DYNAMIC MODELING OF MULTIBODY ROBOTIC MECHANISMS:
Incorporating Closed-Chains, Friction, Higher-Pair Joints, and Unactuated and Unsensed Joints*

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ABSTRACT

We introduce a novel formulation for the dynamic modeling of multibody robotic mechanisms to incorporate closed-chains, higher-pair joints, friction, (including stiction, Coulomb, rolling and viscous friction) and unactuated and unsensed joints. Although we have developed this formulation for the dynamic modeling of *Uranus*, an *omnidirectional* wheeled mobile robot (WMR) designed and constructed in the Robotics Institute of Carnegie Mellon University, our methodology is directly applicable to a spectrum of multibody robotic mechanisms. Our methodology is based upon Newtonian dynamics, our kinematic methodology [10], and the concepts of *force/torque propagation* and *frictional coupling* at a joint which we introduce in this paper. Our extensible *matrix-vector* dynamics formulation allows the application of classical methodologies for the solution of systems of linear algebraic equations (e.g., inverse and forward dynamic solutions [13]). To illustrate the procedure, we apply our dynamics formulation to a planar double pendulum and a biped in the frontal plane.

1. Introduction

The Mobile Robot Laboratory of the Robotics Institute of Carnegie Mellon University has designed and built *Uranus*, an omnidirectional wheeled mobile robot, as a testbed to study servo-control, vision, navigation and sensing [9]. A goal of our servo-control research is to determine whether existing manipulator servo-controller designs (e.g., resolved motion rate control [22], computed torque control [7], and robust computed torque control [21]) are applicable to wheeled vehicles. We thus require a dynamic model of Uranus for the design of *dynamics-based* servo-controllers. We have identified the following *five* salient characteristics of WMRs which require special consideration in the dynamic modeling process: (1) Closed-chains; (2) Friction; (3) Higher-pair joints; (4) Unactuated joints; and (5) Unsensed joints.

Conventional stationary manipulators are open-chain mechanisms; whereas the wheels of a WMR form a *closed-chain* when in contact with the surface-of-travel. WMR mobility stems from the translational *friction* between the wheels and the surface-of-travel; whereas friction is oftentimes neglected in comparison with the inertial and gravitational forces/torques of stationary manipulators. Lack of sufficient friction at the wheel point-of-contact leads to wheel slippage, a problem not encountered in stationary manipulator operation. Friction at the wheel bearings also has a significant effect. For a WMR with the kinematic structure of Uranus, the roller bearing frictions in the omnidirectional wheels can dissipate as much as 80% of the total available energy [1], in direct contrast to manipulator bearing friction which is typically small in comparison with the inertial and gravitational forces/torques [18].

All stationary manipulator joints, prismatic and revolute, are lower-pairs. The WMR joint between each wheel and the surface-of-travel is a *higher-pair*. A lower-pair allows a common surface contact between adjacent links providing holonomic (positional) constraints; whereas a higher-pair allows point or line contact providing nonholonomic (velocity) constraints. To control the motion of an open-chain, all of the joints must be actuated and sensed. In contrast, a closed-chain mechanism may be adequately controlled with some joints *unactuated*, and its motions may be adequately discerned with some joints *unsensed* [10]. Moreover, the WMR higher-pair wheel joints do not allow the actuation and sensing of the rotational θ degree-of-freedom of each wheel about the point-of-contact with the surface-of-travel.

Our dynamics methodology, unlike existing dynamics methodologies, is applicable to the modeling of multibody robotic mechanisms exhibiting these five characteristics. We highlight in Section 2 existing dynamics formulations and subsequently introduce in Section 3 our dynamic modeling procedure. To illustrate our dynamic modeling procedure, we apply our dynamics formulation to a planar double pendulum (in Section 4.1) and a biped in the frontal plane (in Section 4.2). The complete development of our dynamics formulation (discussed in Section 3) and the dynamic model of Uranus, including solutions of the dynamic equations-of-motion and extensions of our dynamics methodology, are documented in our companion technical report [11].

2. Existing Dynamics Formulations

The Lagrange [18] and Newton-Euler [6] formulations are the two dynamics methodologies most widely applied to stationary manipulator modeling [14]. Neither the Lagrange nor the Newton-Euler formulations are adequate for WMR modeling. Both model open-chains containing lower-pair joints. Because WMRs are closed-chains the kinematic and dynamic equations-of-motion must be computed in *parallel*, thus disabling the direct application of the recursive Newton-Euler dynamics algorithm. When dry friction is incorporated [18], the frictional force/torque is added to the actuator force/torque for each joint. This dry friction modeling procedure does not generalize to chains containing *unactuated* joints since dry friction at the unactuated joints does not affect the computed actuator forces/torques. The dependence of the dry frictional forces/torques on the normal force is also neglected in such a model. Viscous friction has been

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incorporated for the actuators, but not for the robot links [19]. Finally, application of these dynamics formulations to robot servo-control requires that *all* of the joint positions (angles) and velocities be sensed.

