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Analysis of Actuation and Dynamic Balancing For a Single Wheel Robot

Yangsheng Xu[1,2], Kwok Wai Au[1], Gora C. Nandy[3] and H. Ben Brown[2]
[1]Department of Mechanical and Automation Engineering,
The Chinese University of Hong Kong, HK
[2]The Robotics Institute, Carnegie Mellon Unversity, Pittsburgh, PA 15213, USA
[3]Mechanical Engineering Department, REC, Durgapur-713 209, India

Abstract

In this paper, we develop a dynamic model of the steering and actuation mechanism of Gyrover, a single-wheel robot which can be considered as a single wheel, actuated through a spinning flywheel attached through a two-link manipulator at the wheel bearing and a drive motor. The spinning flywheel acts as a gyroscope to stabalize the robot, and at the same time it can be titled to achieve steering. In this paper, we develop dynamic model, investigate its motion equation and nonholonomic constraints, and present simulation study. The work is significance in understanding this type of dynamically stable but statically unstable system, and in developing automatic control of the system.

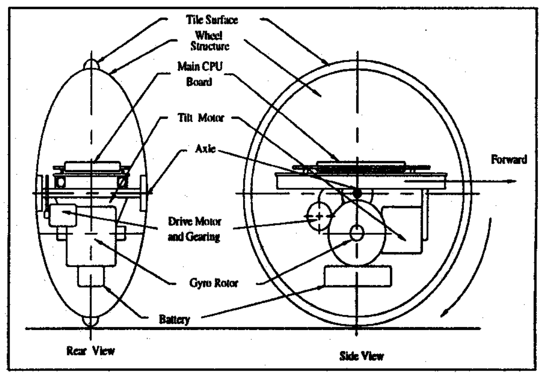

Figure 1: The basic Gyrover configuration composing a wheel, gyro and tilt mechanism.

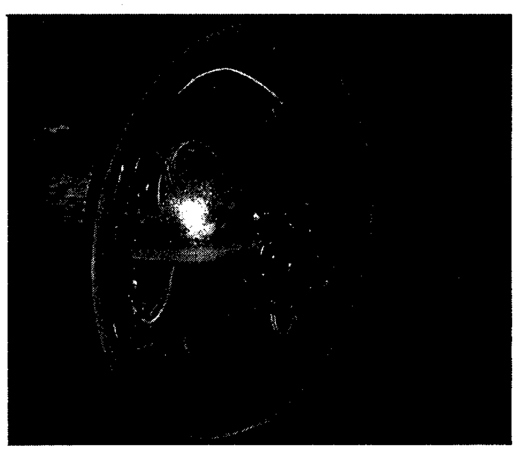

Figure 2: Photograph of the third prototype of Gyrover.

1 Introduction

Gyrover is a novel, single wheel gyroscopically stabilized robot, originally developed at Carnegie Mellon Universty [1]. Three prototypes have already been developed; Figures 1 and 2 show a schematic and photograph of the third prototype. Essentially, Gyrover is a sharp-edged wheel, with an actuation mechanism fitted inside the wheel. The actuation mechanism consists of three separate actuators: (1)*a spin motor*, which spins a suspended flywheel at a high rate, imparting dynamic stability to the robot; (2)*a title motor*, which controls the steering of Gyrover; and (3)*a drive motor*, which causes forward and/or backward acceration, by drivinq the single wheel directly.

The behavior of Gyrover is based on the principle of gyroscopic precession as exhibited in the stability of a rolling wheel. Because of its angular momentum, a spinning wheel tends to precess at right angles to an applied torque, according to the fundamental equation of gyroscopic precession:

$$T = J \times \omega \times \Omega \tag{1}$$

where ω is the angular speed of the wheel, Ω is the wheel's precession rate, normal to the spin axis, J is the wheel polar moment of inertia about the spin axis, and T is the applied torque, normal to the spin and precession axes. Therefore, when a rolling wheel leans to one side, rather than just fall over, the gravitationally induced torque causes the wheel to precess so that it turns in the direction that it is leaning. Gyrover supplements this basic concept with the addition of an internal gyroscope — the spinning flywheel — nominally aligned with the wheel and spinning in the direction of forward motion. The flywheel's angular momentum produces lateral stability when the wheel is stopped or moving slowly.

