On Position Compensation and Force Control Stability of a Robot with a Compliant Wrist

Yangsheng Xu

Richard P. Paul

Department of Mechanical Engineering and Applied Mechanics
University of Pennsylvania
Philadelphia, PA 19104

ABSTRACT

A compliant wrist instrumented between a robot and its end effector provides a necessary compliance for assembly operations, and the displacement and force information generated from the wrist sensor can be utilized to actively control the end effector. The paper discussed the position compensation for the deflection of the compliant wrist due to the gravity load and other external forces at the unconstrained space, and the force control as the robot is constrained with the environment. The system stability problem and dynamic performance are investigated for the different control laws, various wrist parameters, and environment models. By analysis and simulation, some meaningful conclusions are obtained. The results are useful for either design of the compliant wrist device and determination of compensator law in the feedback loop.

1. Introduction

The use of the passive compliance and active accommodation for the problems of robot assembly has been well addressed in the literatures.

However, the general manipulator control problem has two main phases; the positioning of the unconstrained arm in space and the control of generated contact force while the manipulator is partially constrained, as during assembly operations.

In the first case, the manipulator included a compliant element could greatly degrade the manipulator stiffness during gross positioning operations. Therefore, the compliance introduced in the wrist has to be restricted at a certain extent and compensating the deflection by utilizing the information generated in the wrist sensor is desirable. But, as the displacement of the end effector causes the arm to move in the opposite direction so as to restore the initial position, is the system always stable? What kind of control law of the compensator could be applied and how does it effect on the system performance? How does the wrist compliance interrelate with controller parameters, and thereby the system stability and performance of the total robotic system? All those questions may be raised as the researchers are going to implement the technique to the realistic robot system.

In the second case as the manipulator attempt to perform assembly operation or such a task as grinding and welding, there are some similar problems. What is the affect of the wrist stiffness on the stability and the performance of the system? Can we use a non-damping structure for the compliant wrist? If not how much of damping is optimal? How do the environment characteristics and the controller in the feedback loop affect the closed loop system behavior.

The paper is to answer those questions and present some necessary quantitative analysis for a single link manipulator model. The different control laws are compared with one another under various conditions in the sense of stability and system steady-state behavior. Some useful conclusions are obtained and the results are essential for design of the compliant wrist and determination of the feedback controller.

2. Compliant Robot Position Compensator

At first, we discuss the case that the robot moves in unconstrained space. An active compensating control can be developed in the straightforward way by considering the single link manipulator position control system shown in Fig.1 as in the paper [3]. Since the stiffness of the wrist we discussed in the paper has a range from very small value to infinite, the results developed here is also feasible for the rigid wrist case.

The system of Fig.1 shows an actuator controlling the motion of a link and thereby controlling the motion of the end effector through a compliant wrist sensor device which is attached between the end effector and link. The feedback control loop is used to make the end effector position reach a command position against the influence of an external force which could be load gravity, harmonic force, or random excitation.

We will assume the following: the link drive train is rigid compared with the compliance of the wrist, the contribution from the viscous damping and static friction of the actuator is negligible, the rotational inertia of the actuator and the link is J, the proportional feedback gain and rate gain of PD controller are K_p and K_v respectively, the load and end effector mass is m, the stiffness and damping of the wrist sensor are K and C respectively.

A system block diagram of the single joint manipulator is shown in Fig.2. The wrist sensor records the difference of the motions between link and end effector. These signals are the input to the compensating controller H, together with the input command translation motion X_c (corresponding to the angular displacement) which drives the system controller and thereby the actuator.

It is our aim to determine the form of the compensating controller H so that the deflection of the end effector due to external forces applied to the compliant wrist can be compensated. In other words, the response of the end effector becomes independent of the force and compliance of the system.