The Lagrange and Newton-Euler formulations and their extensions to closed-chains and nonholonomic systems (e.g., Draganoiu [2], Kane [4], Luh and Zheng [8], Orin and Oh [17] and Wittenburg [23]) are inadequate to achieve our WMR dynamic modeling goals. Although existing formulations model nonholonomic contraints and closed-chains, none address unactuated and unsensed joints, and none are amenable for incorporating dry friction (i.e., stiction, and Coulomb and rolling friction at the wheel point-of-contact and at the bearings) and viscous friction [11].

3. Our Dynamics Formulation

In our companion technical report [11], we develop a dynamics methodology which satisfies the WMR dynamic modeling requirements outlined in Section 1. Our approach is to construct the conceptually complex dynamic robot model from conceptually simple force/torque models by a conceptually simple force/torque manipulation method. Newtonian dynamics form the basis for modeling inertial and gravitational force/torques, and computing the dynamic equations-of-motion. We introduce the method of *force/torque propagation* to compute the effect at coordinate system B (within the robot mechanism) of forces/torques which originate at coordinate system A. We introduce the viewpoint that dry friction (i.e., stiction, Coulomb friction and rolling friction) is a force/torque *coupling* phenomenon in contrast with the conventional view of dry friction as a force/torque source originating at a joint. Newtonian dynamics, force/torque propagation, frictional couplings and our kinematic modeling methodology [10] thus provide the foundation for our dynamic modeling methodology. We formulate the kinematic and dynamic models independently. In this section we expound upon the concepts of force/torque propagation and frictional coupling at a joint and their roles in our dynamic modeling formulation.

Our dynamics formulation is designed for the *simple closed-chain* mechanical system of rigid bodies depicted in Figure 1. Orin and Oh [17] describe a simple closed-chain mechanism as "*one in which the removal of a particular member of the system breaks all closed-chains*".

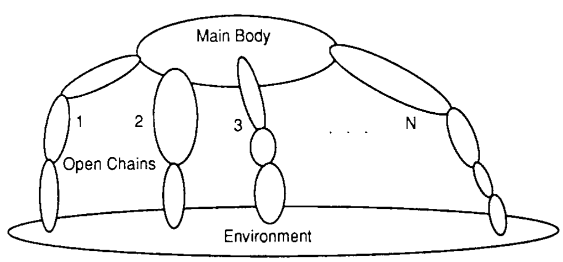

Figure 1: A Simple Closed-Chain Mechanical System of Rigid Bodies

The mechanical system in Figure 1 consists of a main body in contact with N open-chains of rigid bodies. Each pair of adjoining bodies contact at a joint, and the distal rigid body of each open-chain contacts the environment (i.e., a body external to the system). The mechanical configuration in Figure 1 applies to a spectrum of conventional robotic mechanisms [11], including m-DOF robotic manipulators, multi-manipulator systems, WMRs, legged robots, and robotic hands. For each of these robotic mechanisms, *our goal is to formulate the dynamic equations-of-motion of the main body* as a function of the motion (i.e., the positions, velocities and accelerations) of all of the bodies in the system and the actuator and environmental forces/torques.

We utilize the following notation throughout our development: lower case letters denote scalars (e.g., m), lower case bold letters denote vectors (e.g., **f**), upper case letters denote coordinate systems (e.g., A), upper case italics letters denote bodies (e.g., *A*), and upper case bold letters denote matrices (e.g., **M**). Pre-superscripts denote reference coordinate systems. For example, $^{A}\mathbf{f}$ is the vector **f** in the A coordinate system. The pre-superscript may be omitted if the coordinte system is transparent from the context. Post-subscripts denote coordinate systems, bodies, or components of a vector or matrix, as indicated in each application. We place the three force components f_x, f_y, and f_z and the three torque components τ_x, τ_y, and τ_z in the force/torque *six*-vector $\mathbf{f}=\begin{pmatrix} f_x & f_y & f_z & \tau_x & \tau_y & \tau_z \end{pmatrix}^T$. Linear and angular positions, velocities and accelerations are similarly placed in six-vectors with the x, y, and z rotations according to the the roll-pitch-yaw convention [18].

Coordinate systems play a key role in dynamic modeling. We *fix* each coordinate system with a body within the system so that the motion of the coordinate system is exactly that of the body with which it is fixed. Positions $^{A}\mathbf{p}_B$, velocities $^{A}\mathbf{v}_B$, and accelerations $^{A}\mathbf{a}_B$ always denote the motion of the coordinate system B relative to the coordinate system A. To specify the position, velocity or acceleration of body *C*, we thus specify the position $^{A_1}\mathbf{p}_{C_1}$, velocity $^{A_1}\mathbf{v}_{C_1}$, or acceleration $^{A_1}\mathbf{a}_{C_1}$ of coordinate system C_1 which is fixed with body *C* relative to coordinate system A_1 which is fixed with body *A*. An *instantaneously coincident coordinate system* $\bar{X}$ coincides with the coordinate system X but is fixed with the absolute (i.e., stationary) coordinate system at the instant of interest [10]. We assign coordinate systems to joints according to the Sheth-Uicker convention [20] which is applicable to both lower and higher-pairs.

