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Dead Reckoning Navigation for Walking Robots

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ABSTRACT: Autonomous and teleoperated mobile robots require an accurate knowledge of their spatial location in order to accomplish many tasks. Many mobile robots make use of dead reckoning navigation because of its simplicity, low cost and robustness. Although dead reckoning navigation has been used for centuries for ships and wheeled vehicles, the application to a walking machine is novel. Since walking machines differ greatly from ships and wheeled vehicles, a new approach to dead reckoning was developed to solve this problem. This paper discusses the problem, a solution, preliminary test results and future goals for dead reckoning navigation. Experiments were done with the Ambler, an autonomous, six-legged walking robot, but the results are general and apply to any statically stable walking robot. The current results show a systematic bias of two percent of body advance in the direction of travel. Although the cause of this bias is unknown, it is corrected in the position estimation routines.

I. Introduction

Autonomous and teleoperated mobile robots require an accurate knowledge of their spatial location in order to accomplish many tasks. Many mobile robots make use of dead reckoning navigation because of its simplicity, low cost and robustness. Although dead reckoning navigation has been used for centuries for ships and wheeled vehicles, the application to a walking machine is novel. Since walking machines differ greatly from ships and wheeled vehicles, a new approach to dead reckoning was developed to solve this problem. This paper discusses the problem, a solution, preliminary test results and future goals for dead reckoning navigation. Experiments were done with the Ambler, an autonomous, six-legged walking robot, but the results are general and apply to any statically stable walking robot.

Dead reckoning is defined as:

> The determination, without the aid of external observations, of the position and orientation of a vehicle from the record of the courses travelled, the distance made, and the known estimated drift. [1]

A vehicle, starting from an initial position, P_0, travels along a path segment defined by d_1 and θ_1, see Fig. 1. The new position, P_1, can be *estimated* from the course and distance travelled. The accuracy of the location is dependent on the accuracy of the data, since there is no way to *verify* the new location. Therefore, the proper representation of the new location is a position with an associated uncertainty, which is represented as a circle in Fig. 1. Furthermore, at each subsequent location, the positional uncertainty increases.

The implementation of dead reckoning for a vehicle such as a car (travelling over smooth terrain), boat or plane is straight forward. The instrumentation required is a speedometer, a compass and a clock (or alternatively an odometer and a compass). The drift can be estimated by the known precision and accuracy of the devices and an estimation of non-measurable displacement, such as ocean currents. For a car, assuming the initial heading is known, the compass can be eliminated by keeping track of the change in heading of the vehicle. This technique has been used by mariners for centuries when stellar navigation was rendered impossible by weather conditions. The technique is used in modern times by competitors in automobile road-rallies and the sport of orienteering.

An important limitation of dead reckoning navigation, is that it can not account for changes in the vehicle's position which do not affect its measuring devices. For instance, if a vehicle were picked up from one location and placed down in another, dead reckoning would not report a change in position. This limitation does not hold for systems which require external observations, since any change in position is readily apparent.

Although prone to error and generally not as accurate as other methods of navigation, dead reckoning can be more robust than other navigators since it will continue to function when others fail. Furthermore, dead reckoning navigation is less expensive, typically by several orders of magnitude, than other navigational devices. Although in certain special instances dead reckoning navigation alone may provide sufficient navigational accuracy, in general, it should be combined with other navigational techniques to provide greater accuracy. See Section VI for further details.

For the reasons elucidated above, dead reckoning navigation was implemented on the Ambler, an autonomous, six-legged walking robot for planetary exploration. Fig. 2 is a diagram of the Ambler. References [2] and [12] provide a description of the machine, its capabilities and dynamics.

Since wheeled vehicles maintain continuous contact with the ground, the distance the vehicle travels can be calculated from the number of wheel revolutions. (This assumes benign terrain. In rough terrain, the body displacement can be estimated by averaging the rotations of all wheels, however this is typically not

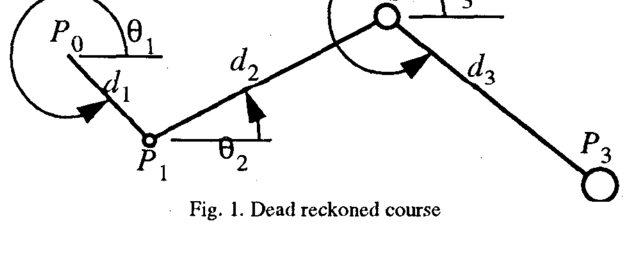

Fig. 1. Dead reckoned course

[1] Modified from Webster's Dictionary, which restricts the definition to the position of ships and planes.

