unless the contact is frictionless and* Γ *is a circle centered at the object's center of mass.*

Proposition 5 implies that three force directions are sufficient for small-time local controllability in the absence of gravity.[2] In contrast, a force closure grasp requires at least 4 unilateral force directions. Further, we cannot attain a force closure grasp of any frictionless disk, but if the disk is not centered at its center of mass, we have small-time local controllability by point contact pushing. Dynamic pushing is therefore a more complete primitive for planar manipulation.

Theorems 2 and 3 imply that a two-degree-of-freedom robot, such as a point which can translate arbitrarily in the plane, is sufficient to make the object small-time locally controllable on its three-dimensional set of zero velocity states.

4.2 With Manipulator Constraints

The results of the previous section address the theoretical capabilities of dynamic nonprehensile manipulation from the viewpoint of the object alone. Just as important, however, are properties of the manipulator which is controlling the object. While an object may be controllable by point contact, the manipulator may not be able to achieve the contacts and motions necessary to bring the object to the desired state.

Ideally, we would like to understand the global reachability properties of a manipulator/object system—given the kinematic and dynamic specifications of a manipulator $\mathcal{M}$, the mass, radius of gyration, and shape of an object $\mathcal{O}$, the friction between them, and their initial state, where can the manipulator take the object? Unfortunately such a question appears to be very difficult to answer, so we are forced to look

[2] Recent work by Lewis [9] indicates that a single bidirectional force (equivalently, two opposing unilateral forces) is insufficient, meaning that three unilateral force directions are also necessary.

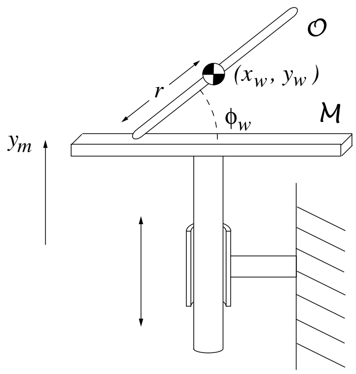

Figure 3: A one-degree-of-freedom prismatic robot manipulating a rod. The rod represents an arbitrary polygon in vertex contact.

locally. Because the contact is nonprehensile, in any neighborhood of a manipulator/object state, the best we can hope for is small-time accessibility in the absence of gravity.

Proposition 1 says that a single control force direction, fixed in the object frame, is sufficient for small-time accessibility. This indicates that a one-degree-of-freedom manipulator may be sufficient for small-time accessibility. This would be an interesting result, as it implies that even the simplest of robots is capable of performing interesting dynamic manipulation in the plane—the object's reachable state space is a full-dimensional subset of its six-dimensional state space.

We examine this possibility for a single prismatic joint and a single revolute joint.[3]

4.2.1 Example 1: Single Prismatic Joint

Consider the system of Figure 3. The manipulator $\mathcal{M}$ is a single prismatic joint, and the object $\mathcal{O}$ is a unit mass rod in point contact with $\mathcal{M}$ at an angle $\phi_w \in (0, \pi)$. The distance from the contact to the rod's center of mass is r and the rod's radius of gyration is ρ. The rod represents an arbitrary polygon in vertex contact with the manipulator.

The configuration of the system is $\mathbf{q}' = (\mathbf{q}, y_m) \in \mathcal{C}'$, where y_m is the position of $\mathcal{M}$ in the world frame $\mathcal{F_W}$. The state space of the system is the eight-dimensional manifold $T\mathcal{C}' = \mathbf{R}^3 \times S^1 \times \mathbf{R}^4$. We assume that the manipulator stays in contact with the rod endpoint at all times, and it may apply zero force (simply "following" the rod) or apply a nonzero force. The three-dimensional submanifold of contact configurations is $\{\mathbf{q}' \in \mathcal{C}' \mid F(\mathbf{q}') = y_w - y_m - r\sin\phi_w = 0\}$.

