Path Generation for Robot Vehicles Using Composite Clothoid Segments

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CMU-RI-TR-90-31 2

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December, 1990

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Abstract

The response of an autonomous vehicle in tracking a reference path, depends partly on the nature of the path. The condition for paths that are intrinsically amenable to follow are briefly presented, and then a method for the generation of amenable paths is proposed in this paper. Previous path generation methods have sought to simplify a path by using arcs, superarcs, polynomial curves, and clothoid curves to round corners, which result from poly-line fits through a given sequence of points. The developed method consists of two steps: First, a sequence of postures is obtained using given points, then each pair of neighboring postures is connected with three clothoid curve segments. In the second step, a completely general method to connect a path of clothoid curves between two completely arbitrary postures was not envisioned and a method for a pair of adjacent postures was developed. By virtue of the property of clothoid curves, a generated path is continuous with respect to position, tangent direction, and curvature, and is linear in curvature. Aside from the properties innate to clothoid curves, the generated paths transition smoothly into turns, pass through all the way points, and sweep outside the corners. For interpolating around obstacles that are commonly inside the corner, these properties are especially useful.

1 Introduction

The robot vehicle navigation problem can be divided into four subproblems: *world perception, path planning, path generation,*[1] and *path tracking,* which can be solved separately (Figure 1). World perception senses the world and symbolizes it into features. Path planning typically uses these features to obtain an ordered sequence of *objective* points that the robot must attain. Path generation converts the sequence of objective points into a path. Then, path tracking controls the vehicle to follow the path.

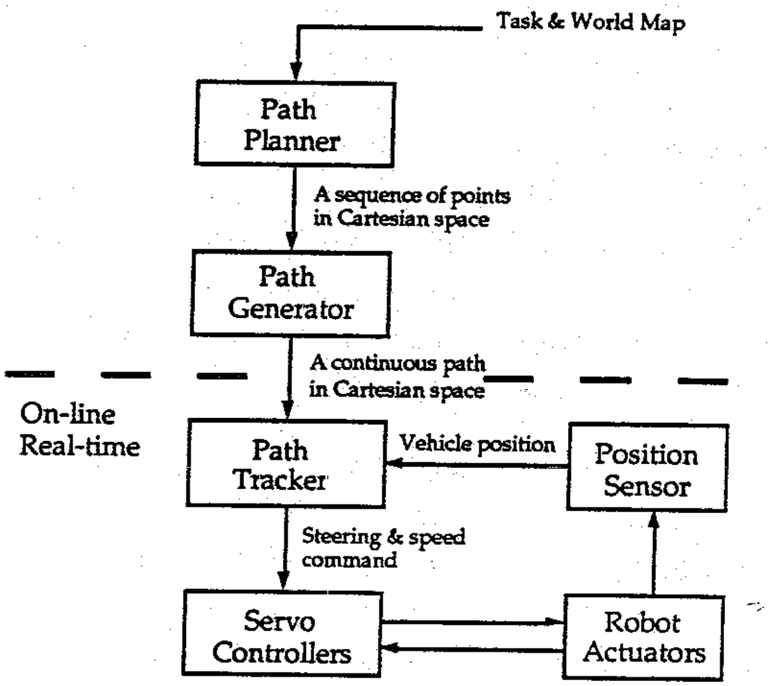

Figure 1: Control Structure for the Autonomous Vehicle

As with any control system, the response of an autonomous vehicle in tracking a reference path depends partly on the nature of the reference path. Actuators are not perfect; thus, if a path is easier for a robot to follow, there will be fewer errors.

[1] Path planning and path generation together are often referred just as path planning. Some navigation schemes do not generate a path, but compute the safe instant motion of the vehicle as path planning [Khatib85, Krogh84, Feng89]

not perfect; thus, if a path is easier for a robot to follow, there will be fewer errors. For a robot vehicle whose guide point is at the center of its rear axle, discontinuities are troublesome because an infinite acceleration of the steering wheel is required to track the reference path. A real robot (with finite acceleration) traveling through such transition points will experience an offset error along the desired path. In addition, the extent to which steering motions are likely to keep the vehicle on the desired path can be correlated to the linearity of curvature of the path, since linear curvature along a path requires linear steering velocity. In this paper, the condition for paths that are intrinsically amenable to follow is briefly presented, and then a method for the generation of amenable paths is proposed.

Numerous path representation methods have appeared in the literature. In the simplest method, a path is composed of a sequence of straight lines and circular arcs; others have used polynomial splines or clothoid curves. Previous methods have, in general, sought to simplify a path by rounding corners, which result from straight lines connecting a given sequence of points [Kanayama85, Nelson88]. The method developed in this paper, however, generates a continuous path with clothoid curves directly from the given points. In addition, the method guarantees continuity of position, heading , and curvature along the generated path. Further, the steering rate is piecewise constant.

The following section presents criteria for paths that are suited to path tracking. The performance of a vehicle tracking a path that was designed without concern for ease of tracking is compared with performance on a path that is intrinsically easier to track. Section 3 presents a method for generating paths with clothoid curves. Then, in Section 4, the results are shown to evaluate the performance of the proposed method. Finally, Section 5 advances the conclusions and suggests the directions for future research.

2 Continuous Paths

2.1 Guide Point

For our analysis a bicycle model is used as an archetype for modeling robot vehicles with two degrees of freedom, steering, and propulsion. The guide point is the point of the vehicle that is controlled to follow the given path. The choice of the guide point is an important decision—it affects the desired steering and propulsion functions required to follow the given path and speed. We have chosen the guide point to be at the midpoint of the rear axle (Figure 2) resulting in the following advantages:

- The steering angle at any point on the path is determined geometrically, independent of speed, in the following manner :

$$\tan\phi = \frac{l}{r} \; ; \qquad \phi = \tan^{-1} c\, l \tag{1}$$

where l is the wheelbase of the vehicle and c is the curvature of the path.