Furthermore, a conventional bicycle/unicycle needs to steer the front wheel or shift the mass of the rider to one side to achieve steering. Gyrover, however, steers by titling the spinning flywheel. Because titling the spinnig flywheel generates a yaw torque against a reaction mass (e.g. the reaction wheel or external wheel to lean), the gyroscopic effect yields a precession about the roll axis, causing the wheel to lean. This allows the entire system to be enclosed within the wheel to provide mechanical and environmental protection for the equipment and actuation mechanism. Moreover, it is not necessary to consider pitching stability, as it is the case for a unicycle. Besides, by titling the flywheel, we may upright the robot even though it fall on the ground without rolling. Torques generated by the drive motor — reacting against the internal mechanism hanging as a pendulum from the wheel's axis — producing thrust for acceleration and braking.

Thus far, Gyrover robot has been controlled only manually, using two joysticks to control the drive and tilt motors through a radio link. A complete dynamic model is necessary to develop automatic control of the system. In our previous work [2], we have developed a 3-dimensional dynamic model of a single gyroscopic wheel. Because the motion between the flywheel and the single wheel is highly coupled with each other. We need to consider the dynamics of the single wheel and flywheel at the same time. In this paper, we derive the dynamic model using the constrained generalized Lagrangian formulation. We investigate the dynamic behaviour and nonholonomic constraints of the system. Finally, we present simulate study at different initial conditions and titling angles of flywheel.

2 Kinematics of Gyrover

2.1 Coordinate frame

In derivation of the equations of motion of the robot, we assume that the wheel is a rigid, homogeneous, disk which rolls over a perfectly flat surface without slipping. We model the actuation mechanism, suspended from the wheel bearing, as a two-link manipulator, with a spinning disk attached at the end of the second link (Figure 3). The first link of length l_1 represents the vertical offset of the actuation mechanism from the axis of the Gyrover wheel. The second link of length l_2 represents the horizontal offset of the spinning flywheel and is relatively smaller compared to the vertical offset.

Next, we assign four coordinates frames: (1) the inertial frame $\sum_O$, whose $x-y$ plane is anchored to the flat surface, (2) the body coordinate frame $\sum_B$ $\{x_B, y_B, z_B\}$, whose origin is located at the center of the single wheel, and whose z-axis represents the axis of rotation of the wheel, (3) the coordinate frame of internal mechanism $\sum_C$ $\{x_c, y_c, z_c\}$, whose center is located at point D, and whose z-axis is always parallel to z_B, and (4) the flywheel coordinates frame $\sum_E$ $\{x_a, y_a, z_a\}$, whose center is located at the center of the Gyrover flywheel, and whose z-axis represents the axis of rotation of the flywheel. Note that y_a is always parallel to y_c. The definition and configuration of system and varibles are shown in Table 1 and Figure 3.

Table 1: Parameters definition	
α, α_a	Precession angles of the wheel and for the actuator (i.e. flywheel), respectively, measured about the vertical axis
β	Lean angles of single wheel
β_a	angle between the link l_1 and z_a-axis of flywheel
γ, γ_a	Spin angles of the wheel and flywheel respectively
θ	Angle between link l_1 and x_B-axis of the wheel
m, m_i, m_f	Mass of the wheel, internal mechanism and flywheel respectively
R, r	Radius of the wheel and flywheel respectively
g	Gravitational acceleration
$I_{xxw}, I_{yyw}, I_{zzw}$	Moment of inertia of the wheel about x, y and z direction
$I_{xxf}, I_{yyf}, I_{zzf}$	Moment of inertia of the flywheel about x, y and z direction