The conditions that $\mathcal{O}$ remain in contact with $\mathcal{M}$ are

$$\begin{aligned} \frac{dF(\mathbf{q}'(t))}{dt} &= 0 \\ \frac{d^2F(\mathbf{q}'(t))}{dt^2} &= 0. \end{aligned}$$

Erdmann [5, 6] refers to these constraints as the *First and Second Variation Constraints*, respectively. These constraints state that the velocity and acceleration of the system normal to the constraint surface must be zero. For the system of Figure 3, these constraints are written

$$\begin{aligned} \frac{dF(\mathbf{q}'(t))}{dt} &= \dot{y}_w - \dot{y}_m - r\dot{\phi}_w \cos\phi_w = 0 \\ \frac{d^2F(\mathbf{q}'(t))}{dt^2} &= \ddot{y}_w - \ddot{y}_m + r(\dot{\phi}_w^2 \sin\phi_w - \ddot{\phi}_w \cos\phi_w) = 0. \end{aligned} \tag{1}$$

(The state variables' dependence on time is omitted for clarity.) If these equations are satisfied at the initial state for some manipulator control $\ddot{y}_m$, they remain satisfied for any $\ddot{y}_m$ that does not "pull away" from the rod.

Assuming the contact is frictionless, the acceleration of $\mathcal{O}$ must satisfy the constraints

$$\ddot{x}_w = 0 \tag{2}$$

$$\ddot{y}_w r\cos\phi_w + \rho^2\ddot{\phi}_w = 0. \tag{3}$$

Equation (2) constrains the direction of the contact force, and Equation (3) constrains the force to pass through the contact point. Using Equations (1)–(3), we solve for the acceleration of $\mathcal{O}$ as a function of the manipulator control $\ddot{y}_m$ and the system state $(\mathbf{q}', \dot{\mathbf{q}}')$:

$$\begin{aligned} \ddot{x}_w &= 0 \\ \ddot{y}_w &= \rho^2 K \\ \ddot{\phi}_w &= -rK\cos\phi_w, \end{aligned}$$

$$K = \frac{\ddot{y}_m - \dot{\phi}_w^2 r\sin\phi_w}{\rho^2 + r^2\cos^2\phi_w}.$$

Treating K as the control input, we can write the drift vector field $X_0(\mathbf{q}, \dot{\mathbf{q}})$ and the control vector field $X_1(\mathbf{q}, \dot{\mathbf{q}})$ (which are the projections of the vector fields on the system state space $T\mathcal{C}'$ to vector fields on the object's state space $T\mathcal{C}$) as follows:

$$\begin{aligned} X_0(\mathbf{q}, \dot{\mathbf{q}}) &= (\dot{x}_w, \dot{y}_w, \dot{\phi}_w, 0, 0, 0)^T \\ X_1(\mathbf{q}, \dot{\mathbf{q}}) &= (0, 0, 0, 0, \rho^2, -r\cos\phi_w)^T. \end{aligned}$$

The drift field is written without a gravity term, but one can be included in any direction without changing the results.

It is clear that $\mathcal{O}$ is not accessible as Equation (2) integrates to yield the velocity constraint $\dot{x}_w = c_1$ and the position constraint $x_w = c_1 t + c_2$. The situation is (locally) similar with slipping contact and nonzero friction, except the applied force is constrained to act along a friction cone edge instead of the contact normal.

We might ask instead if the rod is small-time accessible on its reduced state space $(y_w, \phi_w, \dot{y}_w, \dot{\phi}_w)$. Constructing the vector fields $X_2 = [X_0, X_1]$, $X_3 = [X_1, [X_0, X_1]]$, and $X_4 = [X_1, [X_0, [X_0, X_1]]]$ and projecting to the reduced state space, we see that these vector fields span if $-4K^6\rho^4 r^4\cos^2\phi_w \sin^2\phi_w \neq 0$. This determinant is zero if $K = 0$ (the manipulator cannot accelerate into the rod), $\rho = 0$ (the rod has zero inertia, so there is no coupling between force and torque), or $r = 0$ (the contact is coincident

[3] Sign errors were propagated through equations in this section in (Lynch [11]).