We define that G_c is the transfer function of the actuator, PD controller and rigid link. H is the transfer function of position compensator in the feedback loop. G_1 is the transfer function of end effector motion/command position, and G_2 is the transfer function of end effector motion/applied external force. An equivalent block diagram is shown in Fig.3. From Fig.3, we set $F = 0$ and $X_c = 0$ respectively, to obtain:

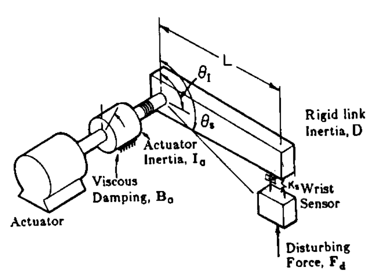

Fig.1 Single joint robot with a compliant wrist

CH2555-1/88/0000/1173$01.00

$$\frac{X(s)}{X_c(s)} = \frac{G_1(s)G_c(s)}{1-H(s)G_c(s)+H(s)G_c(s)G_1(s)} \quad (1)$$

$$\frac{X(s)}{F(s)} = \frac{(1-H(s)G_c(s))G_2(s)}{1-H(s)G_c(s)+G_1(s)G_c(s)H(s)} \quad (2)$$

where

$$G_1 = \frac{C\,s+K}{m\,s^2+C\,s+K} \quad (3)$$

$$G_2 = \frac{1}{m\,s^2+C\,s+K} \quad (4)$$

$$G_c = \frac{K_p}{J\,s^2+K_v\,s+K_p} \quad (5)$$

Therefore, the characteristic equation of the closed loop system is

$$1-H(s)\,G_c(s)+G_1(s)\,G_c(s)\,H(s)=0 \quad (6)$$

or,

$$(ms^2+Cs+K)(Js^2+K_v s+K_p)-H(s)K_p ms^2=0 \quad (7)$$

The steady state characteristics of the closed loop system can be determined by setting s to zero.

$$\lim_{s\to 0}\frac{X}{X_c} = \frac{G_c(s)G_1(s)}{1-H(s)G_c(s)+H(s)G_c(s)G_1(s)} = \frac{KK_p}{KK_p-H(0)K_pK+H(0)KK_p} = 1 \quad (8)$$

$$\lim_{s\to 0}\frac{X}{F} = \frac{[1-H(s)G_c(s)]G_2(s)}{1-H(s)G_c(s)+H(s)G_c(s)G_1(s)} = \frac{K_p-H(0)K_p}{KK_p-H(0)K_pK+H(0)K_pK} = \frac{1-H(0)}{K} \quad (9)$$

To compensate all deflection of the sensor and reach the exact command position, the desirable system should be

$$\lim_{s\to 0}\frac{X(s)}{X_c(s)} = 1 \quad (10)$$

$$\lim_{s\to 0}\frac{X(s)}{F(s)} = 0 \quad (11)$$

In order that (11) be true, the compensator transfer function $H(s)$ needs to reach unity as s goes to zero, (or the time domain function $h(t)$ reaches unity as t goes to infinite).

3. Various Compensators and Stability

3.1. Proportional Compensator

The simplest control law for the compensator is proportional feedback. From (9), the deflection can be compensated if $H(s)=K_h=1$. We consider two cases; in one case, the inertia of link and actuator is neglected, which is reasonable for the slow robot motion, or a light weight robot. In the other case, the inertia is considered.

For the first case, from Routh-Hurwitz Criterion, the system is stable if

$$K_h < 1-\frac{KK_v}{CK_p+KK_v}+\frac{CK_v}{mK_p} \quad (12)$$

From here, we may understand the following two points.

(a) If the sensor damping is small, CK_p and CK_v can be neglected. From (12), for a stable system, K_h can only be chosen as negative value. If $K_h<0$, from (9), $\frac{X(0)}{F(0)} > \frac{1}{K} = \frac{X_0}{F_0}$ (static deformation/applied force), which means that the deformation of the sensor increases. In other words, the system is made softer, The desirable situation is to make the system stiffer in this case. Therefore, K_h has to be positive. In order to use positive feedback and make the stable system, the only choice is to provide some damping in the wrist system so that (12) be satisfied.