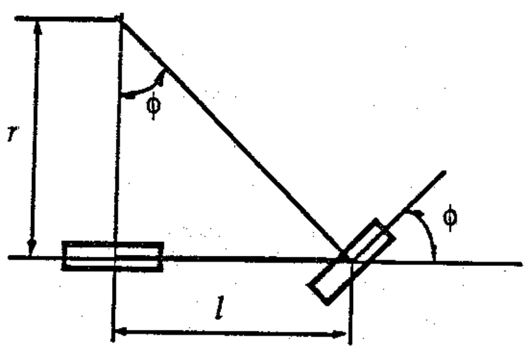

Figure 2: Geometry of a Bicycle Model

Also, the angular velocity of the driving rear wheel is determined only by the vehicle speed (v) and the radius of the wheel (R_w):

$$\omega = \frac{v}{R_w} \tag{2}$$

If the guide point is placed elsewhere, expressions for ϕ and ω are more complex than shown in Equations 1 and 2. The reference steering angle and angular velocity of the driving wheel must then be obtained by numerical integration.

- The vehicle is able to follow the minimum turning radius for the maximum steering angle [Nelson88]. In other words, the peak steering angle is smaller than that for any other guide point.
- The heading of the vehicle is aligned with the tangent direction of the path. This gives a more reasonable vantage point for a vision camera or a range scanner mounted at the front of the vehicle, as in Figure 3, and a smaller area is swept by the vehicle.

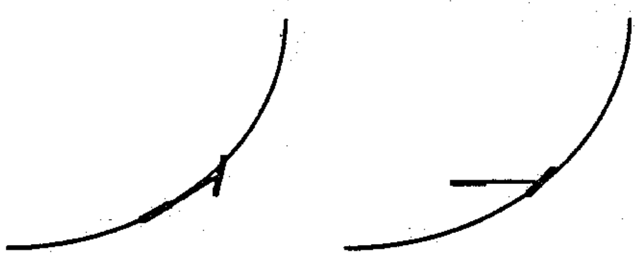

Figure 3: The Heading Alignment along a Path

For the guide point at the center of the rear axle, paths with discontinuities of curvature will require infinite acceleration of the steering wheel. Consider the motion of the vehicle along a path which consists of two circular arcs (Figure 4). There is a discontinuity in curvature at the point where the two circular arcs meet. An infinite acceleration in steering is required for the vehicle to stay on the specified path provided that it does not come to a stop at the transition point, simply because it takes a finite amount of time to switch to the new curvature. In reality, moving through a transition point with non-zero velocity results in an offset error along the desired path. Likewise, rotary body inertia requires continuity of heading, and steering inertia requires continuity of curvature.

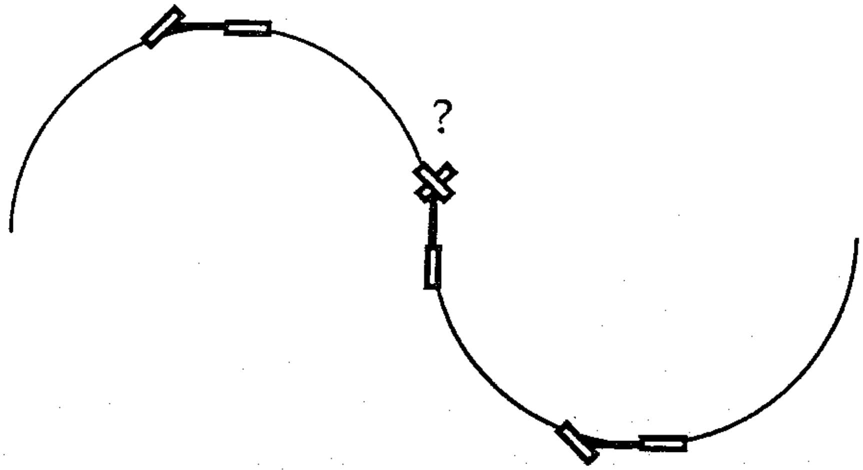

Figure 4: Steering along Circular Arcs

This is not the case if the guide point is moved from the rear axle, for example, to the front wheel. However, this choice loses the advantages discussed above.

Selection of the guide point is important to the formulation of the path tracking problem. When the guide point is chosen on the center of the rear axle, steering and driving reference inputs are computed from the path parameters and vehicle speed, respectively, as in Equation (2). This enables the planning and controlling of steering along a path to be independent of speed. It is referred to as *path* tracking, in contrast to trajectory tracking in which steering and speed are controlled along a time history of position [Shin90].

2.2 Vehicle Path Modeling

The configuration of a conventionally steered vehicle moving on a plane surface can be described completely by a set of coordinates (x, y, θ_V, ϕ); vehicle position, heading, and steering angle, respectively. The continuity of (x, y, θ_V, ϕ) is recommended, because the vehicle has inertia and finite control response, which preclude discontinuous motions. Otherwise, tracking errors will be greater. Since a path specifies an ideal motion of a conventionally steered vehicle, a path must guarantee the continuity of (x, y, θ_V, ϕ) of a vehicle.

A path can be parameterized in terms of path length s as $(x(s), y(s))$. Tangent direction $(\theta(s))$ and curvature $(c(s))$ can be derived along a path:

$$\theta(s) = \frac{dy(s)}{dx(s)} \tag{3}$$

$$c(s) = \frac{d\theta(s)}{ds} \tag{4}$$

The continuity of (x, y) is guaranteed if a path is continuous. But the effect of the continuity of vehicle heading and steering on the path depends on the guide point. Since the guide point is chosen at the center of the rear axle, as mentioned in Section 2.1, the heading of the vehicle θ_v is aligned with the tangent direction of the path θ, and the steering angle ϕ is determined by the curvature of the path, as in equation (1) Thus, the continuity of (x, y, θ_v, ϕ) is tantamount to the continuity of (x,y,θ,c). If we define *posture* as the quadruple of parameters (x,y,θ,c), a posture describes the state of a conventionally steered vehicle, and a path is required to be posture-continuous for easy tracking.[3]

Further, the rate of change of curvature (sharpness) of the path is important, too, since the linearity of curvature of the path dictates the linearity of steering motion along the path (1). If we assume that less control effort is required for an actuator to provide a linear velocity profile than an arbitrary nonlinear one, the extent to which steering motions are likely to keep a vehicle on a desired path can be correlated to the linearity of curvature of the path.