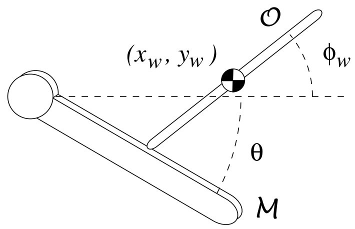

Figure 4: A one joint revolute robot manipulating a rod.

with the object's center of mass). If none of these conditions holds, then we need only consider the cases $\cos\phi_w = 0$ and $\sin\phi_w = 0$. Taking higher order brackets shows that the rod is small-time accessible on its reduced state space unless $\dot{\phi}_w = 0$ and $\cos\phi_w = 0$ (the rod is perpendicular to the robot's surface). In this case, the rod cannot be rotated.

If there is nonzero friction at the contact between the rod and the manipulator, and the contact is not initially slipping, then in general the rod is small-time accessible on its full state space. This derives from the fact that the direction of force applied to the rod, within the friction cone, is a function of the manipulator's acceleration (and the state of the system). This gives control over the *direction* of the applied force, not just the magnitude as in the slipping case. Of course, the rod must undergo both sticking and slipping contact phases for it to be small-time accessible; otherwise the configuration of the rod is confined to a two-dimensional set where the endpoint is pinned to the manipulator.

4.2.2 Example 2: Single Revolute Joint

Now consider the system of Figure 4. The manipulator $\mathcal{M}$ is a single revolute joint, and the object $\mathcal{O}$ is a rod as before. The configuration of the system is $\mathbf{q}' = (\mathbf{q}, \theta) \in \mathcal{C}'$, where θ is the angle of the revolute joint. Assuming the single link is thin, the three-dimensional submanifold of contact configurations is given by $\{\mathbf{q}' \in \mathcal{C}' \mid F(\mathbf{q}') = \cos\theta(y_w - r\sin\phi_w) + \sin\theta(r\cos\phi_w - x_w) = 0\}$. After collecting acceleration, centrifugal, and Coriolis terms, the second variation constraint is written

$$\begin{aligned}\frac{d^2F(\mathbf{q}'(t))}{dt^2} = 0 = &\\ &\ddot{x}_w(-\sin\theta) + \ddot{y}_w(\cos\theta) + \ddot{\phi}_w(-r\cos(\phi_w-\theta)) +\\ &\ddot{\theta}(r\cos(\phi_w-\theta) - x_w\cos\theta - y_w\sin\theta) +\\ &\dot{\phi}_w^2(r\sin(\phi_w-\theta)) +\\ &\dot{\theta}^2(r\sin(\phi_w-\theta) + x_w\sin\theta - y_w\cos\theta) +\\ &\dot{\phi}_w\dot{\theta}(-2r\sin(\phi_w-\theta)) +\\ &\dot{\theta}\dot{x}_w(-2\cos\theta) + \dot{\theta}\dot{y}_w(-2\sin\theta). \qquad (4)\end{aligned}$$

Assuming the contact is frictionless, we have constraints on the direction and point of application of the contact force:

$$\ddot{x}_w\cos\theta + \ddot{y}_w\sin\theta = 0 \qquad (5)$$
$$\ddot{y}_w r\cos(\phi_w-\theta) + \ddot{\phi}_w\rho^2\cos\theta = 0. \qquad (6)$$

Solving Equations (4)–(6) for the acceleration of the object in terms of the control $\ddot{\theta}$ and the system state $(\mathbf{q}', \dot{\mathbf{q}}')$, we get

$$\ddot{x}_w = \rho^2 K\sin\theta \qquad (7)$$
$$\ddot{y}_w = -\rho^2 K\cos\theta \qquad (8)$$
$$\ddot{\phi}_w = rK\cos(\phi_w-\theta), \qquad (9)$$

$$\begin{aligned}K = &(\ddot{\theta}(r\cos(\phi_w-\theta) - x_w\cos\theta - y_w\sin\theta) +\\ &\dot{\phi}_w^2(r\sin(\phi_w-\theta)) +\\ &\dot{\theta}^2(r\sin(\phi_w-\theta) + x_w\sin\theta - y_w\cos\theta) +\\ &\dot{\phi}_w\dot{\theta}(-2r\sin(\phi_w-\theta)) +\\ &\dot{\theta}\dot{x}_w(-2\cos\theta) +\\ &\dot{\theta}\dot{y}_w(-2\sin\theta))/(r^2\cos^2(\phi_w-\theta)+\rho^2).\end{aligned}$$

Equations (7)–(9) have the structure we expect—the force is normal to the manipulator link and the torque about the center of mass of $\mathcal{O}$ depends on the relative angle between the object and the manipulator. The equations also show that even for this simplest of systems, the force is a complex function of the state and control.