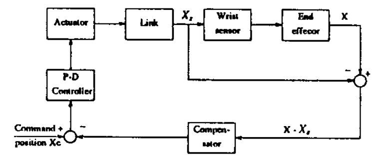

Fig. 2 Block diagram of the single joint manipulator system

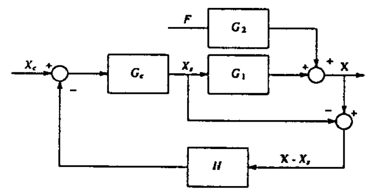

Fig. 3 Block Diagram of the Closed Loop System

(b) If one chooses $K_h=1$,the deflection can be compensated at steady state. Let the gain ratio of the regular PD controller $\tau_{vp}=\frac{k_v}{k_p}$, and suppose that we have a unit mass, the stability relation between the sensor damping ratio and the sensor stiffness can be obtained.

$$\zeta > -\frac{1}{4}K^{1/2}\tau_{vp}+\frac{1}{4}(K\tau_{vp}^2+4)^{\frac{1}{2}} \quad (13)$$

The sensor damper and stiffness determine the stability as shown in Fig.4 for the various the gain ratio. The region beyond the curve is stable as shadow area for the ratio 0.1 in Fig.4. The sensor compliance is restricted by the amount of the damping ratio. The stronger the damping, the more compliant the wrist can be made. The gain ratio of the PD controller has to be chosen as reasonably large, although a too large gain ratio system may cause a slow response.

To illustrate the behavior of the closed loop system under the unit compensator , some numerical simulations were performed. The parameters of the system were chosen as follows: m = 1 kg (load and end effector mass); K = 4000 N/m (sensor stiffness); K_p = 36 (proportional feedback gain in PD controller); K_v = 3.6 (rate gain in PD controller); ζ = 0.2 — 1.2 (sensor damping ratio); The end effector position response under step force with various damping is shown Fig.5. The damping is of great significance in order that the response converges to zero quickly. The end effector position response under unit-step command motion is almost identical to that of the open-loop system.

In the second case, the inertia of the link and actuator is considered. the stability condition is

$$(JC+mK_v)(JK+CK_v)(CK_p+KK_v) > KK_p(JC+mK_v)^2+Jm(CK_p+KK_v)^2 \quad (14)$$

Using the same parameters as in the previous example, the stability regions are computed as shown in Fig.6. The dotted curve in Fig.6 is for that the inertia is neglected. From Fig.6, if the sensor stiffness is large than a certain value, when the inertia has to be included, the necessary damping is much smaller than that it can be neglected. However, for a soft sensor, the conclusion is different and more damping is required. The position response for various force and damping has been investigated and the results is similar as the case that the inertia is neglected.

3.2. Proportional-Derivative Compensator

If the Proportional-Derivative control law is chosen as the compensator $H(s)=K_1+K_2s$, the steady state behavior of the closed loop system is

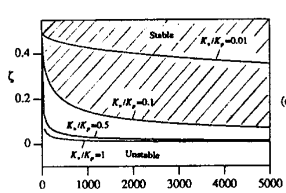

Fig.4 Stability condition for sensor stiffness and sensor damping ratio with P compensator

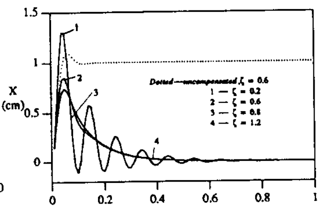

Fig.5 End-effector position response under $4kg$ step force with P compensator (The inertia is not included)