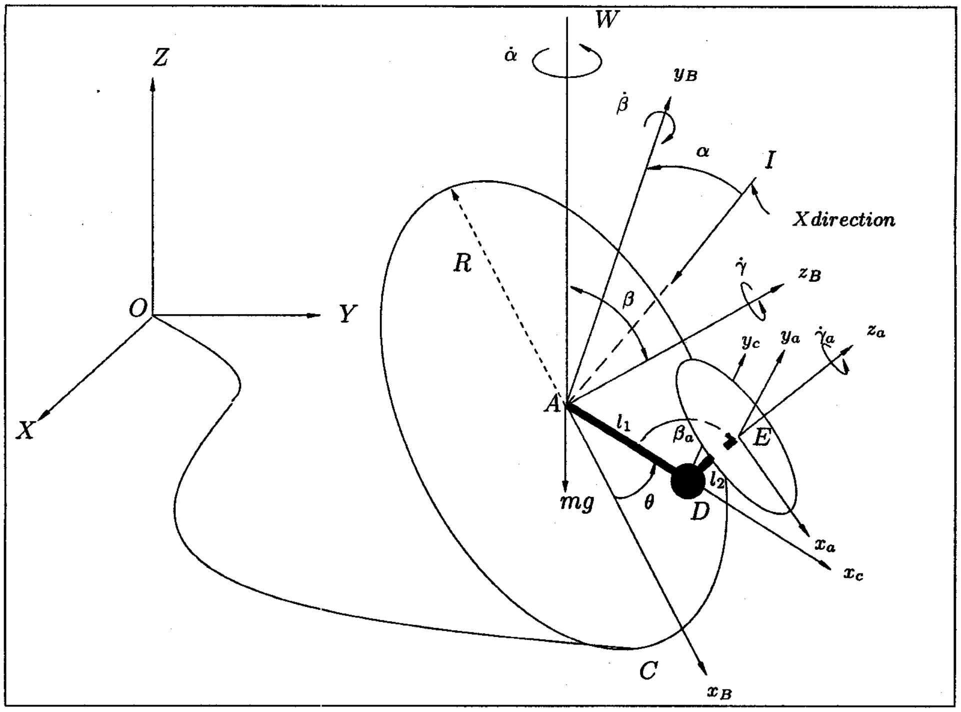

Figure 3: Definition of system variables for Gyrover

2.2 Nonholonomic constraints

Rolling without slipping is a typical example of a nonholonomic system, since in most cases, some of the constrained equations for the system are nonintegrable. Thus, Gyrover is a similar type of the nonholonomic system. We first derive the constraints of the single wheel, and then derive the dynamic model of Gyrover based on this constraints. We define (i, j, k) and (l, m, n) to be the unit vectors of the coordinate system $XYZO(\sum_O)$ and $x_B y_B z_B A(\sum_B)$, respectively. The transformation between the two coordinate frames given by

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = R_B^O \begin{bmatrix} l \\ m \\ n \end{bmatrix} \tag{2}$$

where R_B^O is the rotation matrix from $\sum_O$ to $\sum_B$.

$$R_B^O = \begin{bmatrix} -\sin\alpha\cos\beta & -\cos\alpha & -\sin\alpha\sin\beta \\ \cos\alpha\cos\beta & -\sin\alpha & \cos\alpha\sin\beta \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \tag{3}$$

Let v_A and ω_B denote the velocity of the center of mass of the single wheel and its angular velocity w.r.t. inertia frame $\sum_O$. Then, we have

$$\omega_B = -\dot{\alpha} s\beta l + \dot{\beta} m + (\dot{\gamma} + \dot{\alpha} c\beta) n \tag{4}$$

The constraints results from the requirement that the disk rolls without slipping on the horizontal plane, i.e, the velocity of the contact point on the disk is zero at any instant

$$v_c = 0, \tag{5}$$

where v_c is the velocity of contact point of the single wheel. Now, we can express v_A as

$$v_A = \omega_B \times r_{AC} + v_c \tag{6}$$

where $r_{AC} = -Rl$ represents the vector from the frame C to A in Figure 3. Substituting (4) and (5) in (6) gives

$$v_A = \dot{X} i + \dot{Y} j + \dot{Z} k, \tag{7}$$

where,

$$\dot{X} = R(\dot{\gamma} c\alpha + \dot{\alpha} c\alpha c\beta - \dot{\beta} s\alpha s\beta) \tag{8}$$

$$\dot{Y} = R(\dot{\gamma} s\alpha + \dot{\alpha} c\beta s\alpha + \dot{\beta} c\alpha s\beta) \tag{9}$$

$$\dot{Z} = R\dot{\beta} c\beta \tag{10}$$

Equations (8) and (9) are nonintegrable and hence are nonholonomic. Equation (10) can be integrable, i.e,

$$Z = R\sin\beta. \tag{11}$$

Therefore, Gyrover can be represented by through seven(e.g. $X, Y, \alpha, \beta, \gamma, \beta_a, \theta$), rather than eight independent coordinates.

