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Under-Actuated Robot Systems: Dynamic Interaction and Adaptive Control

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Abstract

An under-actuated robot manipulator is a serial mechanism, in which the number of pints is greater than the number of actuators. Making use of the dynamic interaction between the passive joints and actuated joints, the robot can provide desirable motion and forces dynamically. In comparison to a fully actuated robot, the under-actuated system will be more compact in size and lower weighted due to less actuaton, and more efficient due to less energy consumption. In this paper, we intend to answer the following two questions: (1) What is the dynamic coupling of the system and how to control the system by using its dynamic coupling? (2) When the dynamic parameters u e uncertain/unknown in practice, and kinematics relationship is thus not accurate, what adaptive control scheme is feasible for this nonlinear system where linear parameterization does not hold and linear structured adaptive control scheme is not valid?

1 Introduction

For a conventional robot manipulator, the number of joints is equal to the number of actuators, or actuated joints; such a fully driven serial mechanism is called a full-actuated system. If the total number of joints is greater than the number of actuators in the mechanism, the system is referred to as an under-actuated system. As well known that the martial-arts superstar Bruce Lee used to play a pair of numbchucks (three-link-sticks) as his favorite weapon. Each numbchuck is composed of two passive pints that connect three hardwood segments together. It is fascinating to see that by manipulating the bottom segment, Bruce Lee could fast project the tip of each numbchuck to a target, producing a tremendous impact force acting on the target, as shown in Figure 1. Now. consider Bruce Lee as an actuated manipulator and the sticks as passive linkages; the system becomes a typical under-actuated system. This shows that by use of the dynamic interaction of the under-actuated system, the position or force of the system may still be controlled.

More examples of such under-actuated systems can be found in a variety of applications. although they may not be referred to as the under-actuated systems, nor be realized as a class of popular mechanisms. An inverted-pendulum (cart-pole system) is the simplest example of a two-joint under-actuated system. McGeer's passive walker is known as a fully passive system which completely takes advantage of dynamic coupling of all the links [13]. Another example is the body control of a gymnast. He/she does not control all joints of his/her body in an exercise; instead, the gymnast only controls fewer joints to balance the whole body [14]. The McGeer's walking robot daign and Takashima's gymnut motion analysis show that the under-actuated systems have great potential in robotic research and applications.

Fig.1 Bruce Lee and his numbchucks

There are a number of advantages to the use of the under-actuated systems. First. reducing the number of actuators for a robot manipulator will niinimizc cncrgy consumption, and will be potentially attractive to the applications where energy efficiency is a major concern, such as for space robots. Second, eliminating some actuators will allow more compact design leading to both overall size and total weight reductions. This will ultimately reduce the manufacturing cost and running power.

Not only the under-actuated system is useful in practice, but also the concept is important in analysis of a class of systems that can be considered as virtual under-actuated systems. For example, a free-flying space robot system [16] is useful for maintenance tasks and EVA missions in a space station and/or a satellite. The concept of under-actuated systems provides an approach to modeling dynamic systems with either fret bases, free ends, or free joints. Some of these mechanisms can be potentially utilized in space and underwater applications [12].

To study an under-actuated system, two primary questions should be addressed: "Under what conditions can it be controlled?" and "How is it controlled?" Arai and Tachi [11] discussed the controllability issue of the passive joints based on linear system theory and dynamic equilibrium. However, their conclusion has certain limitations due to the linear system assumption, while most under-actuated systems are obviously nonlinear. Vukobratovic and his t e m [17] presented a control scheme to control unknown statu by dynamic equilibrium and applied it to biped postual stabilization. Based on the same principle, Arai and Tachi (11) proposed another control scheme and applied it to a two-DOF system when the passive joint was braked or released. The most interesting work has been done by Nakakuki, Fujimoto and Yamafuji [15] recently. They actually built a model of a three link under-actuated system and performed three interesting experiments: (1) filling down and then standing up, (2) locomotion by peristaltic motion, and (3) ascending a step. Their prototype was the first experimental setup for testing the under-actuated system.