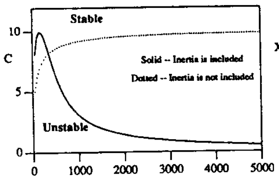

Fig.6 Stability Condition for the sensor damper and stiffness with P compensator

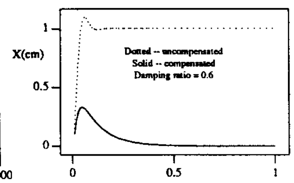

Fig.7 End-effector position under $4kg$ step force with PD compensator (The inertia is not included)

$$\lim_{s \to 0} \frac{X}{F} = \frac{K_p - K_1 K_p}{K_p K} = \frac{1-K_1}{K} \tag{15}$$

In order to compensate for all deflection, K_1 should be chosen as unity. The stability conditions are

$$K_2 < \frac{K_v}{K_p} \tag{16}$$

$$K_1 < 1 + \frac{CK_v}{K_p m} \tag{17}$$

$$K_1 < 1 + \frac{CK_v}{mK_p} - \frac{K(K_v - K_p K_2)}{KK_v + K_p C} \tag{18}$$

Compared (18) with (12), one may see the stability condition with PD compensator is better than that with proportional compensator. The critical damping for a stable system is fairly small. Therefore, for the small damping sensor case, the PD compensator is much better than the P compensator in terms of stability. Simulation was performed with the same parameters as in previous simulations and $K_1 = 1, K_2 = 0.095$, $\zeta = 0.6$ as shown in Fig.7.

For the case in which the inertia of actuator and link are considered, the similar process was performed and it is observed that a bigger shock or serious vibration may occur if the damping is weak. In other words, the more damping is required in this case.

3.3. Lead-lag Network Compensator

Compensating also can be achieved using a simple lead-lag network in the feedback loop. The lead-lag network can be represented in the form $H(s) = K_{p_1}(1+K_{v_1}s)/(1+K_I s)$. If the inertia is neglected in the system, the transfer function is

$$\frac{X}{F} = \frac{(1+K_I s)(K_p + K_v s) - K_p K_{p_1}(1+K_{v_1})}{(1+K_I s)(ms^2+Cs+K)(K_p+K_v s) - ms^2 K_p K_{p_1}(1+K_{v_1}s)} \tag{19}$$

$$\frac{X}{X_c} = \frac{(Cs+K)(1+K_I s)K_p}{(1+K_I s)(ms^2+Cs+K)(K_p+K_v s) - ms^2 K_p K_{p_1}(1+K_{v_1}s)} \tag{20}$$

The characteristic equation is

$$(mK_I K_v)s^4 + (mK_I K_p + mK_v + K_I CK_v - mK_p K_{v_1} K_{p_1})s^3$$
$$+ (CK_I K_p + K_v C + K_v K_I K)s^2 + (K_I K K_p + CK_p + K_v K)s + KK_p = 0 \tag{21}$$

The gain K_{p_1} has to be chosen as unity for the steady-state compensation. Simulations with $K_{p_1} = 1, K_{v_1} = 0.095, K_I = 0.1$, and same parameters as in previous examples is shown as Fig.8. The damping needed for the stable system is weaker than that needed in the P compensator, but stronger than that in PD compensator.

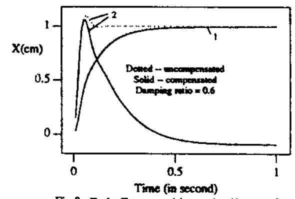

Fig.8 End-effector position under $4kg$ step force and unit-step command motion with Lead-Lag Network compensator
1 -- command response 2 -- force response

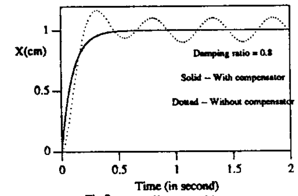

Fig.9 End-effector position response under unit-step command motion and harmonical force with $1/G_c$ compensator

3.4. Compensator as an inverse of G_c

From the equations (1) (2), an interesting fact is that if the compensator transfer function is an inverse of the transfer function of the actuator, the PD controller, and the link dynamics, $H(s) = 1/G_c(s)$, the ratio of X/F will be made zero at any time, and the ratio of X/X_c will be equal to G_c and tend to unity in the steady state. The system will become independent of the external force and sensor parameters. This is just an ideal situation. Although in the realistic system, such a compensator is nonlinear and varies in time. But the approximate value is possible and the idea is prospective.