Certain spline curves [Nelson88] are good candidates for good paths, because they guarantee posture-continuity along paths. However, these spline curves do not guarantee linear gradients of curvature. Clothoid curves, by contrast, do not have the same properties of curvature and vary linearly. In the following subsection, clothoid curves are introduced and, are compared the performances of a vehicle tracking a clothoid path and an arc path.

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[3] Omni directional locomotion systems, such as an ilonator wheel system [Daniel85] and legged systems, can follow paths with discontinuities of tangent direction and curvature, since they can subtend pure rotation or pure lateral motion. However, continuity of curvature is still recommended because it provides smooth changes in the centrifugal force: $F_{cg} = \frac{v^2}{r} = c\,v^2$

2.3 Clothoid Curves

Clothoid curves [Kanayama85, Yates52] are a family of curves that are posture-continuous, and are distinct in that their curvature varies linearly with the length of the curve:

$$c(s) = k\,s + c_i \tag{5}$$

where k is the rate of change of curvature (sharpness) of the curve and subscript i denotes the initial state. A clothoid curve segment is shown in Figure 5.

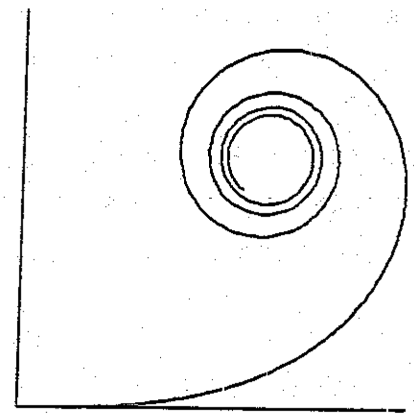


Figure 5: A Clothoid Curve

Given an initial posture, sharpness of the clothoid segment, and the distance along that segment, position, orientation, and curvature at any point along the curve are calculated as (5, 6, 7, and 8):

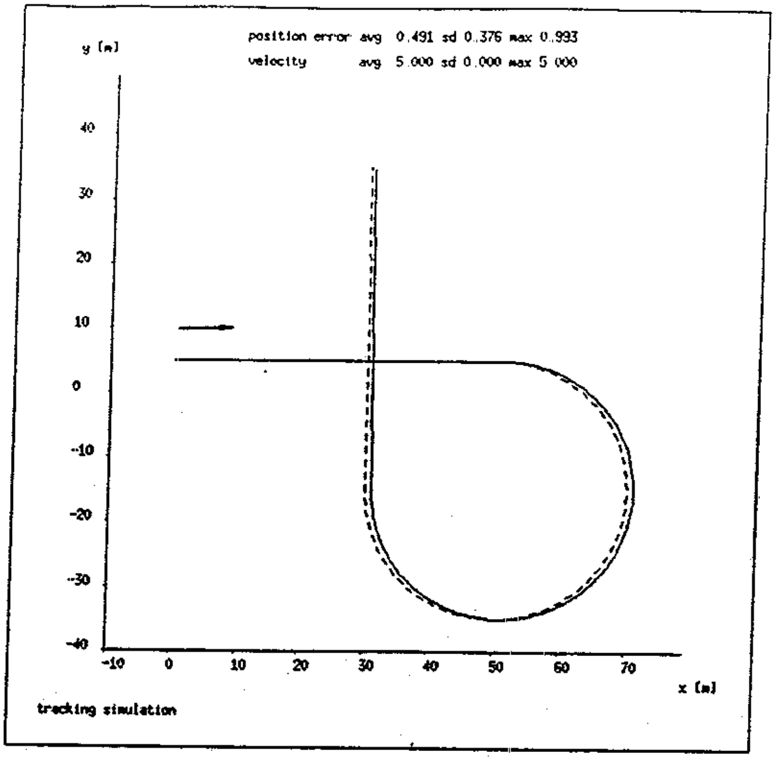
$$\theta(s) = \theta_i + \int_0^s c(\xi)\,d\xi = \frac{k}{2}s^2 + c_i\,s + \theta_i \tag{6}$$

$$x(s) \;=\; x_i + \int_0^s \cos\theta(\xi)\,d\xi \;=\; \int_s^0 \cos\{\frac{k}{2}\xi^2 + c_i\,\xi + \theta_i\}\,d\xi + x_i \qquad (7)$$

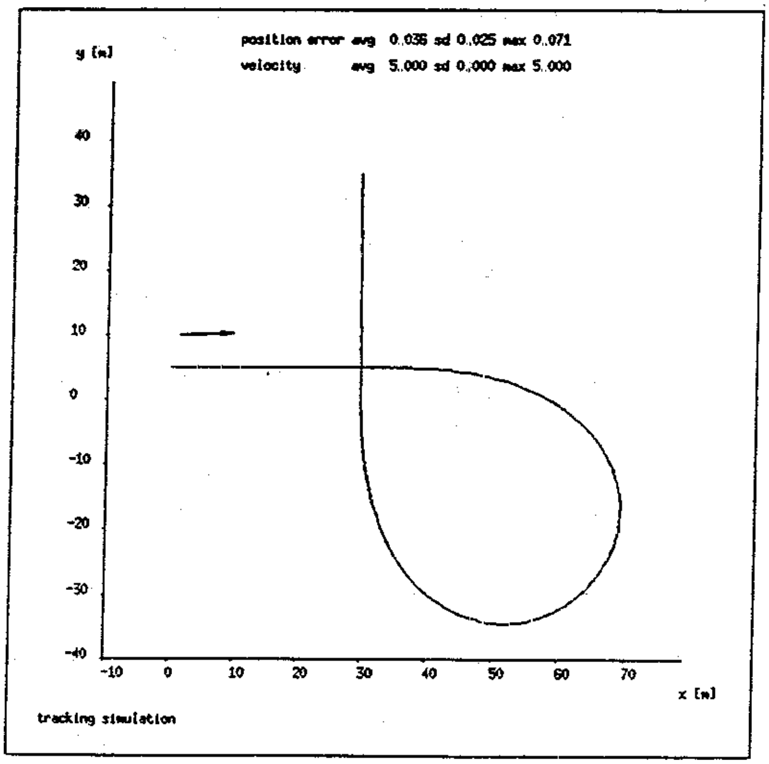
$$y(s) \;=\; y_i + \int_0^s \sin\theta(\xi)\,d\xi \;=\; \int_s^0 \sin\{\frac{k}{2}\xi^2 + c_i\,\xi + \theta_i\}\,d\xi + y_i \qquad (8)$$