Much more research work, however, must be done. One of the most serious problems is that the kinematic relationship for an under-actuated system is dynamically dependent. In other words, the kinematic mapping from a given Cartesian space specification to the joint space is a function of dynamic parameters, such as the masses, centroid coordinates and inertia moments of the links. To cope with the parameter uncertainty, an adaptive control scheme is needed [7]. However, most existing adaptive control schemes developed for full-actuated robot systemr are not applicable to the under-actuated syrtems due to the nonlinear parameterization [16]. In this paper, we intend to deal with this problem for precisely controlling the under-actuated systems under parameter uncertainty.

It has been shown that a fully actuated manipulator system is always exactly linearizable [1,2]. However, for an under-actuated dynamic system, due to the fact of deficient input channels, the entire system is unlinearizable. This results in a fundamental difficulty inherent in the control of under-actuated dynamic systems, Nevertheless, an under-actuated system can be decomposed into two subsystems: a linearizable one and an unlinearizable one which is also called an *internal dynamics* [1,2]. It can be observed that two subsystems are virtuilly inseparable. This phenomenon has already been revealed in classical mechanics, and is often referred to as a *non-holonomic constraint* problem [5,6]. For a non-holonomic system, how does the internal dynamics interfere the entire system stability? To answer this question and to resolve the aforementioned parameter uncertainty problem, we will propose an extended dynunic model, and based on this model, develop a normal form augmentation approach to adaptively control the under-actuated systems with parameter uncertainty.

2 Extended Dynamic Model

Consider an n-joint robot manipulator, in which some joints are passive. To model such an under-actuated system, we extend it to be a fully actuated system with zero torques on all the passive joints. This is called an extended dynamic model. The number of input channels, in such an extended model, is equal to the number of active joints, while the dimension of the system state space is twice the total number of joints because the relative degree in each output channel of the system is *two.*

For analysis simplicity, consider that an under-actuated system consists of two serial submechanisms, one with all passive joints and the other one with all actuated joints. If the upper body is passive, it is called a *lower-actuated system*, while the lower body is passive, it is called an *upper-actuated system*. We assume that the lower body has l joints and the upper body has m pints, and thus the total number of joints is $l+m$. It is noted that any under-actuated system with mixed passive/actuated joints can be constructed by assembling a set of such models in series. Therefore, the analysis presented here will, in general, be applicable to any under-actuated system.

Let $q = (q_1 \cdots q_n)^T \in \Re^n$ be the joint position vector for an under-actuated system with n-joints. The dynamic equation can be written as

$$W\ddot{q} + C\dot{q} - \tau_g = \tau, \qquad (1)$$

where the n by n matrix W is known as the *inertial matrix* of the system and is positive-definite and symmetric, and

$$C = \frac{1}{2}\dot{W} + \frac{1}{2}(W_d^T - W_d) \qquad (2)$$

with an n by n matrix W_d defined by

$$W_d = (I \otimes \dot{q}^T)\frac{\partial W}{\partial q} = \begin{pmatrix} \dot{q}^T \frac{\partial W}{\partial q_1} \\ \vdots \\ \dot{q}^T \frac{\partial W}{\partial q_n} \end{pmatrix}. \qquad (3)$$

In (3), I is the n by n identity matrix, and $\otimes$ is the Kronecker product operator between matrices. It is clearly seen that the first term $\dot{W}$ in (2) is symmetric, and the second term $\frac{1}{2}(W_d^T - W_d)$ is skew-symmetric. Thus, the matrices C in (2) and $\dot{W}$ have the following identity for any $z \in R^n$:

$$z^T C z = \frac{1}{2} z^T \dot{W} z. \qquad (4)$$

3 Inertial Matrices

With the extended dynamic model, the n by n inertial matrix W for an under-actuated dynunic system can be partitioned into four blocks,

$$W = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}, \tag{5}$$

where W_{11} is an l by l symmetric submatrix attributed by the lower body, W_{22} is ur m by m symmetric block as an inertial matrix of the upper body with respect to the fixed base, and $W_{12} = W_{21}^T$ is an l by m submatrix representing the interaction between the lower and upper bodies of the under-actuated system.