The forces/torques acting on each rigid body within the system originate from inertial, gravitational, actuation, viscous friction, and environmental contact. These forces/torques may be applied at a point (as with actuation and environmental forces/torques) or may be distributed over the mass (as with inertial and gravitational forces/torques) or surface (as with viscous friction forces/torques) of the body; however, these forces/torques are all conventionally modeled as originating at a point. We assign a coordinate system which is fixed with the body and located at that point-of-application as a reference for labeling the forces/torques. The dynamic model of each of these forces/torques is simplified conceptually in this particular *natural* coordinate system. For example, the inertial forces acting on a rigid body are conceptually simple to model in a natural coordinate system located at the center-of-mass of the body and aligned with the principal axes [5]. We utilize the conceptually simple dynamic models of all forces/torques as the modular building blocks for the systematic formulation of the dynamic model of the complex mechanical system in Figure 1.

The dynamic model of the system is then formulated by *propagating* all forces/torques acting on all of the bodies within the system to a common coordinate system. Even though we may model each force/torque at any coordinate system fixed with the body on which the force/torque is acting, the components of the force/torque vector depend upon the location of the coordinate system. We may therefore model a force/torque acting on body *A* at two distinct coordinate systems A_1 and A_2 both fixed with body *A*. The force/torque vector at coordinate systems A_1 and A_2 are then ${}^{A_1}\mathbf{f}_A$ and ${}^{A_2}\mathbf{f}_A$, respectively. The two force/torque vectors describe the same force/torque at different coordinate systems. The force/torque ${}^{A_1}\mathbf{f}_A$ applied at coordinate system A_1 thus has the identical effect on body *A* as the force/torque ${}^{A_2}\mathbf{f}_A$ applied at coordinate system A_2. The force/torque ${}^{A_2}\mathbf{f}_A$, which is a *linear* function of the force/torque ${}^{A_1}\mathbf{f}_A$, is computed according to ${}^{A_2}\mathbf{f}_A = {}^{A_1}\mathbf{L}_{A_2}^T\,{}^{A_1}\mathbf{f}_A$, where the *link Jacobian matrix* ${}^{A_1}\mathbf{L}_{A_2}$ is computed directly from the position six-vector ${}^{A_2}\mathbf{p}_{A_1}$ [11,18]. We refer to computing the force/torque ${}^{A_2}\mathbf{f}_A$ at coordinate system A_2 from the force/torque ${}^{A_1}\mathbf{f}_A$ at coordinate system A_1 as *force/torque propagation.*

Forces/torques propagate through joints according to the *coupling* characteristics of the joint. Forces/torques in directions which do not correspond to joint degrees-of-freedom propagate across the joint as if the joint were a rigid link because no relative motion is possible. Forces/torques aligned with the degrees-of-freedom of the joint propagate across the joint according to the frictional characteristics. For example, Coulomb friction couples a normal force exerted by one body to a force opposing the motion of the contacting body. Since the force/torque due to Coulomb friction would not exist without the normal force, we consider Coulomb friction a coupling phenomenon rather than a force/torque source. Forces/torques ${}^{A_2}\mathbf{f}_A$ on body *A* at joint coordinate system A_2 are coupled through the joint to adjoining body *B* according to ${}^{A_2}\mathbf{f}_B = {}^{A_2}\mathbf{C}_{BA}\,{}^{A_2}\mathbf{f}_A$, where the *coupling matrix* ${}^{A_2}\mathbf{C}_{BA}$ is formulated according to the degrees-of-freedom and the frictional characteristics of the joint [11]. We incorporate friction as an integral component of our dynamics formulation to unify the static and dynamic modeling of forces/torques.

We *cascade* transposed link Jacobian and coupling matrices to propagate force/torques from their *natural* coordinate system A_1 fixed to body *A* to coordinate system Z_2 fixed to body *Z* separated from body *A* by intermediate links and joints. For example, ${}^{Z_2}\mathbf{f}_Z = {}^{Z_2}\mathbf{P}_{A_1}\,{}^{A_1}\mathbf{f}_A$ where the *propagation matrix* ${}^{Z_2}\mathbf{P}_{A_1} = {}^{Z_1}\mathbf{L}_{Z_2}^T\,{}^{Y_2}\mathbf{C}_{ZY}\,{}^{Y_1}\mathbf{L}_{Y_2}^T\,{}^{X_2}\mathbf{C}_{YX} \ldots {}^{A_1}\mathbf{L}_{A_2}^T$. In analogy with the propagation of light and sound waves through a medium, forces/torques originating at a point within a mechanical system propagate their effects throughout the system.