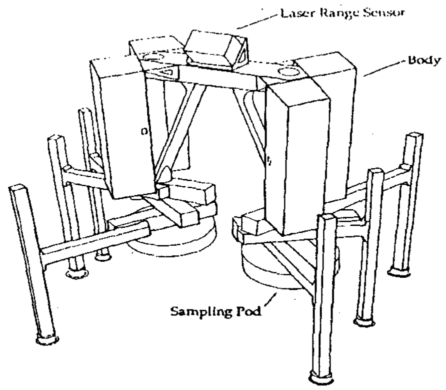

Fig. 2. The Ambler

very accurate.) However, when the Ambler body moves, the feet remain stationary (see discussion in Section VI), therefore, there is no direct means of measuring the Ambler's displacement. This necessitates a new methodology for dead reckoning estimation. Furthermore, the authors claim that this new methodology works equally well on benign terrain as on severe terrain, and that the dead reckoning for this class of walking machines will be more accurate than wheeled machines over severe terrain.

This paper is organized as follows: Section II describes dead reckoning navigation that has been previously implemented for other robotic systems. Section III presents a formal statement of the problem. Section IV describes the approach implemented. Section V presents some test results. The final sections discuss future work, improvements and conclusions.

II. Previous Work

Since few mobile robots exist, there are few published articles pertaining to the use of dead reckoning navigation for this type of system. The paper by Smith and Cheeseman, [16], deals with a robot on a planar surface and discusses the effects of the accumulation of error. This paper assumes some external sensing devices, such as acoustic sensors or cameras. Similarly, the papers by Watanabe and Shin'ichi, [19], and Crowley, [4], assume external sensors as well as dead reckoned position information.

The paper by Amidi [1], provides a discussion of an implementation of dead reckoning navigation for wheeled vehicles, and its actual implementation on the NavLab. The paper presents equations for determining the dead reckoned position of a wheeled robot on benign terrain. The accuracy of the model was determined by comparing dead reckoned position estimates with positioning information obtained from a Global Positioning Satellite system. In addition, there is a discussion of sensor fusion for improving overall system navigation.

The author is not aware of any work discussing dead reckoning navigation for legged vehicles. This may be due in part to the very small number of legged vehicles in existence today.

III. Problem Statement

The problem is to determine the position and orientation of a walking robot at any given time. To do this a coordinate system, called the *body frame*, will be assigned to the robot. Another coordinate system, called the *world frame*, will be assigned to a frame of reference. Points in one frame are mapped into the other frame by a translation (by a vector **T**) and a rotation (by a matrix **R**). Fig. 3 below shows the world frame, the body frame and a point, *j*, fixed in the body frame.

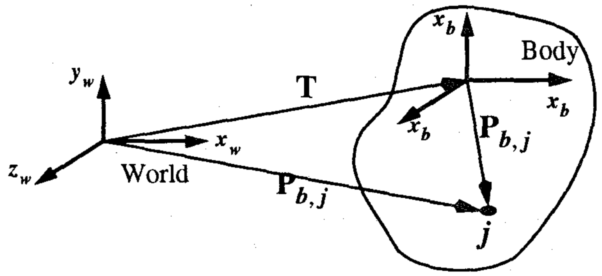

Fig. 3. Relation between world and body frames

If the position of a point fixed on a rigid body and orientation of that body were known for all time in some frame, then the position of any other point on that body, with respect to the specified frame, is readily calculated for all time. But this will not work for the Ambler for several reasons: First, the Ambler has automatic body leveling, which permits continual reflexive changes in body orientation. Second, structural deformation will also lead to unmodellable changes in orientation. Third, the technique used to set a foot on the ground causes unintentional changes in body orientation. Fourth, small soil deformations can also cause unreported changes in body orientation as well as in foot position. (See the discussion in Section VI.) These reasons preclude the possibility of storing body orientation and using a simple scheme based solely on the addition of displacements from the initial configuration. Instead, only the positions of the feet are stored and are used as the basis for position estimation.