3.1 Inertial Matrix of Upper-Actuated Systems

Consider **an upper-actuated system with l passive joints** in the lower body. Using the partitioned **form (5), we** write the inverse of the inertial matrix W [10] as $W^{-1} =$

$$\begin{pmatrix} W_{11}^{-1} + W_{11}^{-1}W_{12}\tilde{W}_{22}^{-1}W_{12}^{T}W_{11}^{-1} & -W_{11}^{-1}W_{12}\tilde{W}_{22}^{-1} \\ -\tilde{W}_{22}^{-1}W_{12}^{T}W_{11}^{-1} & \tilde{W}_{22}^{-1} \end{pmatrix}, \tag{6}$$

where

$$\tilde{W}_{22} = W_{22} - W_{12}^{T}W_{11}^{-1}W_{12}, \tag{7}$$

which is referred to as an *effective inertid matrix* of the upper-actuated system. We can show that this matrix is positive-definite and symmetric, and thus always invertible [8,9].

3.2 Inertial Matrix of Lower-Actuated Systems

If a lower-actuated system with m passive joints in the upper body is considered, then using the **same** partitioning form of the inertial matrix W in **(5),** but through an alternative inversion [10], we can write the inverse of W for a lower-actuated system as $W^{-1} =$

$$\begin{pmatrix} \tilde{W}_{11}^{-1} & -\tilde{W}_{11}^{-1}W_{12}W_{22}^{-1} \\ -W_{22}^{-1}W_{21}\tilde{W}_{11}^{-1} & W_{22}^{-1} + W_{22}^{-1}W_{21}\tilde{W}_{11}^{-1}W_{12}W_{22}^{-1} \end{pmatrix}, \tag{8}$$

where

$$\tilde{W}_{11} = W_{11} - W_{12}W_{22}^{-1}W_{21} \tag{9}$$

which is called the effective inertial matrix of the lower-actuated dynamic system. Likewise, we can show that $\tilde{W}_{11}$ is also positive-definite and symmetric.

4 Input-Output **Linearization**

Based on derivation of the extended dynamic model **equation** in Section 2, the upper-actuated and the lower-actuated system equations can be written, respectively, as follows:

$$W\ddot{q} + C\dot{q} - \tau_g = \begin{pmatrix} 0 \\ u_{up} \end{pmatrix}, \tag{10}$$

and

$$W\ddot{q} + C\dot{q} - \tau_g = \begin{pmatrix} u_{lo} \\ 0 \end{pmatrix}, \tag{11}$$

where $u_{up} \in \Re^m$ and $u_{lo} \in \Re^l$ are the actuated pint torque inputs for the upper-actuated urd lower-actuated systems, respectively. Clearly, the location of the zero vector, **0,** on the right-hand side of the above equations is the main distinction between the two cases.

The kinematic relationship of an under-actuated system can be developed based on the extended model. Suppose an m-dimensional Cartesian displacement of the system end-effector with respect to the fixed base is chosen as an output **vector** which is a differentiable function of the joint position $q = (q_{lo} \;\; q_{up})$, urd b denoted by $y = h(q) \in \Re^m$. The Jacobian matrix of y is determined by

$$J = \frac{\partial h}{\partial q} = (J_1 \;\; J_2), \tag{12}$$

where $J_1 = \partial h/\partial q_{lo}$ is of m by l, $J_2 = \partial h/\partial q_{up}$ is of m **by m, and $q_{lo} \in \Re^l$ and $q_{up} \in \Re^m$ are the** joint **position vectors for the** lower body and the upper body of the system, respectively. Similarly to the effective inertial matrix definition, we define an ***effective Jacobian rnatrir*** by

$$\tilde{J}_2 = JT_{12} = J_2 - J_1W_{11}^{-1}W_{12} \tag{13}$$

for the upper-actuated system, and define

$$\tilde{J}_1 = JT_{21} = J_1 - J_2W_{22}^{-1}W_{21} \tag{14}$$

for the lower-actuated system, in which the output $y = h(q)$ is 1-dimensional.