We have implemented the aforementioned force/torque propagation and frictional coupling concepts to formulate a *step-by-step* dynamic modeling procedure for the simple closed-chain multibody mechanical system in Figure 1 [11]. The forces/torques acting on the system are the inertial (i), gravitational (g), actuation (a), viscous friction (v) and environmental (e) forces/torques originating from all of the bodies within the system. We propagate all of these forces/torques to the center-of-mass coordinate system M(*M*) of the main body *M*. We then equate the sum of these propagated forces/torques to zero (according to Newton's equilibrium law) to formulate the six *primary* dynamic equations-of-motion:

$$\sum_X \sum_s \left({}^{M(M)}\mathbf{P}_{N(s,X)}\,{}^{N(s,X)}\mathbf{f}_{sX} \right) = \mathbf{0}\,. \quad (1)$$

In (1), the inner summation is over all force/torque sources (i.e., s = i, g, a, v, and e) and the outer summation is over all bodies *X* within the system, and N(s,*X*) is the natural coordinate system for source s acting on body *X*. The force/torque acting on body X from source s is ${}^{N(s,X)}\mathbf{f}_{sX}$ and the corresponding propagation matrix is ${}^{M(M)}\mathbf{P}_{N(s,X)}$.

The constraints between forces/torques along the degrees-of-freedom of the joint axes are not included in the formulation of the six primary dynamic equations-of-motion in (1). We must, therefore, include these force/torque constraints to complete our dynamic model of the system. We thus formulate N_s *secondary* dynamic equations-of-motion where N_s is the number of joint degrees-of-freedom in the system. The secondary dynamic equations-of-motion for the joint between bodies *A* and *B* at joint coordinate system B_1 are formulated according to:

$$[\mathbf{I}\text{-}{}^{B_1}\mathbf{C}_{BA}\;{}^{B_1}\mathbf{C}_{AB}]\sum_Y \sum_s \left({}^{B_1}\mathbf{P}_{N(s,Y)}\,{}^{N(s,Y)}\mathbf{f}_{sY} \right) = \mathbf{0}. \quad (2)$$

In (2), the outer summation is over all bodies *Y* within the system between the joint and the environment, and the coupling matrix ${}^{B_1}\mathbf{C}_{AB}$ is computed by negating the nondiagonal elements of the coupling matrix ${}^{B_1}\mathbf{C}_{BA}$. From the six equations in (2), we obtain (6-d_{AB}) null equations and d_{AB} secondary force/torque equations-of-motion where d_{AB} is the number of DOFs of the joint between bodies *A* and *B*. We formulate (2) for all of the joints in the system.

The complete dynamic model consisting of the six *primary* and the N_s *secondary* dynamic equations-of-motion in (1) and (2) are *linear* in the actuator and environmental forces/torques and the accelerations of the center-of-mass of all of the system bodies. The velocities and accelerations in the model are referenced to natural instantaneously coincident coordinate systems. We thus apply our kinematic methodology [10] to compute these instantaneously coincident velocities and accelerations from the velocities and accelerations of the joints and the main body. We then substitute *componentwise* these separate kinematic computations into the dynamic equations-of-motion to formulate the closed-form dynamic model [11].

4. Examples

4.1 Planar Double Pendulum

We apply our dynamics formulation to the planar double pendulum (an *open-chain* mechanism) sketched in Figure 2.

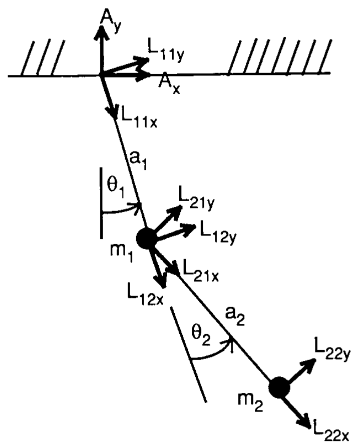

Figure 2: A Planar Double Pendulum

The four position six vectors of the planar double pendulum are:

$${}^{A}\mathbf{p}_{L_{11}}=\left(0\ 0\ 0\ 0\ 0\ (\theta_1-90°)\right)^T \qquad {}^{L_{11}}\mathbf{p}_{L_{12}}=\left(a_1\ 0\ 0\ 0\ 0\ 0\right)^T$$
$${}^{L_{12}}\mathbf{p}_{L_{21}}=\left(0\ 0\ 0\ 0\ 0\ \theta_2\right)^T \text{ and } {}^{L_{21}}\mathbf{p}_{L_{22}}=\left(a_2\ 0\ 0\ 0\ 0\ 0\right)^T;$$

the coupling matrices are

$${}^{A}\mathbf{C}_{EL_1} = {}^{L_{12}}\mathbf{C}_{L_1L_2} = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0 \end{pmatrix};$$