The response of a robot vehicle to different reference paths was compared. Open loop responses of a vehicle following a path composed of arcs and straight lines, and another composed of clothoid segments, are shown in Figures 6, and 7: The vehicle is made to follow a set of steering commands, which are obtained from the path specification using (1), without trying to compensate for tracking errors. The dynamic responses of the vehicle are modeled as first order lag systems with hard limits on acceleration. In each case, the reference path is indicated by dashed lines, while the actual path is indicated by solid lines. Simulation results show that an arc path results in larger steady state tracking errors caused by discontinuities in curvature, as well as the parameter variation of actuator and vehicle speed.

While the results shown above support the claim that "good" paths are intrinsically easier to track, it turns out that when using an optimized closed scheme and fast actuators, tracking performance is only marginally better for smooth, linear-curvature paths than for paths composed of arcs and straight lines, especially at low speeds. At low speeds, the compensating controller makes up for errors from curvature discontinuities, non-linearities, and imperfect actuators. For tracking paths with significant curvature, it is essential to ensure curvature continuity at high speeds or with slow actuators.

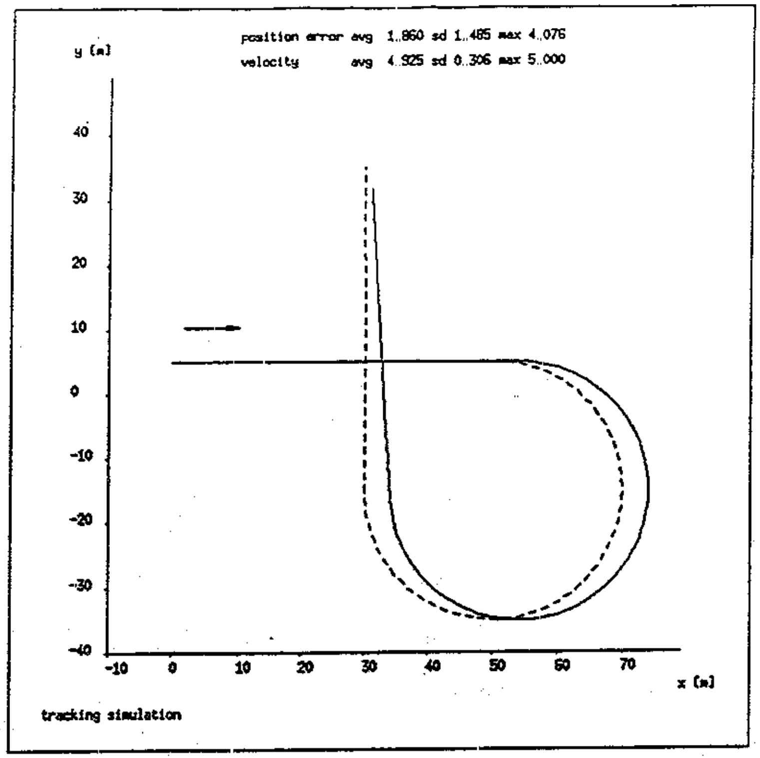


(A) Along a path composed of arcs and straight lines

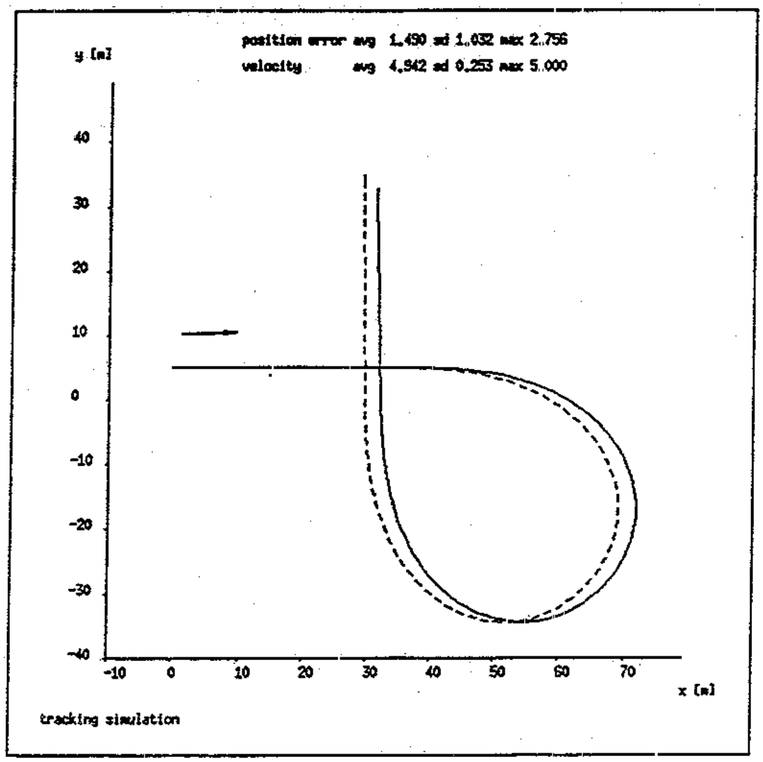


(B) Along a path composed of clothoid segments

Figure 6: Open-loop Performance:Time Constant = 0.1 s, Vehicle Speed = 5 m/s



(A) Along a path composed of arcs and straight lines



(B) Along a path composed of clothoid segments

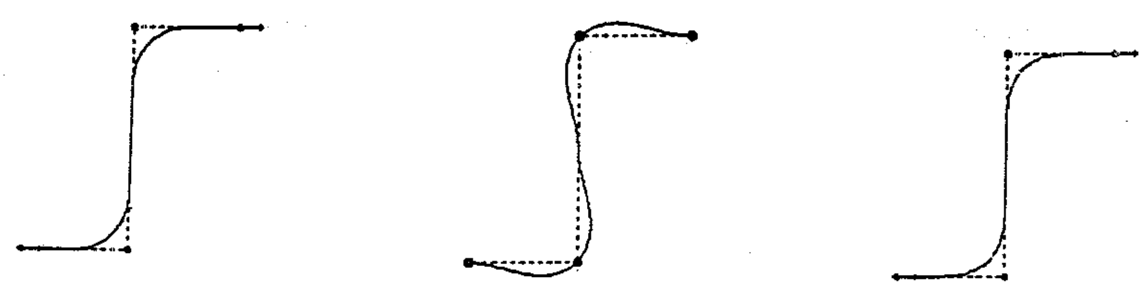
Figure 7: Open-loop Performance:Time Constant = 0.5 s, Vehicle Speed = 2 m/s

3 Generating Posture-Continuous Paths

The previous section discussed the condition for a path that is intrinsically easy to follow. This section presents a method to generate such paths. The problem posed in generating paths is: how to produce a unique, easily trackable, continuous path from a given sequence of points. This problem is similar to those addressed by previous path generation methods [Kanayama85, Nelson88].

3.1 Existing Path Generation Methods

Hongo et al. proposed a method to generate continuous paths composed of connected straight lines and circular arcs from a sequence of objective points (Figure 8) [Hongo85]. While paths composed solely of arcs and straight lines are easy to compute, such a scheme leaves curvature discontinuities at the transitions of the segments, as discussed previously.



(A) By Arc method (B) By Cubic Spline Method (C) By Kanayama's Method

Figure 8: Paths Generated by Existing Methods