The definitions of the effective-Jacobian matrix $\tilde{J}_2$ or $\tilde{J}_1$ and the effective inertial matrix $\tilde{W}_{22}$ or $\tilde{W}_{11}$ show that the motion of under-actuated systems, unlike a full-actuated dynamic system, is determined by not only the actuated body itself, but also the interaction from the passive body motion.

Using the effective Jacobian matrix concept dong with the dynamic equations (10) and (11), we can deduce

$$J\ddot{q} + JW^{-1}(C\dot{q} - \tau_g) = JW^{-1}\begin{pmatrix} 0 \\ u_{up} \end{pmatrix}$$

$$= \tilde{J}_2\tilde{W}_{22}^{-1}u_{up} = D_2(x)u_{up} \tag{15}$$

for the upper-actuated systems, and

$$J\ddot{q} + JW^{-1}(C\dot{q} - \tau_g) = JW^{-1}\begin{pmatrix} u_{lo} \\ 0 \end{pmatrix}$$

$$= \tilde{J}_1\tilde{W}_{11}^{-1}u_{lo} = D_1(x)u_{lo} \tag{16}$$

for the lower-actuated systems. In the above two equations, **$D_2(x) = \tilde{J}_2\tilde{W}_{22}$ is of m by m and $D_1(x) = \tilde{J}_1\tilde{W}_{11}$ is of l by l, both are called the *decoupling matrix*,** while $x = (q^T \;\; \dot{q}^T)^T \in$ **$\Re^{2n}$ is the state vector.**

In order to linearize the under-actuated systems by using the ***input-output linearization procedure*** [1,2], let us define a new input $v = \ddot{y} = J\ddot{q} + \dot{J}\dot{q}$. Substituting $J\ddot{q} = v - \dot{J}\dot{q}$ into either (15) or (16) results in

$$u = \alpha(x) + \beta(x)v, \tag{17}$$

where

$$\alpha(x) = D^{-1}(x)[JW^{-1}(C\dot{q} - \tau_g) - \dot{J}\dot{q}]$$
$$\beta(x) = D^{-1}(x). \quad (18)$$

Equations **(17)** and (18) are notation-unified for both the upper- and lower-actuated systems, i.e., u b either u_{up} or u_{lo}, and $D(x)$ is either $D_2(x)$ or $D_1(x)$.

As the counterpart of (18), a full-actuated system which is exactly lineuizable is shown that

$$\alpha_o(x) = C\dot{q} - WJ^{-1}\dot{J}\dot{q}$$
$$\beta_o(x) = WJ^{-1}. \quad (19)$$

Comparing (18) to (19), we conclude that for an under-actuated system, due to the existence of non-trivial internal dynamics, the property of linear parameterization in $\alpha(x)$ and $\beta(x)$ is no longer valid. This results in a fundamental difficulty to design an adaptive control scheme [7,12,16] for any type of under-actuated dynamic system. Furthermore, the existence of internal dynamics also raises difficulty in justifying the control system Stability if the control law **(17)** with (18) is adopted. To overcome the problems, a ***normal form* augmentation *approach*** will be investigated in the next section.

5 Normal Form Augmentation Approach

Since the operating task for an under-actuated ryskm is usually specified in terms of Cartesian displacement of ita end-effector, we may choose the Cartesian displacement as a system output $y = h(q) \in \Re^m$ for an upper-actuated system, or $y = h(q) \in \Re^l$ for a lower-actuated ryrtem. Now, for an upper-actuated system with totally $n = l + m$ pints, since there are $2n - 2m = 2l$ unobservable variables that constitute states of the internal dynamics, the l joint positions in q_{lo} of the passive, lower body and their time-derivatives may be the best choice of states to represent the internal dynamics. Thur, we are motivated to define an augmented output vector $y_a = \begin{pmatrix} y \\ q_{lo} \end{pmatrix} \in \Re^n$, and ita time-derivative