and the nine force/torque vectors are

$${}^{L_{12}}\mathbf{f}_{iL_1} = \left(-m_1\,{}^{L_{12}}a_{L_{12}x}\quad -m_1\,{}^{L_{12}}a_{L_{12}y}\quad 0\quad 0\quad 0\quad 0\right)^T$$
$${}^{L_{22}}\mathbf{f}_{iL_2} = \left(-m_1\,{}^{L_{22}}a_{L_{22}x}\quad -m_1\,{}^{L_{22}}a_{L_{22}y}\quad 0\quad 0\quad 0\quad 0\right)^T$$
$${}^{L_{12}}\mathbf{f}_{gL_1}=\left(0\ -m_1g\ 0\ 0\ 0\ 0\right)^T \qquad {}^{L_{22}}\mathbf{f}_{gL_2}=\left(0\ -m_2g\ 0\ 0\ 0\ 0\right)^T$$
$${}^{A}\mathbf{f}_{eL_1}=\left(f_x\ f_y\ 0\ 0\ 0\ \tau_z\right)^T \qquad {}^{L_{11}}\mathbf{f}_{aL_1}=-{}^{A}\mathbf{f}_{aL_1}=\left(0\ 0\ 0\ 0\ 0\ \tau_1\right)^T$$
$$\text{and} \qquad {}^{L_{21}}\mathbf{f}_{aL_2}=-{}^{L_{21}}\mathbf{f}_{aL_2}=\left(0\ 0\ 0\ 0\ 0\ \tau_2\right)^T.$$

The constants m_1 and m_2 are the masses at the ends of links L_1 and L_2, respectively, g is the gravitational constant, and τ_1 and τ_2 are the actuator torques applied at the bases of links L_1 and L_2, respectively. Each actuator produces a torque on the link it is driving and an equal and opposite reactional torque on the link to which it is mounted. We obtain the six primary dynamic equations-of-motion by propagating all of the forces/torques to the end of the pendulum (coordinate system L_{22}), and one secondary equation-of-motion at each joint; i.e., the coordinate systems L_{12} and A. The six primary dynamic equations-of-motion lead to the three non-trivial scalar dynamic equations-of-motion:

$$-c_2m_1\,{}^{L_{12}}a_{L_{12}x}-s_2m_1\,{}^{L_{12}}a_{L_{12}y}-m_2\,{}^{L_{22}}a_{L_{22}x} +c_{12}m_1g+c_{12}m_2g+s_{12}f_x-c_{12}f_y = 0 \quad (3)$$

$$s_2m_1\,{}^{L_{12}}a_{L_{12}x}-c_2m_1\,{}^{L_{12}}a_{L_{12}y}-m_2\,{}^{L_{22}}a_{L_{22}y} -s_{12}m_1g-s_{12}m_2g+c_{12}f_x+s_{12}f_y = 0 \quad (4)$$

$$-a_2s_2m_1\,{}^{L_{12}}a_{L_{12}x}+a_2c_2m_1\,{}^{L_{12}}a_{L_{12}y} +a_2s_{12}m_1g+\tau_2-a_2c_{12}f_x-a_2s_{12}f_y = 0 \quad (5)$$

where $s_i = \sin(\theta_i)$, $c_i = \cos(\theta_i)$, $s_{ij} = \sin(\theta_i+\theta_j)$, and $c_{ij} = \cos(\theta_i+\theta_j)$. The two secondary dynamic equations are:

$$\tau_1-\tau_2-a_1c_1f_x-a_1s_1f_y = 0 \quad (6)$$

$$-\tau_1+\tau_z = 0\ . \quad (7)$$

The four acceleration equations required to complete (3)-(7) are:

$${}^{L_{12}}a_{L_{12}x} = -a_1\omega_1^2 \quad (8)$$

$${}^{L_{12}}a_{L_{12}y} = a_1\alpha_1 \quad (9)$$

$${}^{L_{22}}a_{L_{22}x} = -a_1c_2\omega_1^2 + a_1s_2\alpha_1 - a_2(\omega_1+\omega_2)^2 \quad (10)$$

$${}^{L_{22}}a_{L_{22}y} = a_1s_2\omega_1^2 + a_1c_2\alpha_1 + a_2(\alpha_1+\alpha_2)\ . \quad (11)$$

In (8)-(11), ω_1 and ω_2 are the angular velocities and α_1 and α_2 are the angular accelerations of the joints. We solve (3)-(11) for the two joint torques τ_1 and τ_2 and obtain the classical inverse dynamic model of the planar double pendulum [12].