Kanayama proposed pairs of clothoids segments as a method to interpolate between given points [Kanayama85]. The problem with clothoids is that though they guarantee smoothness and linearity, it is non-trivial to generate clothoid segments for arbitrary starting and ending postures; the expressions for these curves are underconstrained, and further, no closed form expression is available. (The expression must be evaluated using numerical methods.) Kanayama's scheme is tenable under the simplification made by the requirement that the starting and the ending curvature at the via points are always zero. This model is adequate for simple paths for which symmetric pairs of clothoid curves can be found or for paths with mild curvatures that can be easily broken down in separate clothoid segments.

Certain polynomial spline curves [Mortenson85] are candidates for path segments, because they guarantee continuity of posture. Nelson proposed quintic spline and polar spline curves [Nelson88] obtained from two-point boundary conditions. However, these spline curves do not guarantee linear gradients of curvature. Clothoid curves, by contrast, do vary linearly with the distance along the curve.

3.2 Path Generation from a Sequence of Points

The following two-step method generates a unique posture-continuous path from a sequence of points. The first step is to derive a sequence of unique postures from the objective points; the second, is to interpolate between those postures with clothoid segments. Heading and curvature at the starting and ending positions are presumed from the configuration of the vehicle, using equation (1).

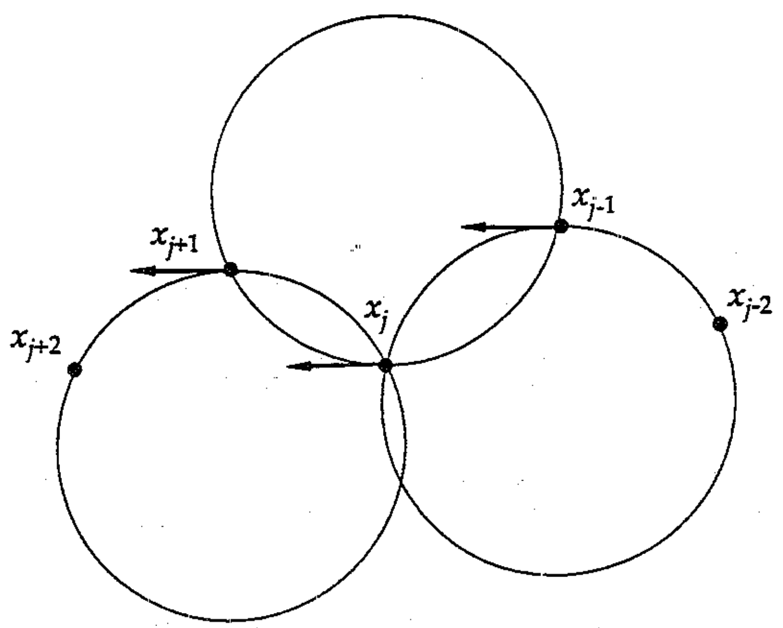


Figure 9: Obtaining Postures and Associated Circles from Objective Points

Let $[X = (X_0, \ldots, X_n) = ((x_0, y_0), \ldots, (x_n, y_n))]$ be a sequence of objective points. The *associated circle* at X_j is defined to be the circle which passes through points X_{j-1}, X_j, X_{j+1} as in Figure 9. Then the heading of the vehicle at X_j is taken as the direction of the tangent to the associated circle at X_j, and the curvature is the reciprocal of the osculating radius of the associated circle, denoting the posture thus obtained as the *associated postures*, The next step is to connect neighboring postures with clothoid segments.

It is not always possible to connect two neighboring postures with one clothoid curve segment, because four governing equations (5, 6, 7, and 8) cannot be satisfied simultaneously with only the two parameters (sharpness k and length s) that a clothoid curve provides. To satisfy these four equations, at least two clothoid segments are needed. However, the general problem cannot be solved with only two clothoid segments. Figure 10 shows two pairs of associated postures and their associated circles. Let P_i, P_f denote the starting and the ending postures, respectively. And C_i, C_f denote the associated circles corresponding to the curvatures at P_i, P_f. They are drawn by solid lines and dotted lines, respectively.