3 Dynamic model of Gyrover

We now derive the equation of motion by calculating the Lagrangian $L = T - P$ of the system, where T and P are the kinetic energy and potential energy of the system respectively. We divide the system into three parts: 1) single wheel, 2) internal mechanism, 3) spinning flywheel.

3.1 Dynamic model of the single wheel

The kinetic energy of the single wheel is given by,

$$\begin{aligned} T_w &= \frac{1}{2}m\left[\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right] \\ &\quad + \frac{1}{2}\left[I_{xxw}\omega_x^2 + I_{yyw}\omega_y^2 + I_{zzw}\omega_z^2\right] \end{aligned} \quad (12)$$

Substituting Equations (4) and (10) in (12) yields

$$\begin{aligned} T_w &= \frac{1}{2}m\left[\dot{X}^2 + \dot{Y}^2 + (R\dot{\beta}c\beta)^2\right] + \frac{1}{2}\left[I_{xxw}(\dot{\alpha}s\beta)^2\right. \\ &\quad \left. + I_{yyw}\dot{\beta}^2 + I_{zzw}(\dot{\alpha}c\beta + \dot{\gamma})^2\right] \end{aligned} \quad (13)$$

The potential energy of the single wheel is

$$P_w = mgRs\beta \quad (14)$$

3.2 Dynamic model of Internal Mechanism and the Spinning Flywheel

We need to compute the translational and rotational parts of kinetic energy for the internal mechanism and flywheel respectively. We assume that l_2 is very small compared with l_1,

$$l_2 \simeq 0 \quad (15)$$

Thus, the flywheel's center of mass (E) coincides with that of the internal mechanism (D).
Let $\{x_f, y_f, z_f\}$ be the center of mass of the internal mechanism and the flywheel w.r.t. $\sum_O$. The transformation from the center of mass of single wheel to the flywheel can be described:

$$\begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + R_B^O \begin{bmatrix} l_1 c\theta \\ l_1 s\theta \\ 0 \end{bmatrix} \quad (16)$$

Let T_f^t denote the translational kinetic energy of the flywheel and the internal mechanism.

$$T_f^t = \frac{1}{2}(m_i + m_f)[\dot{x}_f^2 + \dot{y}_f^2 + \dot{z}_f^2] \quad (17)$$

Differentiating (16) and substituting it in (17), we obtain T_f^t. We observed that the internal mechanism swings slowly, so it should not contribute highly to the rotational kinetic energy. Let ω_f be the angular velocity of flywheel w.r.t. $\sum_O$. We then have

$$\omega_f = R_B^E \omega_B + \begin{bmatrix} 0 \\ \dot{\beta}_a \\ \dot{\gamma}_a \end{bmatrix} \quad (18)$$

where R_B^E is the transformation from $\sum_B$ to $\sum_E$.

$$R_B^E = \begin{bmatrix} c\theta s\beta_a & -s\theta s\beta_a & c\beta_a \\ s\theta & c\theta & 0 \\ c\theta c\beta_a & -c\beta a s\theta & s\beta_a \end{bmatrix} \quad (19)$$

The rotational kinetic energy of the flywheel is now given by,

$$T_f^r = \frac{1}{2}\left[(\omega_{fx})^2 I_{xxf} + (\omega_{fy})^2 I_{yyf} + (\omega_{fz})^2 I_{zzf}\right] \quad (20)$$

The flywheel is assumed to be a uniform disk, the principle moments of inertia are $I_{xxf} = I_{yyf} = \frac{1}{4}m_f r^2, I_{zzf} = \frac{1}{2}m_f r^2$. The potential energy of the flywheel and internal mechanism is

$$P_f = (m_i + m_f)(Rs\beta - l_1 c\theta s\beta) \quad (21)$$

3.3 Lagrangian of the system

The lagrangian of the system thus is

$$L = [T_w + (T_f^t + T_f^r)] - (P_w + P_f) \quad (22)$$

Substituting (12), (17), (20), (14) and (21) in (22), L is determined. There are only two control torques avaiable on the system. One is drive torque (u_1) and the other is the tilt torque (u_2). Consequently, using the constrained Lagrangian method, the dynamic equation of entire system is given by,

$$M(q)\ddot{q} + N(q, \dot{q}) = A^T\lambda + Bu; \quad (23)$$

Where $M(q) \in R^{7\times7}$ and $N(q, \dot{q}) \in R^{7\times1}$ are the inertia matrix and nonlinear terms respectively.