$$\dot{y}_a = \begin{pmatrix} \dot{y} \\ \dot{q}_{lo} \end{pmatrix} = \begin{pmatrix} J_1 & J_2 \\ I & O \end{pmatrix} \begin{pmatrix} \dot{q}_{lo} \\ \dot{q}_{up} \end{pmatrix} = J_{aq2}\dot{q}, \quad (20)$$

where I is the 6 by 6 identity matrix and O is the 6 by m zero matrix. The n by n square Jacobian matrix J_{aq2} defined in **(20)** can be inverted to be

$$J_{aq2}^{-1} = \begin{pmatrix} O & I \\ J_2^{-1} & -J_2^{-1}J_1 \end{pmatrix} \quad (21)$$

if J_2 in $J = (J_1 \;\; J_2)$ is nonsingular. Using $\ddot{y}_a = J_{aq2}\ddot{q} + \dot{J}_{aq2}\dot{q}$ and substituting $\ddot{q} = J_{aq2}^{-1}(\ddot{y}_a - \dot{J}_{aq2}\dot{q})$ into (10), we obtain

$$WJ_{aq2}^{-1}\ddot{y}_a - WJ_{aq2}^{-1}\dot{J}_{aq2}\dot{q} + C\dot{q} - \tau_g = \begin{pmatrix} 0 \\ u_{up} \end{pmatrix}. \quad (22)$$

Premultiplying (22) by $J_{aq2}W^{-1}$ yields

$$\ddot{y}_a - \dot{J}_{aq2}\dot{q} + J_{aq2}W^{-1}(C\dot{q} - \tau_g) = J_{aq2}W^{-1} \begin{pmatrix} 0 \\ u_{up} \end{pmatrix}. \quad (23)$$

The above equation can be decomposed into two parts

$$\ddot{y} - \dot{J}\dot{q} + JW^{-1}(C\dot{q} - \tau_g) = JW^{-1} \begin{pmatrix} 0 \\ u_{up} \end{pmatrix} = D_2^{-1}(x)u_{up}, \quad (24)$$

and

$$\ddot{q}_{lo} + (I \;\; O)W^{-1}(C\dot{q} - \tau_g) = (I \;\; O)W^{-1} \begin{pmatrix} 0 \\ u_{up} \end{pmatrix}. \quad (25)$$

Equation (24) represents the linearizable subsystem of the rystem, while **(25)** describes the internal dynamics. If the ***static state-feedback* control low (17)** is applied to the subsystem (24), it can be immediately obtained that $\ddot{y} = v$, provided that all link dynamic parameters are known. Therefore, if we define the output error function as $e(t) = y_d(t) - y(t)$, and

$$v = \ddot{y}_d + k_v\dot{e} + k_pe, \quad (26)$$

the dynamics of the linearizable subsystem is equivalent to

$$\ddot{e} + k_v\dot{e} + k_pe = 0, \quad (27)$$

where k_v and k_p are two constant gains which can be chosen such that the linear error equation **(27)** is Hurwitz. However, this justifies only the stability of the linearizable part, and the stability effect due to the internal dynamics is still to be investigated.

Since the complete set of equations including the linearized portion and the internal dynamics is conventionally called the ***normal form*** [1,2], we refer to the definitions of y_a and J_{aq2}, and thederivation of equation **(22) as a normal** *form augmentation approach* [8,9].

Likewise, we can derive the augmented equation for a lower-actuated system by defining

$$y_a = \begin{pmatrix} y \\ q_{up} \end{pmatrix}, \quad \text{and} \quad J_{aq1} = \begin{pmatrix} J_1 & J_2 \\ O & I \end{pmatrix}, \quad (28)$$

and the inverse of J_{aq1} becomes

$$J_{aq1}^{-1} = \begin{pmatrix} J_1^{-1} & -J_1^{-1}J_2 \\ O & I \end{pmatrix}.$$

Then, we obtain

$$\ddot{y}_a - \dot{J}_{aq1}\dot{q} + J_{aq1}W^{-1}(C\dot{q} - \tau_g) = J_{aq1}W^{-1} \begin{pmatrix} u_{lo} \\ 0 \end{pmatrix} \quad (29)$$

for lower-actuated dynamic systems.