Neglecting the inertia of the actuator and link, $H(s) = 1/G_c(s) = 1+K_v s/K_p$ (which is a regular PD controller), with $K_p = 36, K_v = 3.6$, the simulation is shown in Fig.9 for the damping ratio 0.6. Since, in this case, the system response is independent of the external forces theoretically, the solid curve in Fig.9 is not only the response under the unit-step command motion, but also the response under the force and command motion together, which can be compared with the open-loop system response under both of command step and harmonic force in the Fig.9.

In the case that the inertia of the link and the actuator cannot be estimated exactly, or there are some other second order terms neglected in the model, the effect of inertia error has been investigated and the results show that it is not significant.

3.5. Summary

(1) By simulation for a single joint robot, we have shown that the deflection due to external force applied in the compliant wrist sensor can be corrected with a proper compensator in the feedback loop. Stability conditions of the closed loop system are dependent on the compensator control law. The steady state characteristics is determined by the compensator steady state behavior.

(2) For the proportional compensator case, a stable system needs a highly damped sensor. If the sensor damping is weak, the proportional compensator is not a good choice for a stable system. Stability of the system is also dependent on the sensor system natural frequency. The lower the natural frequency of the sensor system, the higher damping that is needed in the sensor. The ratio of proportional gain to derivative gain in the PD controller has an effect on the stability region. When the inertia of actuator and link are considered, higher damping is necessary if the system is to maintain stable.

(3) For the proportional and derivative compensator, the sensor damping required for a stable system is much smaller than that in the case of the proportional compensator. In general, the PD compensator is better than the P compensator in terms of stability, although the parameters have to be chosen carefully. For the Lead-lag network compensator, the sensor damping needed is in between the cases of PD and P compensation.

(4) An ideal compensator control law can be derived as an inverse of the transfer function of actuator, the regular PD controller and the link system. With this compensator all deflections of sensor system can be compensated for all kinds of forces at all frequency regions. The end effector response is not sensitive to inertia estimated error.

4. Force Control -- Rigid Environment

When a compliant robot is constrained by the environment which may or may not be rigid, one concerns the similar stability and compliance problems as in the case that robot is not constrained.

At first, the environment is modeled as rigid. All situation is same as that in previous section except that the end effector is keeping contact with the environment continuously. The viscous damper C_r is represented as the damping force between the rigid body mode to the unattached robot. The wrist device has the stiffness K_w and the damper C_w. The actuator is represented by the input force P. The displacement of the wrist device X_w can be measured by the sensor in the wrist. The system is modeled as a single degree of freedom constrained system as Fig.10.

The open-loop system transfer function is

$$G(s) = \frac{X_w(s)}{F(s)} = \frac{1}{ms^2+(C_r+C_w)s+K_w} \tag{22}$$

The contact force P_c is measured directly by the force sensor or indirectly by the wrist displacement sensor, and can be represented as

$$P_c = K_w X_w \tag{23}$$

Suppose we use the force controller $H(s)$ which is a function of P_d-P_c, the closed-loop system is to be controlled to maintain a desired contact force P_d, and can be shown in the block diagram of the Fig.11.