To solve the dead reckoning problem, it is assumed that at any given time, the Ambler can be treated as a rigid body (in this work, the effects of structural distortion are ignored, see Section V and Section VI for further discussion), which means that the feet do not move unless commanded to do so. With these assumptions, the dead reckoning problem becomes one of finding the rigid body transformation (the matrix **R** and the vector **T**) which satisfies:

$$\mathbf{P}_{w,j} = \mathbf{R}\,\mathbf{P}_{b,j} + \mathbf{T} \qquad (1)$$

where, $\mathbf{P}_{wj}$ is the position of foot *j* in the world frame, and $\mathbf{P}_{bj}$ is the position of foot *j* in the body frame. Both $\mathbf{P}_{wj}$ and $\mathbf{P}_{bj}$ are known at all times since the former are stored and the latter can be calculated from the mechanism kinematics. Since measured data is imperfect and models are not exact, a solution is sought which minimizes the squared error, $q(\mathbf{R},\mathbf{T})$, where:

$$q(\mathbf{R},\mathbf{T}) = \sum_j w_j \mathbf{e}_j^T \mathbf{e}_j \qquad (2)$$

with

$$e = P_{w,j} - R\,P_{b,j} - T \quad (3)$$

where w_j are weights which are related to the observations.

To specify the position and orientation of a rigid body in some frame, the position of three, non-colinear points fixed in the rigid body must be known in that frame. If fewer than three points were known, the problem is insoluble. (If only one point were known, the body would be free to rotate about that point. If only two points were known, the body would be free to rotate about the line defined by the two points.) If more than three points are specified, the problem is over-constrained. In the case of the Ambler, six points, the positions of the feet, are known, thereby over-constraining the problem. One approach to this over-constrained problem would be to discard the extra data, but ignoring valid information does not seem productive. Another approach is to use a method which allows weighting the data according to its reliability. The weights are the w_j which appear in (2). Several methods for doing this are discussed in Section IV.

In summary:

> The Ambler dead reckoning problem is to determine the position and orientation of the Ambler in the world based on the positions of the feet in the world frame. The solution is the rigid body transformation which minimizes the squared error defined by (2). This is an over-constrained problem.

IV. Approach

The implementation of dead reckoning on the Ambler is decomposed into three components: the elimination from consideration of slipped feet, the solution of the rigid body transformation problem and the update of the foot positions. The first step is used to remove erroneous data points so they do not affect the accuracy of the solution. The second step is the actual position determination. The third step is used to update the positions of the feet based on the newly calculated body position.

A. *Checking for "slipped feet"*

It is possible that a foot may make an uncommanded move due to some external force; this would violate our assumptions. To achieve the best position estimate, these positions should be eliminated so they do not corrupt the solution. For any rigid body, the distance between two points fixed on that rigid body is a constant. Using this fact, the foot positions are preprocessed to prune any suspect ones. Using the assumption that the Ambler is a rigid body, the distance between each pair of feet is calculated in the world frame and the body frame, and then compared. In predicate form, this can be expressed as

$$\text{slip(i,j)} = \begin{cases} 1 & \text{if} \left|\, \| P_{w,i} - P_{w,j} \| - \| P_{b,i} - P_{b,j} \| \,\right| > \Delta \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

If a foot slips, the rigid body assumption is violated, and the difference between the distance calculated between two feet in the world frame and the distance calculated between two feet in the body frame will exceed some allowed value Δ. The position of a foot which is found to have slipped is not used for the solution of the rigid body transformation.

B. *Solution of rigid body transformation*

The determination of a rigid body transform is a common problem in computer vision, arising frequently in techniques such as motion analysis and camera calibration. Three different techniques are commonly used for the solution of this problem. The first method ignores the fact that **R** is an orthonormal matrix and treats it instead as a matrix of nine independent unknowns. Once a solution is obtained, **R** is orthonormalized. The other methods treat **R** as an orthonormal matrix from the start, but derive the result by different representations of **R**. The second method leaves the problem in the form of vectors and matrices, while the third makes use of quaternions.