It can be seen from equations **(23)** and **(29)** that by the normal form augmentation approach, an entire under-actuated system can be viewed as a full-actuated system, as if the internal dynamics disappears, as long as the following two conditions hold:

1. All the passive joint positions, velocities and accelerations are measurable; and
2. All the passive joint torques are equal to zero in the equations.

6 Direct Adaptive Control

Motivated by the above point of view, we will develop an adaptive control scheme for an upper-actuated system executing trajectory-tracking tasks against dynamic parameter uncertainty. We will also show that the proposed scheme asymptotically stabilizes the system. The simulation and experimental verification will be carried out in the near future.

Let an augmented output error function between the desired $(y_a)_d = \begin{pmatrix} y_d \\ q_{lo} \end{pmatrix}$ and the actual $y_a = \begin{pmatrix} y \\ q_{lo} \end{pmatrix}$ be $e_a = (y_a)_d - y_a = \begin{pmatrix} e \\ 0 \end{pmatrix}$. Furthermore, let an *extended augmented error* be defined by

$$s = \dot{e}_a + k_v e_a = \begin{pmatrix} \dot{e} + k_v e \\ 0 \end{pmatrix} \in \Re^n, \qquad (30)$$

where $e = y_d - y \in \Re^m$ is the output error function, and $k_v > 0$ is the constant grin. Then, we define a reference output velocity η and a reference output deration $\dot{\eta}$ as follows,

$$\eta = \begin{pmatrix} \dot{y}_d + k_v e \\ \dot{q}_{lo} \end{pmatrix} \quad \text{and} \quad \dot{\eta} = \begin{pmatrix} \ddot{y}_d + k_v \dot{e} \\ \ddot{q}_{lo} \end{pmatrix}. \qquad (31)$$

Comparing (31) with (30) yields

$$s = \eta - \dot{y}_a, \quad \text{and} \quad \dot{s} = \begin{pmatrix} \ddot{e} + k_v \dot{e} \\ 0 \end{pmatrix} = \dot{\eta} - \ddot{y}_a. \qquad (32)$$

We now define $E = \frac{1}{2} s^T M s$ to represent an extended error energy, and then,

$$\dot{E} = s^T M \dot{s} + \frac{1}{2} s^T \dot{M} s = s^T M \dot{\eta} - s^T M \ddot{y}_a + \frac{1}{2} s^T \dot{M} s, \quad (33)$$

where $M = J_{sq}^{-T} W J_{sq}^{-1}$ is called the Cartesian inertial matrix.

Based on (22) and (4), (33) can be derived to be

$$\dot{E} = s^T M \dot{\eta} + s^T G \eta - s^T J_{sq}^{-T} \begin{pmatrix} 0 \\ u_{up} \end{pmatrix} - s^T J_{sq}^{-T} \tau_g. \quad (34)$$

We now define a following control law:

$$\begin{pmatrix} 0 \\ u_{up} \end{pmatrix} = J_{sq2}^T [M_m \dot{\eta} + G_m \eta + \begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix}] - \tau_g, \quad (35)$$

where W_m and C_m represent the inertial matrix W and the matrix C in a model plant, respectively, and

$$M_m = J_{sq2}^{-T} W_m J_{sq2}^{-1}$$

and

$$G_m = J_{sq2}^{-T} C_m J_{sq2}^{-1} - M_m \dot{J}_{sq2} J_{sq2}^{-1}.$$

In (35), H is an m by m positive-definite and symmetric constant weighting matrix.

The rector $\delta \in \Re^l$ in the control law (35) playa a key important role in assurence of the second condition, as stated in the last section. In fact, since

$$J_{sq2}^T = \begin{pmatrix} J_1^T & I \\ J_2^T & O \end{pmatrix},$$

the control law (35) can be splitted into two portions,

$$0 = (W_{11} \;\; W_{12})_m J_{sq2}^{-1} \dot{\eta} + (J_1^T \;\; I) G_m \eta + J_1^T H(\dot{e} + k_v e) + \delta - \tau_{glo} \qquad (36)$$

and

$$u_{up} = (W_{21} \;\; W_{22})_m J_{sq2}^{-1} \dot{\eta} + (J_2^T \;\; O) G_m \eta + J_2^T H(\dot{e} + k_v e) - \tau_{gup}. \qquad (37)$$

It is clear that since δ only appears in (36), δ can simply be evaluated to ensure that (36) vanishs. Therefore, equation (37) becomes the control input to the upper-actuated dynamic system.