4.1. Proportional Control

We consider a simply proportional control law in the force control loop.

$$H(s) = K_p(P_d-P_c) \tag{24}$$

The closed-loop transfer function will become

$$\frac{P_c(s)}{P_d(s)} = \frac{K_p K_w}{ms^2+(C_r+C_w)s+K_w(1+K_p)} \tag{25}$$

The relation of the resulting system error, $E(s) = P_d(s)-P_c(s)$, for a given input $P_d(s)$ is

$$\frac{E(s)}{P_d(s)} = \frac{1}{1+G(s)} \tag{26}$$

The steady-state error can be expressed as

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} \frac{sP_d(s)}{1+G(s)} \tag{27}$$

When a step, ramp, and paraboloid are simple mathematical expressions for the input force, namely, $p_d(t)$ is defined as $U(t)$, $tU(t)$, $t^2U(t)/2$, respectively, where the notation $U(t)$ means a unit step force for $t>0$. The system error at the steady-state is following:

$$e_{ss} = \frac{1}{1+K_p}, \quad as \quad P_d(s) = \frac{1}{s}$$

$$e_{ss} = \infty, \quad as \quad P_d(s) = \frac{1}{s^2}$$

$$e_{ss} = \infty, \quad as \quad P_d(s) = \frac{1}{s^3}$$

The above results are obvious because the open-loop is a zero order system. Therefore, the closed-loop system has a force error at the steady-state under the step command force in this case.

Suppose the wrist device and force controller parameters are $m = 2\,kg$, $K_w = 4000\,N/m$, $C_r = 200\,N/m/s$, $K_p = 30$, the simulation of the step force response is performed for the different wrist dampers as shown in Fig.12. In Fig.12, the dashed line at force level 1 is the desired contact force. Because $K_p = 30$, there is 3.2% force error at the steady state.

Summary

1) From the system transfer function (25), the closed loop system is always stable, no matter how compliant the wrist is, or how high the gain of controller.

2) There is always a force error at the steady-state. The error is inverse proportional to the gain of controller at the step force input, and becomes infinite at the ramp and paraboloid force input.

3) The high gain is desirable in an effort to minimize the steady-state error under the step command force, but the gain value has a contribution of the stiffness of the closed-loop system. Therefore, if the steady-state error is concerned, the wrist device must be made more compliant and more damping is necessary. In other words, if the high wrist stiffness is desired, the gain of controller cannot be chosen too big.

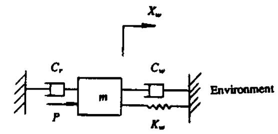

Fig.10 The rigid environment system

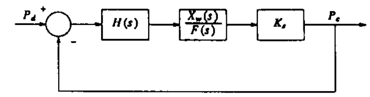

Fig.11 The closed-loop control system block diagram

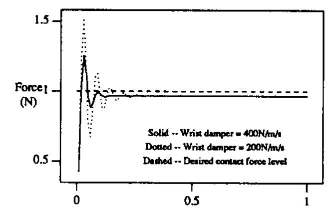

Fig.12 End-effector contact force response under step command force with P controller

4.2. Proportional-Derivative Control

When the PD controller is chosen as $H(s) = K_p(1+K_v s)$, the open-loop transfer function becomes

$$G(s) = \frac{K_p K_w (1+K_v s)}{ms^2+(C_r+C_w)s+K_w} \tag{28}$$

Form Fig.11, the closed-loop system transfer function is of a form as

$$\frac{P_c(s)}{P_d(s)} = \frac{K_p K_w (1+K_v s)}{ms^2+(C_r+C_w+K_p K_w K_v)s+K_w(1+K_p)}$$

Since, from (28), since the open-loop is still a zero order system, the steady-state performance will be identical with that in the P controller.

The simulation is performed for the step force input as shown as Fig.13 with the same parameters as in the previous examples and the rate gain of the controller $K_v = 0.01$. From Fig.13, with the same damper, the system response is much improved because the rate gain in the PD controller has a contribution on the system damping. Therefore, a proper value of the rate gain is beneficial to the improvement of the system behavior.

Summary

1) The system is always stable and independent of the wrist compliance and the controller.

2) Similarly as with the P controller, under the step force input, the steady-state behavior is dependent on the proportional gain in the PD controller. The higher the gain K_p, the smaller the steady-state force error. Under the ramp and paraboloid force input, the

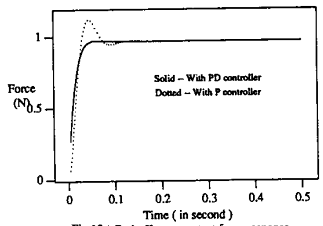

Fig.13 End-effector contact force response under step command force with P and PD controller

steady-state error is infinite.