R as nine independent unknowns

This technique is conceptually the simplest, but probably yields the worst results. The fact that **R** is a rotation matrix is ignored, instead it is treated as a matrix of nine independent unknown quantities. This allows rewriting the right-hand side of (1) as a matrix of constants multiplied by a vector of 12 unknowns (9 from **R** and 3 from **T**). Since each foot position consists of three values (the Cartesian coordinates in the world system), a minimum of four foot positions are required to find a solution using this method. A solution which minimizes the squared error is applied to obtain **R** and **T**, [7]. Once an **R** matrix is obtained, Gram-Schmidt orthonormalization, or equivalent, [14], is applied to make **R** an orthonormal matrix.

This method seems appealing because of the simplicity of the solution. However, the solution may require more computer time than the other methods due to the inversion of a 12×12 matrix and the orthonormalization procedure. In addition, poor results are expected because there are *not* 12 independent unknowns, but rather only six.

R as a rotation matrix

Two techniques are presented which find the solution to (2). The first represents **R** as a matrix, the second represents **R** as a quaternion. Since these techniques make use of the fact that rotation matrices are orthonormal, only three foot positions are required for the solution.

For both of these techniques, auxiliary equations are needed in addition to (1) to ensure the orthogonality of **R**. In the singular value decomposition scheme, these are expressed as additional constraint equations forcing the orthogonality and unit norm of the columns of the matrix. For the quaternion approach, this condition is enforced by normalizing the resultant quaternions. It is important to note, that in both of these cases, as well as the previous case, the solution is obtained without iteration.

The Singular Value Decomposition Technique

The derivations for this method are found in [13] in Appendix B.1. Given a set of *j* measurements with associated weights, w_j, the best estimate for **R** and **T** to minimize (2) is given by:

$$w = \sum_j w_j \qquad (5)$$

$$\mathbf{P}_1 = \sum_j w_j \mathbf{P}_{w,j} \qquad (6)$$

$$\mathbf{P}_2 = \sum_j w_j \mathbf{P}_{b,j} \qquad (7)$$

$$\mathbf{A} = \sum_j w_j \mathbf{P}_{w,j} \mathbf{P^T}_{b,j} \qquad (8)$$

$$\mathbf{E} = \mathbf{A} - \frac{1}{w} \mathbf{P}_1 \mathbf{P}_2^T \qquad (9)$$

and let

$$\mathbf{E} = \mathbf{USV}^T \qquad (10)$$

be the singular value decomposition (SVD) of **E**. Then

$$\hat{\mathbf{R}} = \mathbf{UV}^T \qquad (11)$$

$$\hat{\mathbf{T}} = \frac{1}{w}[\mathbf{P}_1 - \hat{\mathbf{R}}\mathbf{P}_2]. \qquad (12)$$

One important implementational note is that the **U** and **V** matrices returned by the SVD routine must have the same "handedness", that is they must *both* be right-handed or left-handed (that is, the determinants have the same sign), otherwise, the result of (11) will be incorrect.

The Eigenvalue Technique

This solution differs from the singular value decomposition approach in the mathematics only. The underlying principles and assumptions are the same. Faugeras and Hebert first published this solution in [6]. First, (2) is written in the form:

$$q = \sum_j |\mathbf{q} * \mathbf{x}_j - \mathbf{x'}_j * \mathbf{q} - \mathbf{t} * \mathbf{q}|^2 \qquad (13)$$

where $\mathbf{q}$ is a quaternion representing the rigid body rotation, $\mathbf{t}$ is a quaternion representing the rigid body displacement, and $\mathbf{x}$ is a quaternion representing the data points. It can be shown that

$$\mathbf{q} * \mathbf{x}_j - \mathbf{x'}_j * \mathbf{q} = \mathbf{A}_j \mathbf{q} \qquad (14)$$

where $\mathbf{A}_i$ is a 4×4 matrix and * is the quaternion multiplication operator. Defining