To develop a dynamic parameter adaptation law, let ξ be the parameter column vector that lists all real physical objective parameters to be identified. Let ξ_m be the corresponding parameter vector for the model plant of the under-actuated system. Now, substituting the control law (35) into (34), we further obtain

$$\dot{E} = s^T Y \phi - s^T \begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix}, \qquad (38)$$

where $Y\phi = (M - M_m)\dot{\eta} + (G - G_m)\eta$, and Y is a matrix function of q, $\dot{q}$, $\dot{q}_b$, and y_d, $\dot{y}_d$ and $\ddot{y}_d$, and is indepeadcnt of the objective physical parameters, while $\phi = \xi - \xi_m$ is the parameter deviation vector between the real plant and the model plant.

Now, the adaptation law can be defined as

$$\dot{\phi} = -\Gamma Y^T s, \qquad (39)$$

where Γ is a constant *odoptotion gain matrix* and is also positive-definite and symmetric. Then, a following Lyapunov function can be adopted to justify the stability of the system with the control law (35) and the adaptation law (39),

$$V_L = E + \frac{1}{2} \phi^T \Gamma^{-1} \phi = \frac{1}{2} s^T M s + \frac{1}{2} \phi^T \Gamma^{-1} \phi. \qquad (40)$$

Clearly, $V_L > 0$, and $V_L = 0$ only at the equilibrium point of thia adaptive system, i.e., $(e^T \;\; \dot{e}^T)^T = 0$ and $\phi = 0$. Taking time-derivative for V_L, we have

$$\begin{aligned} \dot{V}_L &= \dot{E} + \dot{\phi}^T \Gamma^{-1} \phi = s^T Y \phi - s^T \begin{pmatrix} H(\dot{e} + k_v e) \\ \delta \end{pmatrix} - s^T Y \phi \\ &= -(\dot{e} + k_v e)^T H (\dot{e} + k_v e) \end{aligned} \qquad (41)$$

which is negative-definite and is zero only at the equilibrium point.

Therefore, the control law (37) and the adaptation law (39) *asymptotically stabilize* the entire upper-actuated system to track a desired trajectory described in terms of y_d, $\dot{y}_d$ and $\ddot{y}_d$. Since J_{sq}^{-1} is heavily involved in the control law and adaptation law, the stability **also requires** that J_2 be nonsingular.

Using the same way, the above result can be extended to a lower-actuated system, and the control input u_{lo} is given by

$$u_{lo} = (W_{11} \quad W_{12})_m J_{sq1}^{-1}\dot{\eta} + (J_1^T \quad O)G_m\eta + J_1^T H(\dot{e}+k_v e) - \tau_{glo} \tag{42}$$

where the dimensions of H, e and 2, and η and $\dot{\eta}$ should be accordingly redefined to match the lower-actuated case, and the joint torque $\tau_g = \begin{pmatrix}\tau_{glo}\\ \tau_{gup}\end{pmatrix}$ caused by gravity also has two parts $\tau_{glo} \in \Re^l$ and $\tau_{gup} \in \Re^m$. While the adaptation law for the lower-actuated system cui use the same formula as shown in (39) for the upper-actuated one.

A simulation study and results have been demonstrated in [9] to verify the proposed adaptive control scheme for a space robot system which is an example of the upper-actuated system. Therefore, the normal form augmentation approach can solve two fundamental problems for an under-actuated dynamic system, i.e., the parameter nonlinearity and tlic entire control stability. However, it is necessary to measure the position, velocity and accelerations of all passive joints.