3) The rate gain in the PD controller provides the damping in the system. Therefore, in a slight damped system, an increasing of the rate gain can achieve the same system response as that with a large damping.

4.3. Proportional-Integral Control

The PI controller is represented as $H(s)=K_P(1+\frac{K_I}{s})$. The open-loop transfer function is

$$G(s)=\frac{K_P K_w(1+\frac{K_I}{s})}{ms^2+(C_r+C_w)s+K_w} \tag{29}$$

Now, the open-loop becomes a first order system and the steady-state performance can be obtained as follows.

$$e_{ss}=0, \quad as \quad P_d(s)=\frac{1}{s}$$
$$e_{ss}=\frac{1}{K_P K_I}, \quad as \quad P_d(s)=\frac{1}{s^2} \tag{30}$$
$$e_{ss}=\infty, \quad as \quad P_d(s)=\frac{1}{s^3}$$

Since usually the command force input is taken in the step form, the system has no error at the steady-state. For this point, the PI controller is advantageous.

The stability condition is

$$K_p(C_r+C_w)-mK_I>-(C_r+C_w) \tag{31}$$

Namely, if

$$(C_r+C_w)-mK_I<0 \tag{32}$$
$$K_P<\frac{C_r+C_w}{mK_I-(C_r+C_w)} \tag{33}$$

and, if

$$(C_r+C_w)-mK_I>0 \tag{34}$$
$$K_P>\frac{-(C_r+C_w)}{(C_r+C_w)-mK_I} \tag{35}$$

Since the functions (33) and (35) are identical, if some damper is provided in the wrist device so that (36) is true, the gain K_p can be chosen positive. But, if not, the gain K_p must only be chosen as negative and the system will be made stiffer, namely, the contact force goes the opposite direction from the desired force level. Therefore, the damper is necessary in the wrist system if the PI controller is chosen.

Summary

1) The system maintains stable if some damping is provided in the wrist system so that $C_r+C_w-mK_I$ becomes positive.
2) The system has no error at the steady-state for the step input force case, which is independent of the gain in the controller or the parameters in the wrist.
3) The system stability is independent of the stiffness of the wrist device.

4.4. Proportional-Derivative-Integral Control

We can do the similar analysis for the PID controller

$$H(s)=K_p(1+K_v s+\frac{K_I}{s})$$

as previous section, and list some summary as follows.

Summary

1) The system has no force error at the steady-state for the step force input, as in the case with PI controller.
2) The sensor must be provided some damping so that the system can be made stable and all gains in the controller have a large choice.
3) The gate gain and the integral gain in the controller have to be carefully adjusted so that the desirable performance can be reached.

5. Force Control -- Compliant Environment

When the environment has some compliance, or the workpiece does not have the same value level of the stiffness as the robot arm, the robot system included the compliant wrist element can be modeled as Fig.14. The system also can be interpreted as the block diagram in Fig.11. Similarly as in the case of rigid environment, various controllers have been tested for the system stability and the steady-state behavior.

In this case, the output contact force measured by the wrist sensor is

$$P_c=K_w(X_r-X_w)$$

The open system behavior can be can be represented as

$$G(s)=\frac{H(s)K_w(m_e s^2+C_e s+K_e)}{[ms^2+(C_r+C_w)s+K_w][m_e s^2+(C_e+C_w)s+(K_w+K_e)]-(C_w s+K_w)^2} \tag{36}$$

From (36), the characteristic equation of the open-loop system is

$$(mm_e)s^4+[(C_r+C_w)m_e+(C_w+C_e)m]s^3+[m(K_w+K_e)+m_e K_w$$
$$+C_r C_e+C_w C_r+C_w C_e]s^2+(C_r K_e+C_w K_e+C_r K_w+C_e K_w)s+K_e K_w=0$$

or,

$$f_4 s^4+f_3 s^3+f_2 s^2+f_1 s^1+f_0=0 \tag{37}$$

Since all coefficients of the characteristical equation are positive, the stability condition is

$$f_1 f_2 f_3>f_1^2 f_4+f_3^2 f_0 \tag{38}$$

The above inequality can usually be satisfied unless the damping of the wrist and environment is very small, which could be shown as following process.