$$\mathbf{A} = \sum_j \mathbf{A}_j \mathbf{A}, \qquad (15)$$

$$\mathbf{C} = \sum_j \mathbf{A}_j, \qquad (16)$$

the solution is found to be

$$\mathbf{t} = \mathbf{C}\mathbf{q}_{min} * \bar{\mathbf{q}}/N \qquad (17)$$

where $\mathbf{q}_{min}$ is the eigenvector associated with the minimum eigenvalue of the matrix $\mathbf{A} - \mathbf{C}^T\mathbf{C}/N$, where N is the number of data points. The rotation matrix can be extracted directly from $\mathbf{q}_{min}$, [6] and [8].

Since a rigid body rotation can be described by three numbers, storing the entire **R** matrix is superfluous. Routines are used to uniquely extract three numbers from a rotation matrix, [3] and [11].

C. Update positions of slipped feet

After the position of the body has been calculated, the positions of the feet in the world are calculated based on the new body position and compared to the stored values. Ideally, there should be no difference, but since the algorithms used are based on minimizing the squared error, it is not expected that the difference will be identically zero for any foot. To minimize the accumulation of error, the position of a foot is updated only if the difference exceeds some tolerance value. This can be done since the primary assumption of the dead reckoning technique is that the feet do not move unless commanded to do so.

D. Determination of rotations

A clinometer measures the angle of the gravity vector relative to itself. To understand the need for clinometers for the determination of the tilt angles, the following thought experiment is helpful. Imagine the Ambler standing in a normal stance. If one leg were to fully extend vertically, the Ambler would be at a severe angle, with only three feet making contact with the ground. This violates the assumption that feet which are not commanded to move, do not move. However, in this case, the dead reckoning would erroneously report that the Ambler had placed the foot into a deep hole and that body position had not changed.

To solve this problem, clinometers are used to detect the body tilt angles. After the dead reckoning algorithm has computed the new body position, the clinometers are read, and the values are used in the rotation matrix. Experiments conducted before the installation of the clinometers showed that the tilt angles were never changed by the dead reckoning algorithm. The effect of this was that the dead reckoning reported that the Ambler was walking on a slope, depending on the initial value of the slope that was supplied to the system. The use of clinometers dramatically improves the accuracy of the dead reckoning. Further improvements can be made by using additional sensing information. This will be discussed in more detail in Section VI.

V. Test Results

To date, the only approach implemented on the Ambler for the solution of the rigid body transformation problem is the singular value decomposition technique. Several tests of the dead reckoning accuracy have been conducted. To test the real-world accuracy of the Ambler, a system was devised which measures the position of the body in the world directly. This is used as the ground-truth against which the accuracy of the dead reckoning is compared. As the Ambler moves along a path, the commanded move, the dead reckoned position and the directly measured position are recorded for each step.

To measure the ground truth position of the Ambler, retro-reflectors were mounted on the Ambler in several locations. Their positions in the body frame are known from direct measurement. To determine the position of the Ambler, a surveying instrument is used to locate these points in the world frame. By using the same technique as the dead reckoning algorithm, to find the solution of a rigid body transformation, the position of the Ambler is calculated. This method has a certain degree of inaccuracy, due to errors in measured location of the retro-reflectors, operator inconsistency in taking the measurements, the use of a least-squares technique, etc.; however, each measurement is independent of the previous measurement and the error sources are independent, so there will be no accumulation of ground-truth measurement error.

Testing of the Ambler has shown that absolute positioning accuracy is on the order of tenths of a percent, however, using the previously described procedure, the real-world accuracy of the dead reckoning was determined to be more than 3 cm/body advance, more than six percent of the body advance. Using a calibration procedure, kinematic improvements were incorporated into the Ambler model and another dead reckoning test was conducted. The dead reckoned position and the commanded move were consistent, the errors are small and appear to be random, Fig. 4, but there was a monotonically increasing difference between the dead reckoned position estimate and the ground truth position, Fig. 5.

After the kinematic improvements, the error per step has been reduced to about 1.2 cm/body advance (two percent of body advance). As in the previous test, the dead reckoned position always lags the actual position. There are two possible conclusions that can be drawn: there is still an unmodelled systematic error and/or the Ambler slips as it walks. Instead of trying to determine the source of this bias, it is corrected by simply adding two percent of the body advance in the direction of travel, (see Fig. 6).