4.2 Biped in the Frontal Plane

We next apply our dynamics formulation to the biped in the frontal plane (a *closed-chain* mechanism) depicted in Figure 3 [3]. The twelve position six-vectors of the biped are:

$${}^{A}\mathbf{p}_{A_1}=\left(-d_1/2\ 0\ 0\ 0\ 0\ 0\right)^T \qquad {}^{A_1}\mathbf{p}_{L_{11}}=\left(0\ 0\ 0\ 0\ 0\ -\theta_1\right)^T$$
$${}^{L_{11}}\mathbf{p}_{C_1}=\left(0\ k_1\ 0\ 0\ 0\ 0\right)^T \qquad {}^{C_1}\mathbf{p}_{L_{12}}=\left(0\ (l_1-k_1)\ 0\ 0\ 0\ 0\right)^T$$
$${}^{L_{12}}\mathbf{p}_{L_{21}}=\left(0\ 0\ 0\ 0\ 0\ (\theta_1-\theta_2)\right)^T \qquad {}^{L_{21}}\mathbf{p}_{C_2}=\left(d_2\ k_2\ 0\ 0\ 0\ 0\right)^T$$
$${}^{A}\mathbf{p}_{A_3}=\left(d_1/2\ 0\ 0\ 0\ 0\ 0\right)^T \qquad {}^{A_3}\mathbf{p}_{L_{32}}=\left(0\ 0\ 0\ 0\ 0\ (180°-\theta_3)\right)^T$$
$${}^{L_{32}}\mathbf{p}_{C_3}=\left(0\ k_3\ 0\ 0\ 0\ 0\right)^T \qquad {}^{C_3}\mathbf{p}_{L_{31}}=\left(0\ (l_3-k_3)\ 0\ 0\ 0\ 0\right)^T$$
$${}^{L_{31}}\mathbf{p}_{L_{32}}=\left(0\ 0\ 0\ 0\ 0\ (\theta_3-\theta_2-180°)\right)^T$$
$${}^{L_{22}}\mathbf{p}_{C_2}=\left(-d_2\ k_2\ 0\ 0\ 0\ 0\right)^T.$$

The coupling matrices are:

$${}^{A_1}\mathbf{C}_{EL_1} = {}^{L_{12}}\mathbf{C}_{L_1L_2} = {}^{L_{31}}\mathbf{C}_{L_2L_3} = {}^{A_3}\mathbf{C}_{EL_3} = \begin{pmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

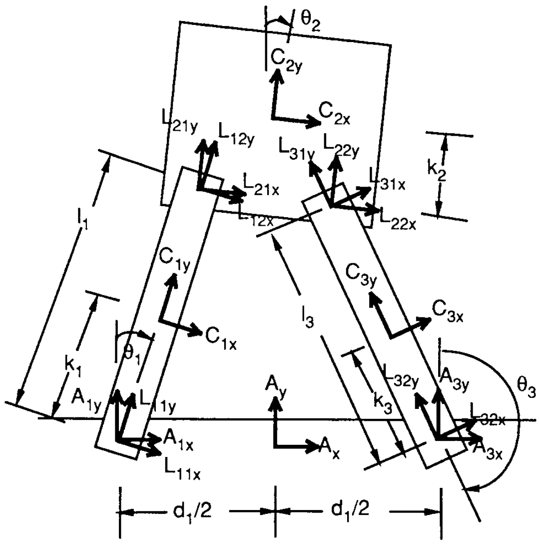

Figure 3: Biped in the Frontal Plane

And the sixteen force/torque vectors are:

$$
{}^{C_1}\mathbf{f}_{iL_1} = \begin{pmatrix} -m_1\,{}^{\bar{C}_1}a_{C_1x} \\ -m_1\,{}^{\bar{C}_1}a_{C_1y} \\ -m_1\,{}^{\bar{C}_1}a_{C_1z} \\ 0 \\ 0 \\ -I_1\,{}^{\bar{C}_1}\alpha_{C_1z} \end{pmatrix} \qquad {}^{C_2}\mathbf{f}_{iL_2} = \begin{pmatrix} -m_2\,{}^{\bar{C}_2}a_{C_2x} \\ -m_2\,{}^{\bar{C}_2}a_{C_2y} \\ -m_2\,{}^{\bar{C}_2}a_{C_2z} \\ 0 \\ 0 \\ -I_2\,{}^{\bar{C}_2}\alpha_{C_2z} \end{pmatrix}
$$

$$
{}^{C_3}\mathbf{f}_{iL_3} = \begin{pmatrix} -m_3\,{}^{\bar{C}_3}a_{C_3x} \\ -m_3\,{}^{\bar{C}_3}a_{C_3y} \\ -m_3\,{}^{\bar{C}_3}a_{C_3z} \\ 0 \\ 0 \\ -I_3\,{}^{\bar{C}_3}\alpha_{C_3z} \end{pmatrix}
$$

$$
{}^{G(L_1)}\mathbf{f}_{gL_1} = \begin{pmatrix} 0 & -m_1 g & 0 & 0 & 0 & 0 \end{pmatrix}^T
$$

$$
{}^{G(L_2)}\mathbf{f}_{gL_2} = \begin{pmatrix} 0 & -m_2 g & 0 & 0 & 0 & 0 \end{pmatrix}^T
$$