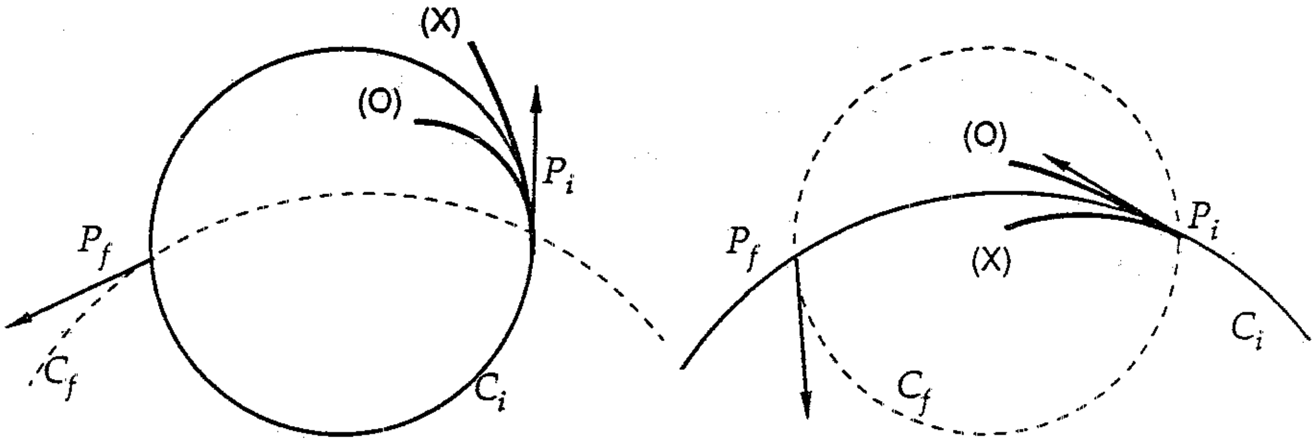


Figure 10: Determination of the Sign of the First Clothoid Segment

As in (A) of Figure 10, if the orientation of P_f is outward from C_i, the ending part of a solution curve should be inside C_i. Then, it is plausible that the starting part of a solution curve also lies inside C_i. Similarly, if the direction of the P_f is inward into C_i, as in (B) of Figure 10, it is plausible that the starting part of a solution curve lies outside C_i. Note that the sign of the sharpness determines the side of the associated circle on which the clothoid segment lies: (1) If the sharpness is zero, then the clothoid curve remains on the associated circle in question. (2) If the sharpness is positive, then the clothoid curve will be in the left side of the associated circle. (3) Otherwise, it will be in the right side of the associated circle. Table 1 lists all the possible cases of relative geometry of the first and last segments of the solution curve to the circles C_i and C_f (The convention used is that negative curvature equates to a right turn and a positive curvature equates to left turn). Hence, a solution curve should satisfy the following proposition:

> If the direction of P_f is outward from C_i, the sign of k of the first clothoid segment is chosen so that the curve lies inside C_i.
>
> If the direction of P_f is inward into C_i, the sign of k of the first clothoid segment is chosen so that the curve lies outside C_i.
>
> Otherwise, k of the first clothoid segment is chosen so that the curve remains on C_i.[4]

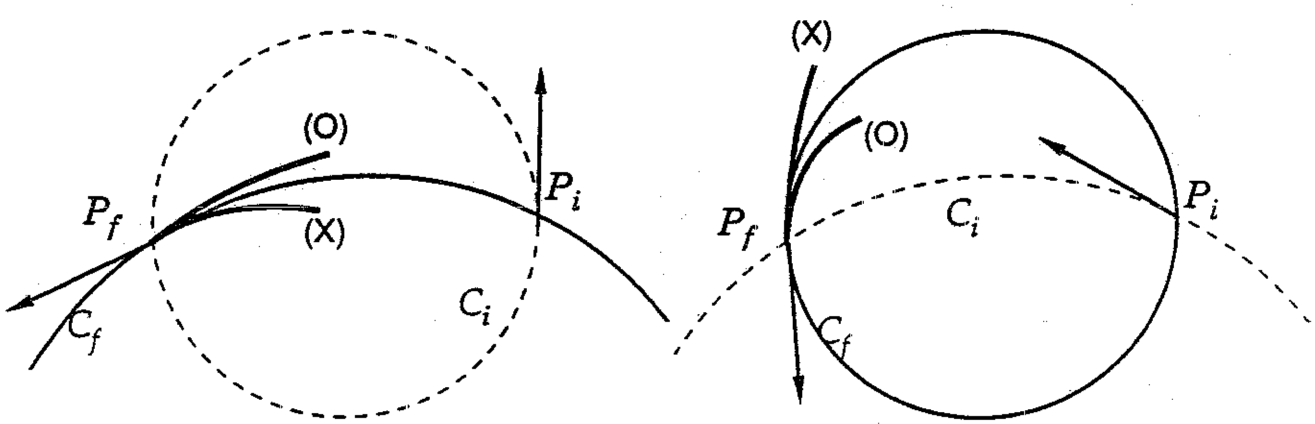


Figure 11: Determination of the Sign of the Last Clothoid Segment

As a corollary to the above proposition, the sign of k of the last clothoid segment can be determined (Figure 11):

[4] In this case, P_i and P_f share the same associated circle, two postures are connected with the part of their associated circle, which is a clothoid curve of zero sharpness.

If the direction of P_i is outward from C_f, the sign of k of the last clothoid segment is chosen so that the curve lies outside C_f.

If the direction of P_i is inward into C_f, the sign of k of the last clothoid segment is chosen so that the curve lies inside C_f.

Otherwise, k of the last clothoid segment is chosen so that the curve lies on C_f.

Figure 12 shows all possible cases of curvature variations between a pair of neighboring postures. Notice that the signs of k for the first and the last segments are the same for each case.[5] However, the sign is the opposite of that required for the curvature variation between the postures for all the cases except (C) and (D) of Figure 12. Thus, the general problem to connect a pair of neighboring associated postures cannot be solved with two clothoid curve segments.

[5] Determination of the sign of sharpness can be quantitatively summarized, as in Appendix.