$$A(q) = \begin{bmatrix} 1 & 0 & -Rc\alpha c\beta & Rs\alpha s\beta & -Rc\alpha & 0 & 0 \\ 0 & 1 & -Rc\beta s\alpha & -Rc\alpha s\beta & -Rs\alpha & 0 & 0 \end{bmatrix} \quad (24)$$

$$q = \begin{bmatrix} X \\ Y \\ \alpha \\ \beta \\ \gamma \\ \beta_a \\ \theta \end{bmatrix}, \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_1 & 0 \\ 0 & 1 \\ k_2 & 0 \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The nonholonomic constraints can be written as,

$$A(\dot{q})\dot{q} = 0. \tag{25}$$

It is noted that *the last two columns of matrix A are all zero* as the nonholonomic constraints only restrict the motion of the single wheel, not the flywheel.The last two columns represent the motion variables of the flywheel. Moreover, *the matrix B only have three rows that are nonzero* since the input torques only drive the tilt angle of the flywheel (β_a) and the rotating angle of the single wheel (γ), so that the fifth and the sixth rows of B are non-zero as they represent the tilting motion of the flywheel and the rotating motion of the single wheel respectively. Furthermore, when the single wheel rotates, the pendulum motion of internal mechanism is introduced, thus θ changes. Therefore, the drive torque of the single wheel will also affect the pendulum motion of the internal mechanism (θ), so that the seventh row of the matrix B is not zero.

4 Normal form of the system

In this section, we will eliminate the Lagrange multipliers so that a minimum set of differential equations is obtained, based on [4]. We first partition the matrix $A(q)$ as $A = [A_1 : A_2]$, where

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} -Rc\alpha c\beta & Rs\alpha s\beta & -Rc\alpha & 0 & 0 \\ -Rc\beta s\alpha & -Rc\alpha s\beta & -Rs\alpha & 0 & 0 \end{bmatrix}$$

Let

$$C(q) = \begin{bmatrix} -A_1^{-1}A_2 \\ I_{3\times 3} \end{bmatrix} \tag{26}$$

Then consider the following relation

$$\dot{q} = C(q)\dot{q}_2 \tag{27}$$

where $q_1 = [X, Y]^T, q_2 = [\alpha, \beta, \gamma, \beta_a, \theta]^T$. Differentiating (27) yeilds

$$\ddot{q} = C(q)\ddot{q}_2 + \dot{C}(q)\dot{q}_2 \tag{28}$$

Substituting (28) into (23) and premultiplying both sides by $C^T(q)$ gives

$$C^T(q)M(q)C(q)\ddot{q}_2$$
$$= C^T(q)\left[Bu - N(q, C(q)\dot{q}_2) - M(q)\dot{C}(q)\dot{q}_2\right] \tag{29}$$

Note that $C^T(q)M(q)C(q)$ is an 5×5 symmetric positive definite matrix function. *Equation (29) only depends on ($\alpha, \beta, \gamma, \beta_a, \theta$).* Using the numerical integration, we can obtain q_2 from $\ddot{q_2}$ in (29), and then obtain (X, Y) by substituting q_2 and $\dot{q}_2$ in (27).

5 Simulation study

In the simulation, we assume the single wheel is a rolling ring with $I_{xxw} = I_{xxy} = \frac{1}{2}MR^2$ and $I_{zzw} = MR^2$. We use the following realistic geometric/mass parameters throughtout our simulations.

Table 2: Parameters values	
Single wheel parameters:	$m = 1.25kg, R = 17cm$
Internal mechanism :	$m_i = 4.4kg,$
Flywheel parameters :	$m_f = 2.4kg, r = 5cm$
Link lengths :	$l_1 = 10cm$

In our initial simulation, the system is similar to a rolling disk if we set the spinning rate of the flywheel and the control torques to be zero. We chose the initial condition for the system below

$$\begin{cases} \dot{\alpha} = \dot{\beta} = \dot{\beta}_a = \dot{\theta} = \dot{\gamma}_a = 0\, rad/s, \dot{\gamma} = 8\, m/s \\ \alpha = \gamma = \theta = 0^o, \beta = 81^o, \beta_a = 90^o \end{cases}$$

, and the simulation takes about 10 sec. Figure 4 show the simulation result in this case. From Fig 4, the robot trajectory is nearly a circular path. As for a rolling disk, if the inclination is not 90^o, it precesses the direction it leans, thus it normally travels in a circular trajectry. This shows that, if the spinning flywheel is not rotating, the characteristic of Gyrover is very similar to a rolling disk.