Using the object acceleration calculated above, we can again write the drift and control vector fields:

$$\begin{aligned}X_0(\mathbf{q}', \dot{\mathbf{q}}') &= (\dot{x}_w, \dot{y}_w, \dot{\phi}_w, \dot{\theta}, 0, 0, 0, 0)^T\\ X_1(K, \mathbf{q}', \dot{\mathbf{q}}') &= (0, 0, 0, 0, \ddot{x}_w, \ddot{y}_w, \ddot{\phi}_w, \ddot{\theta})^T.\end{aligned}$$

We treat K as the control input, which yields simple forms for $\ddot{x}_w$, $\ddot{y}_w$, and $\ddot{\phi}_w$. The angular acceleration $\ddot{\theta}$ is written in terms of K. Note that X_1 is not linear in K.

Because the acceleration of the object is a function of the manipulator state $(\theta, \dot{\theta})$, not just its acceleration $\ddot{\theta}$, we cannot immediately project the vector fields to vector fields on the object's state space $T\mathcal{C}$ as we did with the prismatic joint. We must look at the vector fields X_0 and X_1, and their Lie brackets, on the full state manifold $T\mathcal{C}'$. After we have constructed the Lie brackets, we can look at their projection to $T\mathcal{C}$ to determine the accessibility of $\mathcal{O}$.

As in the proof of Proposition 1, we construct the vector fields $X_2 = [X_0, X_1]$, $X_3 = [X_1, [X_0, X_1]]$, $X_4 = [X_1, [X_0, [X_0, X_1]]]$, $X_5 = [X_1, [X_1, [X_0, [X_0, X_1]]]]$, $X_6 = [X_0, [X_1, [X_1, [X_0, [X_0, X_1]]]]]$. These vector fields are highly complex trigonometric functions; in fact, X_6 consists of thousands of terms and takes more than a minute to compute using Mathematica running on a Sun SPARC 20. To completely answer the question of small-time accessibility, we must consider even higher order bracket terms. Due to the complexity of the bracket terms, we focus on the particular case of zero velocity states $(\mathbf{q}', \mathbf{0})$.

Evaluated at zero velocity and projected to $T\mathcal{C}$, the vector fields $X_1 \ldots X_6$ take the form $(\mathbf{0}, \mathbf{a})^T$, $(-\mathbf{a}, \mathbf{0})^T$, $(\mathbf{0}, \mathbf{b})^T$, $(-\mathbf{b}, \mathbf{0})^T$, $(\mathbf{0}, \mathbf{c})^T$, $(-\mathbf{c}, \mathbf{0})^T$, where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three-vectors and $\mathbf{0}$ is the zero three-vector. Because of this form, it is sufficient to look at $\det(\mathbf{a}\ \mathbf{b}\ \mathbf{c})$ to determine accessibility. The determinant is $4K^6\rho^4r^2e$, where e is a complex trigonometric function of the configuration of the system. Thus, from a zero velocity state, the rod is small-time accessible unless

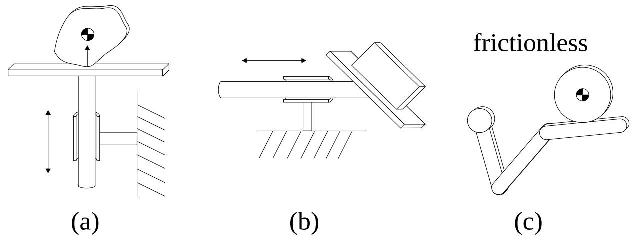

Figure 5: Dynamically singular nonprehensile systems. (a) The system is initially at rest. Any acceleration of the prismatic joint either results in zero force applied to the object, or a force through the center of mass. This dynamic singularity is unstable. (b) If the object is initially at rest in line contact with the manipulator, no acceleration of the manipulator can cause the object to rotate. Even if the robot has two or more prismatic joints, the system may be singular with line contact. (c) If the disk is frictionless and centered at its center of mass, the system is dynamically singular at every state. The angular velocity of the disk cannot be changed.