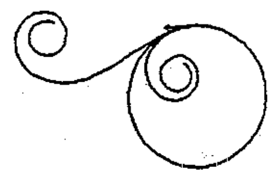
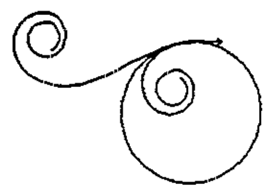
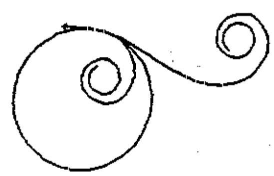
curvature	sharpness	curve location	illustration
$C_i > 0$	$k_i > 0$	inside the associated circle of the starting posture	
	$k_i < 0$	outside the associated circle of the starting posture	
$C_i < 0$	$k_i > 0$	outside the associated circle of the starting posture	
	$k_i < 0$	inside the associated circle of the starting posture	
$C_f > 0$	$k_f > 0$	outside the associated circle of the ending posture	
	$k_f < 0$	inside the associated circle of the ending posture	
$C_f < 0$	$k_f > 0$	inside the associated circle of the ending posture	
	$k_f < 0$	outside the associated circle of the ending posture	

Table 1: Curve Location by Sharpness and Curvature

One adequate solution set of the clothoids is the set of three clothoid segments (k, s_1), $(-k, s_2)$, (k, s_3). The subscripts denote the order of the clothoid segments from P_i. This combination is plausible for the following reasons:

1. The signs of sharpness for the first and last clothoid segments are the same.
2. The sharpness for the second clothoid segment is equal in magnitude and opposite in sign to the first and last segments. This enables the curve of three clothoid segments to satisfy the curvature variation between the starting and the ending postures by varying s_1, s_2, s_3, even though the sign of the first and the last clothoid segments satisfies the curve location requirement.
3. There are four variables in the combination: k, s_1, s_2, s_3. It is possible to find a unique solution satisfying the following four equations which describe the mathematical relationship between the starting and ending postures:

$$c_f = c_i + k\,(s_1 - s_2 + s_3) \tag{9}$$

$$\theta_f = \theta_i + c_i\,(s_1 + s_2 + s_3) + k\,(s_1 s_2 - s_2 s_3 + s_3 s_1) + \frac{k}{2}(s_1^2 - s_2^2 + s_3^2) \tag{10}$$

$$x_f = x_i + \int_0^{s_1} \cos\theta_1(\xi)\,d\xi + \int_0^{s_2} \cos\theta_2(\xi)\,d\xi + \int_0^{s_3} \cos\theta_3(\xi)\,d\xi \tag{11}$$

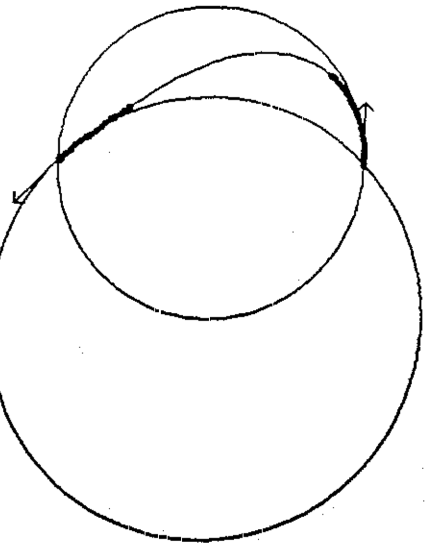
$$y_f = y_i + \int_0^{s_1} \sin\theta_1(\xi)\,d\xi + \int_0^{s_2} \sin\theta_2(\xi)\,d\xi + \int_0^{s_3} \sin\theta_3(\xi)\,d\xi \tag{12}$$

where

$$\theta_1(\xi) = \theta_i + c_i\xi + \frac{k}{2}\xi^2$$

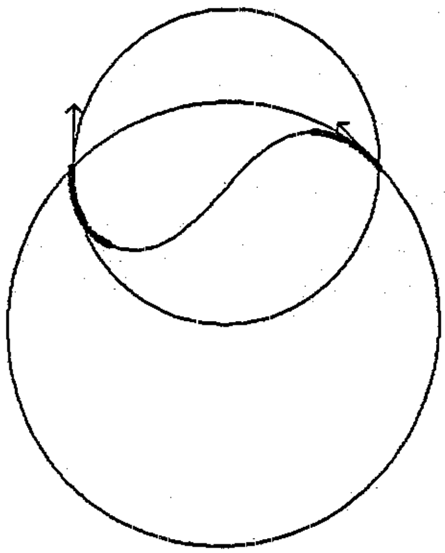
$$\theta_2(\xi) = \theta_i + c_i s_1 + \frac{k}{2}s_1^2 + (c_i + k s_1)\xi - \frac{k}{2}\xi^2$$

$$\theta_3(\xi) = \theta_i + c_i(s_1 + s_2) + k\,(s_1 s_2) + \frac{k}{2}(s_1^2 - s_2^2) + \{c_i + k\,(s_1 - s_2)\}\xi + \frac{k}{2}\xi^2$$