$$
{}^{G(L_3)}\mathbf{f}_{gL_3} = \begin{pmatrix} 0 & -m_3 g & 0 & 0 & 0 & 0 \end{pmatrix}^T
$$

$$
{}^{A_1}\mathbf{f}_{eL_1} = \begin{pmatrix} F_1 & G_1 & 0 & 0 & 0 & H_1 \end{pmatrix}^T
$$

$$
{}^{A_3}\mathbf{f}_{eL_3} = \begin{pmatrix} -F_4 & -G_4 & 0 & 0 & 0 & H_4 \end{pmatrix}^T
$$

$$
{}^{A_1}\mathbf{f}_{aE} = -{}^{L_{11}}\mathbf{f}_{aL_1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & u_1 \end{pmatrix}^T
$$

$$
{}^{L_{12}}\mathbf{f}_{aL_1} = -{}^{L_{21}}\mathbf{f}_{aL_2} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & u_2 \end{pmatrix}^T
$$

$$
{}^{L_{22}}\mathbf{f}_{aL_2} = -{}^{L_{31}}\mathbf{f}_{aL_3} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & u_3 \end{pmatrix}^T
$$

$$
{}^{L_{32}}\mathbf{f}_{aL_3} = -{}^{A_3}\mathbf{f}_{aE} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & u_4 \end{pmatrix}^T.
$$

The principal moment of inertia of body i about the z-axis is I_i, the joint actuator torques are u_i, the environmental forces in the x and y directions are F_i and G_i, respectively, and the environmental torque about the z-axis is H_i. The coordinate systems $G(L_i)$ for i=1,2,3 which are not explicitly drawn in Figure 3 are gravitational coordinate systems located at the center-of-mass of body L_i and aligned with the gravitational field.

We obtain the six primary dynamic equations-of-motion by propagating the sixteen forces/torques to the center-of-mass of the biped body (coordinate system C_2), and one secondary equation-of-motion at each of the four joints;i.e., at the four coordinate systems X = A_1, L_{12}, L_{31}, and A_3. We then substitute the velocities and accelerations relative to the instantaneously coincident coordinate systems (computed from the joint velocities and accelerations) into the dynamic equations-of-motion. Finally, we apply the two positional geometries

$$
l_1 s_1 + 2d_2 c_2 + l_3 s_3 = d_1 \quad \text{and} \quad l_1 c_1 - 2d_2 s_2 + l_3 c_3 = 0
$$

and their derivatives to obtain the Hemami and Wyman dynamic equations-of-motion for the biped in the frontal plane [3].

5. Concluding Remarks and Further Research

We have designed a dynamics formulation to incorporate the special characteristics of WMRs [11]. We model the dynamics of simple *closed chains* by propagating all forces/torques within the system to a common coordinate system. We incorporate dry *frictional* coupling at a joint by introducing coupling matrices. The coupling matrix for a joint contains ones along the diagonal corresponding to joint degrees-of-freedom, and frictional coefficients off-of-the diagonal corresponding to dry friction couplings. *Higher pair joints* are modeled by applying the Sheth-Uicker convention [20] and instantaneously coincident coordinate systems [10]. Actuator forces/torques are incorporated in the dynamic model along with the inertial, gravitational, viscous frictional and environmental contact forces/torques. We thereby model an *unactuated joint* by the absence of an actuator force/torque. We apply our kinematic modeling methodology [10] to compute the *unsensed joint* velocities and accelerations from the sensed joint velocities and accelerations, and therby formulate the dynamic model from sensed joint velocities and accelerations.

We have introduced a dynamics formulation which is conceptually simple to apply. Our matrix-vector dynamics formulation permeated by sparce matrices provides a conceptual framework for the design of computationally efficient algorithms. For example, servo-control algorithms emanating from our dynamics formulation may be designed for efficient computation by eliminating all scalar additions and multiplications by zero and multiplications by plus and minus one [15-16]. Because the dynamic equations are based upon matrix-vector products, the servo-control algorithms may be most amenable to direct implementation on parallel and vector (array) processors.

We are continuing our study of WMR modeling and servo-control. We are applying our kinematic [10] and dynamic WMR models to evaluate servo-control algorithms for real-time WMR trajectory tracking. We are applying the forward dynamic solution to simulate the WMR motion and control the simulation by implementing the inverse kinematic and dynamic solutions with resolved motion rate [22], computed torque [7], and robust computed torque [21] servo-control algorithms. Extensions to our dynamics formulation include: the modeling of flexible links; the modeling of such material effects as stress, strain, fracture, and wear; and the modeling of complex closed-chains.

6. References

[1] D. J. Daniel, "Analysis, Design, and Implementation of Microprocessor Control for a Mobile Platform," Master's Project Report, Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, August 1984.