7 Conclusion

The under-actuated robot systems have a great potential in the applications where energy-effiaency, low weight and compact size are demanded. The concept is also useful as an analytical tool for a variety of hybrid passive/active systems, such as space robots with floating base systems. In this paper, we proposed the extended dynamic model composed of two parts: a lower-body and a upper-body; one of the two bodies is passive. Based on the model, the properties of the inertial matrix and Jacobian matrix were discussed. The model allows us to gain more insight into the dynamic interaction in the system, the effect of the internal dynamics, and the difficulty in the controller design.

Through the input-output linearization on the model, we have shown the non-trivial internal dynamics that makes the fundamental difference between the under-actuated and full-actuated systems, and also reveals the nonlinear parameterization property. The feedback control scheme based on the exact linearization technique is then developed, and a normal form augmentation approach is proposed. This approach makes it possible to attack two fundamental problems in controlling under-actuated system, i.e., the entire control system stability under the exirtence of non-trivial internal dynamics and the adaptive control design under the dynamic parameter nonlinearity.

Finally, we have shown the asymptotical stability of the direct adaptive control scheme developed for both upper- and lower-actuated dynamic systems. The stability is guaranteed if the square Jacobian matrix is invertible and all the passive joint positions, velocities and accelerations are measurable and bounded. As a result of the parameter adaptation, the kinematics relationship can be updated simultaneously, and the mapping from task space to joint space will be more accurate.

References

[1] A. Isidori, *Nonlinear Control Systems*, 2nd Edition, Springer-Verlag, New York, 1989.

[2] J. Slotine and W. Li, *Applied Nonlinear Control*,' Prentice Hall, New Jersey, 1991.

[3] M. W. Spong and M. Vidyasagar, Robot *Dynamics* and *Control* John Wiley & Sons, New York, 1989.

[4] Y. L. Gu and N. K. Loh, "Dynamic Modeling and Control by Utilizing an Imaginary Robot Model", *IEEE Journal of Robotics and Automation*, Vol. 4, No. 5, pp. 532-540, Oct. 1988.

[5] L. D. Landau and E. M. Lifshitz, *Mechanics*, Addison-Wesley, 1960.

[6] V. I Arnold, *Mathematical Methods of Classical Mechanics*, Springer-Verlag, New York, 1976.

[7] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence, and Robustness*, Prentice Hall, 1989.

[8] Y. L. Gu, "A Direct Adaptive Control Scheme for Under-Actuated Dynamic Systems". Pmc. *32nd IEEE Conference on Decision and Control*, San Antonio, TX, Dec. 1993, pp. 1625-1627.

[9] Y. L. Gu and Yangsheng Xu, "A N o d Form Augmentation Approach to Adaptive Control of Space Robot Systems", Proc. *1993 IEEE International Conference on Robotics and Automation*, Atlanta, GA, May 1993, pp. 731-737.

[10] P. Lancaster and M. Tismenetsky, *The Theory of Matrices*, Second Edition, Academic Press, Inc., New York, 1985.

[11] H. Arai and S. Tachi, "Position control of a manipulator with passive joints using dynamic coupling", *IEEE Trans. on Robotics and Automation*, Vol.7(4), 1991.

[12] Y. Nakamura and R. Iwamoto, "Stabilization of the shape of space multibody structure with free joints", in *Proceedings of IROS*, 1993.

[13] T. McGeer, "Passive dynamic walking", *International Journal of Roboticr Research*, Vol.9(2), 1990.

[14] S. Takashima, "Control of gymnast on a high bar", in *Proceedings of IROS*, 1991.

[15] K. Nakakuki and H. Fujimoto and K. Yamafuji, "Motion control of a robot composed of three serial links with curved contour", in *Proceedings of International Conference on Advanced Mechatronics*, 1993.

[16] Y. Xu and T. Kanade, *Space robotics: dynamics and control*, Kluwer Academic Publisher, 1991.

[17] M. Vukobratovic and D. Stokic, "Is dynamic control needed in robotic systems, and if so, to what extent?", *International Journal of Roboticr Research*, Vol.2(2), 1983.