$K = 0$ (the manipulator cannot apply a force, as when the contact point is at the pivot of the manipulator), $\rho = 0$, $r = 0$, or $e = 0$. Because $e = 0$ defines a lower-dimensional manifold of the configuration space, the rod is small-time accessible from a generic configuration.

We conjecture that by taking higher order Lie brackets, a rod with $\rho \neq 0, r \neq 0$ can be shown to be small-time accessible at all states provided the manipulator can apply a force. Unlike the case of a frictionless prismatic joint, we have accessibility with a frictionless revolute joint because the force angle varies with the joint angle θ, giving the robot some control over the direction of the applied force.

4.2.3 Discussion

The example above shows that even a one-degree-of-freedom revolute robot can take an object to a six-dimensional subset of its state space by using slipping and rolling between the robot and the object. We should therefore be able to do interesting planar dynamic manipulation with even the simplest of robots. The four state equality constraints of the robot link (pivot point is fixed) do not translate to state equality constraints for the object.

The examples above consider edge-vertex contacts only. Smooth objects and manipulators can be handled by modifying the constraints. Smooth surfaces may result in other hindrances to small-time accessibility; for example, a frictionless disk centered at its center of mass can never be small-time accessible by any type of contact.

The system is *dynamically singular* at a system state $(\mathbf{q}', \dot{\mathbf{q}}')$ if the set of motion directions of $\mathcal{O}$ loses rank on $T_{(\mathbf{q},\dot{\mathbf{q}})}T\mathcal{C}$. (The object may still be accessible if the system can break the dynamic singularity at some time T.) Figure 5 gives examples of dynamically singular systems.

In short, the object is not small-time accessible if (1) the linear force direction is fixed in the world frame (as with a frictionless prismatic joint); (2) the force always passes through the object's center of mass (as with a frictionless disk, or if the center of mass of the object is coincident with the contact point); or (3) the inertia of the object is zero, giving no coupling between forces and torques.

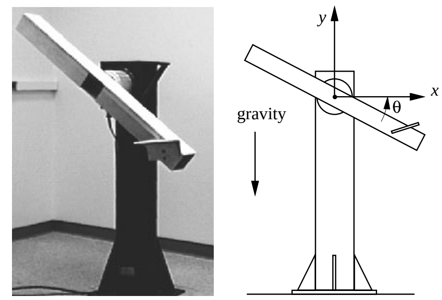

Figure 6: The NSK one joint direct-drive arm.

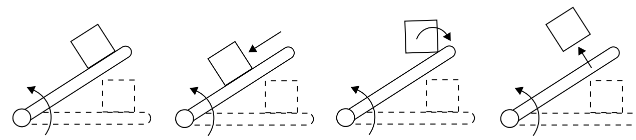

Figure 7: Manipulation phases: dynamic grasp, slip, roll, and free flight.

Although we have shown that the accessible state space may be full dimensional, it appears extremely difficult to calculate the shape of the accessible state space from a given state. Ideally we would have a representation similar to a robot's kinematic workspace.

5 Experimental Setup

To test the manipulation capability of a single joint, we constructed a one joint direct-drive robot to perform experiments in dynamic manipulation (Figure 6). The planning problem is to find an arm trajectory to take the object to a goal state using a sequence of *manipulation phases*, including dynamic grasp, slip, roll, and free flight (Figure 7). (A dynamic grasp is defined as a robot acceleration that keeps the object fixed against it [18].) By sequencing these phases, we can control more degrees-of-freedom of the object.

We have cast the planning problem as a nonlinear optimization. The joint trajectory and manipulation phase switching times are iteratively modified to find a trajectory that takes the object to the goal state using dynamic grasp, roll, and free flight phases (slip is not currently used). Contact friction constraints are also enforced. We have used the planner to solve dynamic tasks such as snatching an object from a table, rolling an object on the surface of the arm, and throwing and catching. The planned trajectories have been successfully implemented on the one joint robot. An example rolling task is shown in Figures 8–10.