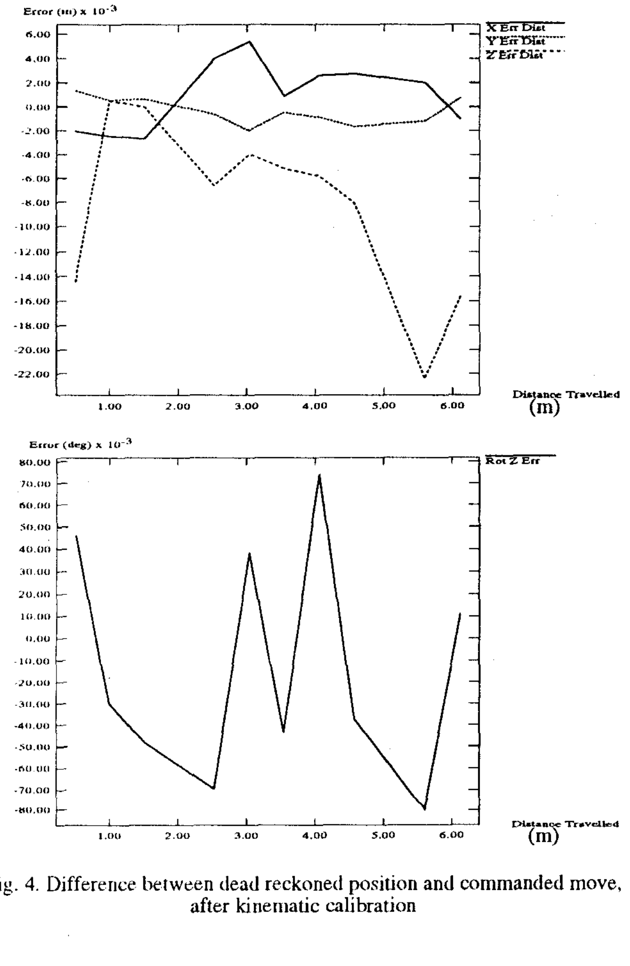

Fig. 4. Difference between dead reckoned position and commanded move, after kinematic calibration

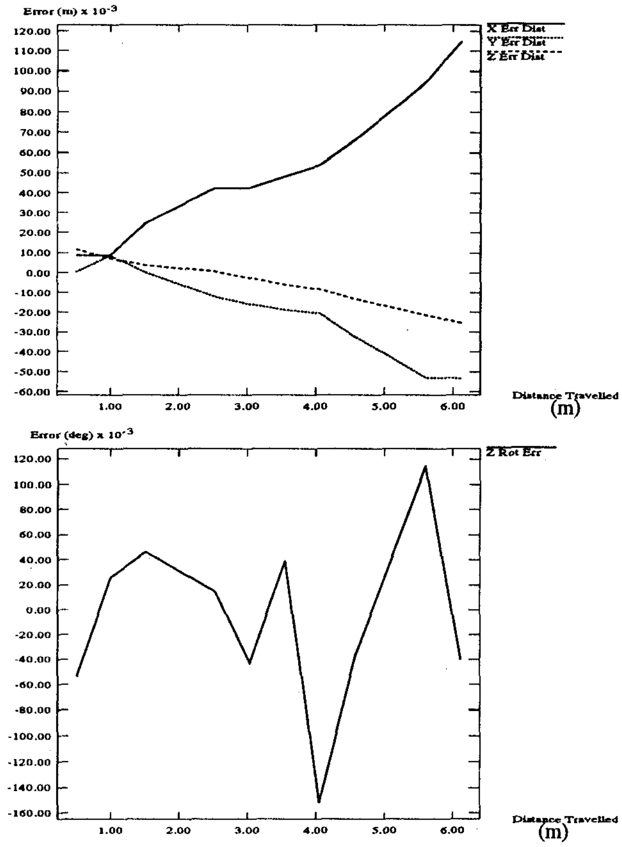

Fig. 5. Difference between dead reckoned position and ground truth positions, after kinematic calibration

The errors in the rotations are of small magnitude and appear to be random, and are probably due to measurement errors.