[2] G. Draganoiu, "Computer Method for Setting Dynamical Model of an Industrial Robot with Closed Kinematic Chains," *Proceedings of the Twelfth International Symposium on Industrial Robots*, Paris, France, June 1982, pp. 371-379.

[3] H. Hemami and B. F. Wyman, "Modeling and Control of Constrained Dynamic Systems with Application to Biped Locomotion in the Frontal Plane," *IEEE Transactions on Automatic Control*, Vol. AC-24, No. 4, August 1979, pp. 526-535.

[4] T. R. Kane, "Dynamics of Nonholonomic Systems," *Journal of Applied Mechanics*, Vol. 83, No. 4, December 1961, pp. 574-578.

[5] L. D. Landau and E. M. Lifshitz, *Mechanics*, Third Edition, Pergamon Press, New York, 1976.

[6] J. Y. S. Luh, M. W. Walker, and R. P. C. Paul, "On-Line Computational Scheme for Mechanical Manipulators," *Journal of Dynamic Systems, Measurement, and Control*, Vol. 102, No. 2, June 1980, pp. 69-76.

[7] J. Y. S. Luh, M. W. Walker, and R. P. C. Paul, "Resolved Acceleration Control of Mechanical Manipulators," *IEEE Transactions on Automatic Control*, Vol. AC-25, No. 3, June 1980, pp. 468-474.

[8] J. Y. S. Luh and Y. Zheng, "Computation of Input Generalized Forces for Robots with Closed Kinematic Chain Mechanism," *IEEE Journal of Robotics and Automation*, Vol. RA-1, No. 2, June 1985, pp. 95-103.

[9] H. P. Moravec (editor), "Autonomous Mobile Robots Annual Report - 1985," Robotics Institute Technical Report No. CMU-RI-MRL-86-1, Carnegie Mellon University, Pittsburgh, PA 15213, January 1986.

[10] P. F. Muir and C. P. Neuman, "Kinematic Modeling of Wheeled Mobile Robots," *Journal of Robotic Systems*, Vol. 4, No. 2, April 1987, pp. 281-340.

[11] P. F. Muir and C. P. Neuman, "Dynamic Modeling of Wheeled Mobile Robots," Robotics Institute Technical Report No. CMU-RI-TR-87-10, Carnegie Mellon University, Pittsburgh, PA 15213, February 1988.

[12] J. J. Murray, *Computational Robot Dynamics*, PhD Dissertation, Electrical and Computer Engineering Department, Carnegie Mellon University, Pittsburgh, PA 15213, September 1986.

[13] C. P. Neuman and P. F. Muir, "From Linear Algebraic Equations to Wheeled Mobile Robot Modeling," in K. S. Narendra (editor), *Proceedings of the Fifth Yale Workshop on Applications of Adaptive Systems Theory*, Yale University, New Haven, CT, May 1987.

[14] C. P. Neuman and J. J. Murray, "Computational Robot Dynamics: Foundations and Applications," *Journal of Robotic Systems*, Vol. 2, No. 4, Winter 1985, pp. 425-452.

[15] C. P. Neuman and J. J. Murray, "Customized Computational Robot Dynamics," *Journal of Robotic Systems*, Vol. 4, No. 4, August 1987, pp. 503-526.

[16] C.P. Neuman and J. J. Murray, "Symbolically Efficient Formulations for Computational Robot Dynamics," *Journal of Robotic Systems*, Vol. 4, No. 6, December 1987.

[17] D. E. Orin and S. Y. Oh, "Control of Force Distribution in Robotic Mechanisms Containing Closed Kinematic Chains," *Journal of Dynamic Systems, Measurement, and Control*, Vol. 102, No. 2, June 1981, pp. 134-141.

[18] R. P. Paul, *Robot Manipulators: Mathematics, Programming and Control*, The MIT Press, Cambridge MA, 1981.

[19] M. S. Pfeifer and C. P. Neuman, "An Adaptable Simulator for Robot Arm Dynamics," *Computers in Mechanical Engineering*, Vol. 3, No. 3, November 1984, pp. 57-64.

[20] P. N. Sheth and J. J. Uicker, Jr., "A Generalized Symbolic Notation for Mechanisms," *Journal of Engineering for Industry*, Series B, Vol. 93, No. 70-Mech-19, 1971, pp. 102-112.

[21] V.D. Tourassis and C. P. Neuman, "Robust Nonlinear Feedback Control for Robotic Manipulators," *IEE Proceedings - D: Control Theory and Applications*, Special Issue on Robotics, Vol. 132, No. 4, July 1985, pp. 134-143.

[22] D. E. Whitney, "Resolved Motion Rate Control of Manipulators and Human Protheses," *IEEE Transactions on Man-Machine Systems*, Vol. MMS-10, No. 2, June 1969, pp. 47-53.

[23] J. Wittenburg, *Dynamics of Systems of Rigid Bodies*, B. G. Teuber Stuttgart, 1977.