Because the term $f_4 f_1^2$ is much smaller than the term $f_0 f_3^2$ in (38), the inequality can be approximately expressed as

$$f_1 f_2>f_0 f_3 \tag{39}$$

When C_e and C_w is small enough to be neglected, (39) becomes

$$m(mK_e+2m_e K_w)>K_w K_e(K_w+K_e) \tag{40}$$

Since the value of the mass is much smaller that that of the stiffness, (40) cannot be satisfied obviously. Therefore, the open-loop could be unstable if the damping of the wrist and environment is weak.

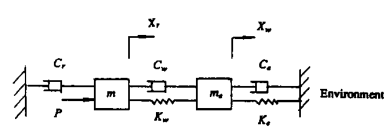

Fig.14 The compliant environment system

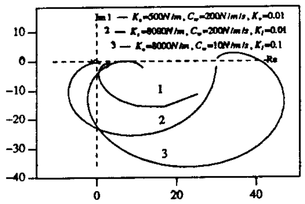

Fig.15 Nquist (Im – Re) curve of the closed-loop system with the PD controller

The Nyquist diagram for the different parameters were computed for the P, PD, PI, PID, and LLW controllers from equation (36). One example is shown in Fig.14. From various simulations of the Nyquist diagram, the open-loop transfer function $G(\omega)$ usually doesn't include the point $(-1, j0)$ at the complex plane. therefore, the closed-loop system can maintain stable if some damping is provided in the wrist system. To save the space of the paper, we may present the main conclusions as follows.

Summary

1) The system which is stable in the rigid environment model could become unstable if the compliance of the environment must be included. The more critical case is that the environment system has no damping.

2) Since the environment usually is weakly damped or has no damping at all, the wrist system must provide some damping. The damping is more important than that in the rigid environment.

3) In the case that the the system damping is small, the alternative way to stablize the system is utilizing the PD controller in the force feedback loop, because the rate gain of the controller can provide some equivalent damping in the closed-loop system.

4) The system performance at the steady-state won't be changed because the order of the transfer function is not changed. When the P, PD, and LLW controllers are applied, the open-loop is a zero order system. The system has a constant error at the steady-state under the step force input, and infinite error under the ramp and paraboloid force input. When the PI and PID controller are applied, the open-loop is a first order system. The system has no error at the steady-state under the step force input and constant error under the ramp force and infinite error under the paraboloid force input.

REFERENCES

[1] S. D. Eppinger and W. P. Seering, "On dynamic models of the robot force control", *Proceedings of the IEEE International Conference on Robotics and Automation*, P.29-34, 1986

[2] S. D. Eppinger and W. P. Seering, "Understanding bandwidth limitations in robot force control" *Proceedings of the IEEE International Conference on Robotics and Automation*, P.904-909, 1987

[3] R. K. Roberts, R. P. Paul, and B. M. Hillberry, "The effect of wrist force sensor stiffness on the control of robot manipulators", *Proceedings of the IEEE International Conference on Robotics and Automation*, P.269-274, 1985

[4] R. P. Paul, "Problems and research issues associated with the hybrid control of force and displacement" *Proceedings of the IEEE International Conference on Robotics and Automation*, P.1966-1971, 1987

[5] D. E. Whitney, "Historical perspective and state of the art in robot force control", *Proceedings of the IEEE International Conference on Robotics and Automation*, P.262-268, 1985