The inaccuracy in the direction of the world Z axis is probably due to the unmodelled effects of the foot terrain interaction. This inaccuracy is caused by the compression of the soil. The Ambler sinks into the soil with each step, but this sinkage is not modelled. Examination of the data shows that the Ambler position estimates are always higher than the ground truth position. The error in angle of the rotation in the plane shows what appears to be a random error, on the order of 0.1 degrees. A larger number of points would have to be studied to determine if the error is truly random.

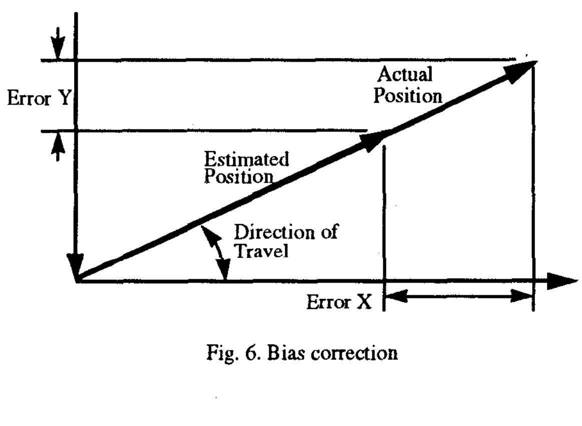

Fig. 6. Bias correction

The current results are sufficiently accurate to be used for near-term testing of the Ambler. However, as the mission duration increases, greater accuracy will be required from the navigation system. This will be achieved by combining the dead reckoned position with other position estimates. To do this, the uncertainty of the dead reckoning must be modelled. The technique presented above for removing the bias is a first step towards this model, but is insufficient. However, if the error in the angle of rotation in the plane is shown to be a random error, then its uncertainty model would be based on the magnitude of the error.

VI. Future Work

There are several areas in which work still needs to be completed. The first is to account for small positional perturbations of the foot positions which are not currently modelled. The second is to implement alternative techniques for the solution of the rigid body transformation problem. The third is to perform more tests of the dead reckoning to determine system accuracy and to help determine an error model. A fourth is to continue testing the system, including turns, point turns, longer distances, different soils, etc.

The most important of these is the first. As stated several times, a primary assumption of the dead reckoning is that the feet do not move unless commanded to do so. However, this is not the case. In the current mode of operation, the Ambler walks by recovering its rear-most foot, placing it in front of the body, loading it to some pre-determined force, then moving the body forward. For dead reckoning purposes, the position of the foot in the world is updated after the foot has placed down, but before the body moves forward. Since most terrains are compliant, as the body moves forward, a greater load is applied to the front legs, causing them to sink into the terrain. Currently, this sinkage is not accounted for and may prove to be a source of inaccuracy. These effects must be studied, and if the results are significant, they should be incorporated into the model. This study should also provide useful information that can be used in other aspects of the project other than navigation, e.g., determining the appropriate amount of force to apply when a foot is set down.

VII. Conclusion

The dead reckoning system implemented thus far is the first step towards the goal of a robust, autonomous navigation system. A key development in the work to date was the recognition that the location of the Ambler can be determined by the solution of the rigid body transform problem, which permits using previously developed methodologies. Initial tests of the system have shown that the system works well and have indicated those areas where improvements are required. The navigation problem takes on increased importance as the goals for the Ambler become more ambitious. The work presented in Section IV is only the beginning of the solution to the navigation problem, a problem which may eventually encompass such topics as soil mechanics and integration of additional navigational devices.

Testing of the Ambler has shown that absolute positioning accuracy is on the order of tenths of a percent. In Section V, the best dead reckoning results presented show a systematic error of two percent of the body advance. The question must be raised is whether more time should be spent improving the accuracy of the dead reckoning, perhaps by further mechanism calibration? The authors feel that this is not necessary, because the error can be quantified and used to combine the results of dead reckoning with other navigational schemes. The final acceptance tests will be to determine the accuracy of the Ambler as it walks outdoors over large distances, elevation changes and obstacles.

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