Then we investigated the case when $\gamma_a \neq 0$. It is noted that, owing to large angular momentum of the spinning flywheel, Gyrover has larger resistance to the change of inclination (β). We also investigated the effect of variation of the tilting angle of flywheel. Figure 5 shows the second simulation result where β_a vares sinusoidally. The initial conditions are

$$\begin{cases} \dot{\alpha} = \dot{\beta} = \dot{\beta}_a = \dot{\theta} = 0\, rad/s, \dot{\gamma} = 8\, m/s \\ \dot{\gamma}_a = 14000\, RPM, \alpha = \gamma = \theta = 0^o, \beta = 90^o, \\ \beta_a = 90^o, u_1 = 0.006N/m \end{cases}$$

In this case, the robot steers left and right alternatively when β_a is changing sinusoidally. We can conclude that when the flywheel tilts in one direction, the robot will lean, so as to achieve steering. Based on this simulation, we noted that if the spinning flywheel rotates, the system will be much stable since a

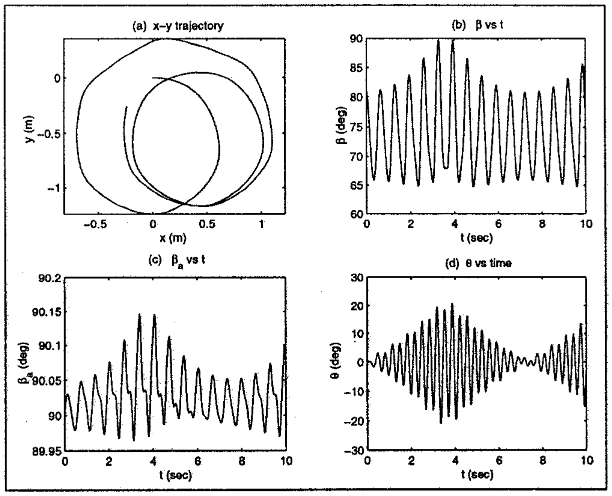

Figure 4: The characteristic of the Gyrover without rotating the flywheel

large momentum is generated by the flywheel. Then inclination of the robot does not vary rapidly. Furthermore, when the tilt angle changes, the inclination of the robot will also change at the same time and in the same direction. The internal mechanism of the robot swings within 30^o.

6 Conclusion

In this paper, we first studied the velocity constraints for Gyrover. Then we developed the dynamic model using the constrained Lagrangian method. Based on the simulation, we found that the property of Gyrover is similar to a rolling disk if its flywheel does not rotate. If the flywheel rotates a high spinning rate, a larger resistance exists with respect to the change of orientation. By tilting the flywheel, the robot can steer to where it intend to go. The simulation study based on the dynamic model of the robot is significant for us to understand the dynamic behaviour of the system, and guide us in the automatic control of the robot.

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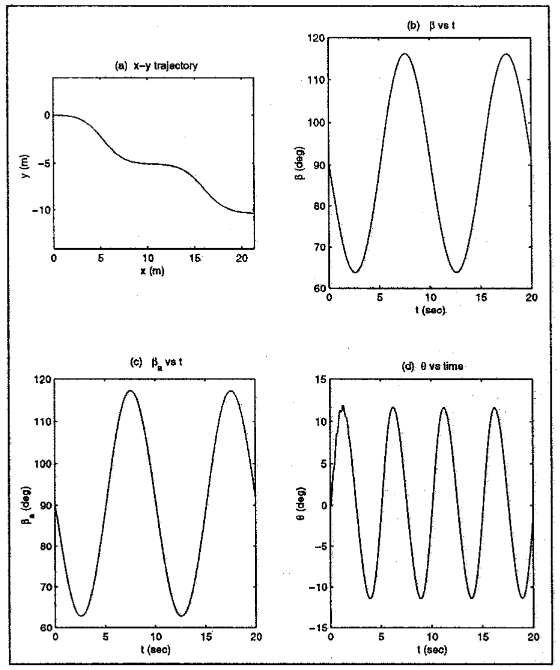

Figure 5: Tilting effect of the spinning flywheel on the robot

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