Details on the planner and experiments can be found in (Lynch [11]; Lynch and Mason [15]; Lynch and Mason [12]).

6 Conclusion

We have shown that by not grasping, a one joint robot can exploit centrifugal and Coriolis forces to control the motion of a planar part. Compared to conventional pick and place, the complexity of the robot system is transferred from hardware to planning and control. Future work should address the geometry of the object's accessible state space and feedback control of the manipulation trajectories.

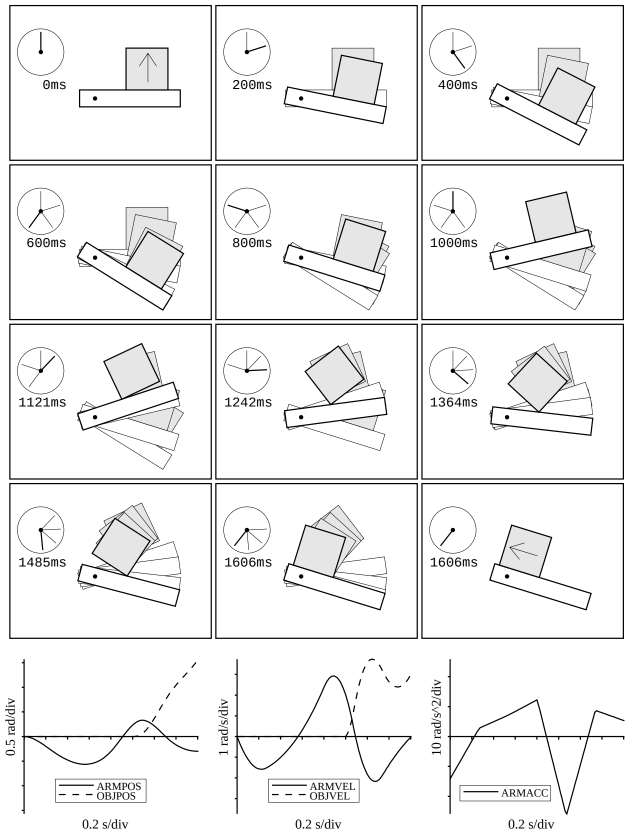

Figure 8: The goal is to roll the square 90 degrees counterclockwise while minimizing the squared impact velocity. The contact friction coefficient is 1.5. The manipulation phases are dynamic grasp, roll, and finally stable equilibrium. The end angle of the arm is limited to make the roll experimentally robust to impact. Otherwise the solution is to end the roll with zero impact velocity and the arm at -45 degrees (with the center of mass balanced over the vertex), which is not robust. Note the windup found by the planner.

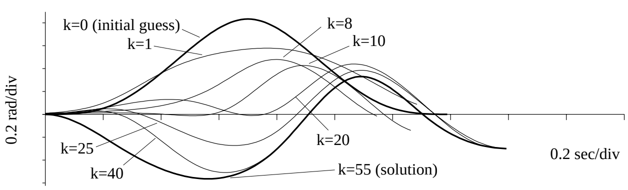

Figure 9: The arm trajectory is represented as a cubic B-spline with nine knot points. Shown here are the initial trajectory guess, the solution, and intermediate iterates.

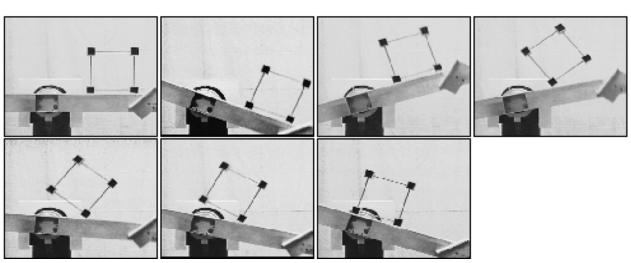

Figure 10: Implementation of the roll on the robot with a wooden 27 cm square frame.

Acknowledgments

This work was funded by NSF under grant IRI-9114208. Thanks to Pradeep Goel and NSK for providing the motor used in the experiments.

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