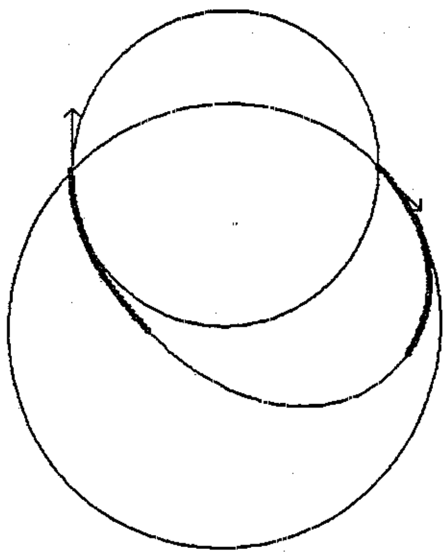
(A) when $c_i > c_f > 0$,

$k_i, k_f > 0$



(B) when $c_i > 0 > c_f$,

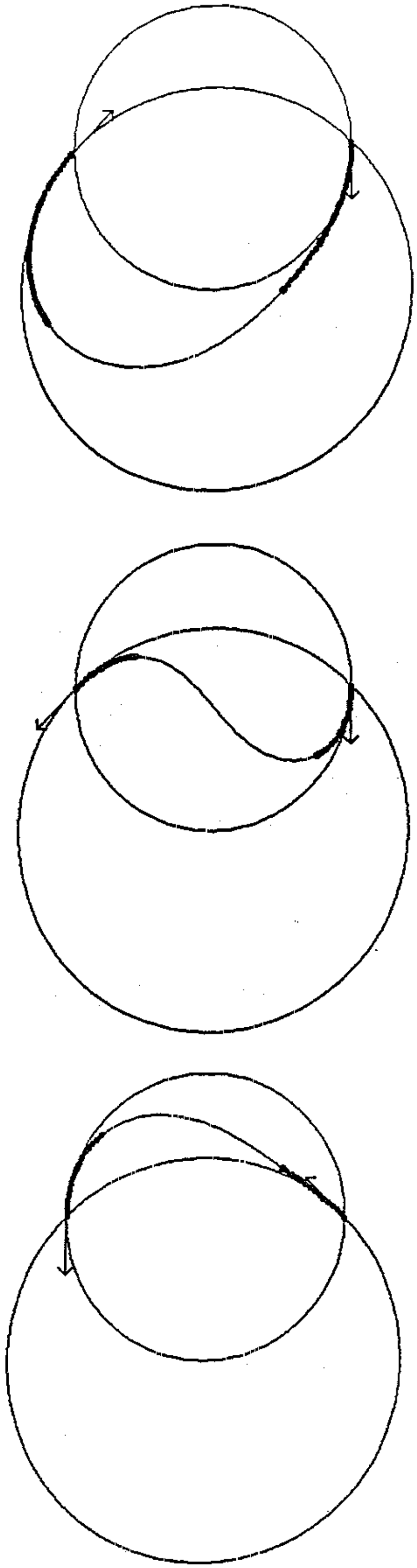
$k_i, k_f > 0$



(C) when $0 > c_i > c_f$,

$k_i, k_f < 0$

Figure 12: Cases by Curvature Variation of P_i and P_f



(D) when $0 > c_f > c_i$,

$k_i, k_f > 0$

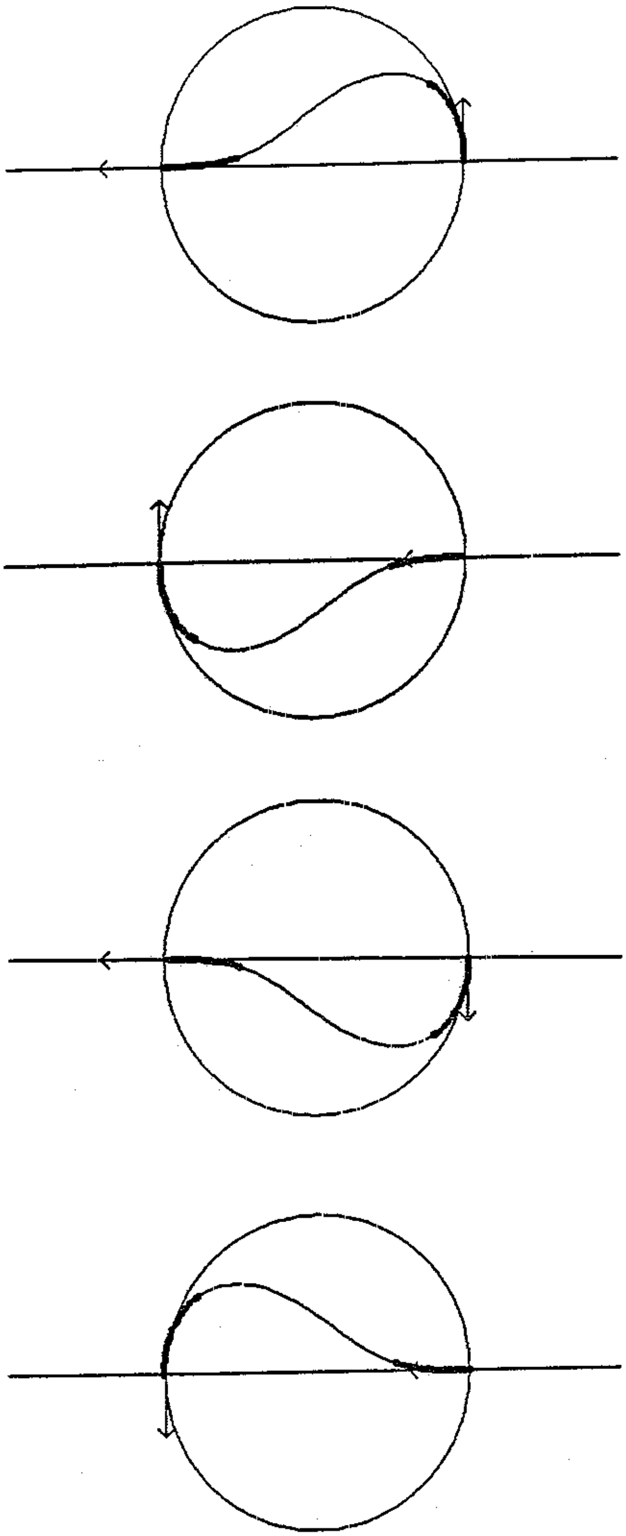
(E) when $c_f > 0 > c_i$,

$k_i, k_f < 0$

(F) when $c_f > c_i > 0$,

$k_i, k_f < 0$

Figure 12: Cases by Curvature Variation of P_i and P_f



(G) when $c_i > c_f = 0$, $k_i, k_f > 0$

(H) when $0 = c_i > c_f$, $k_i, k_f > 0$

(I) when $0 = c_f > c_i$, $k_i, k_f < 0$

(J) when $c_f > c_i = 0$, $k_i, k_f < 0$

Figure 12: Cases by Curvature Variation of P_i and P_f

Since Equations (11) and (12) contain Fresnel integrals, for which there is no closed form solution, the values of k, s_1, s_2, s_3 are computed using the numerical method [Press86] outlined in Figure 13.

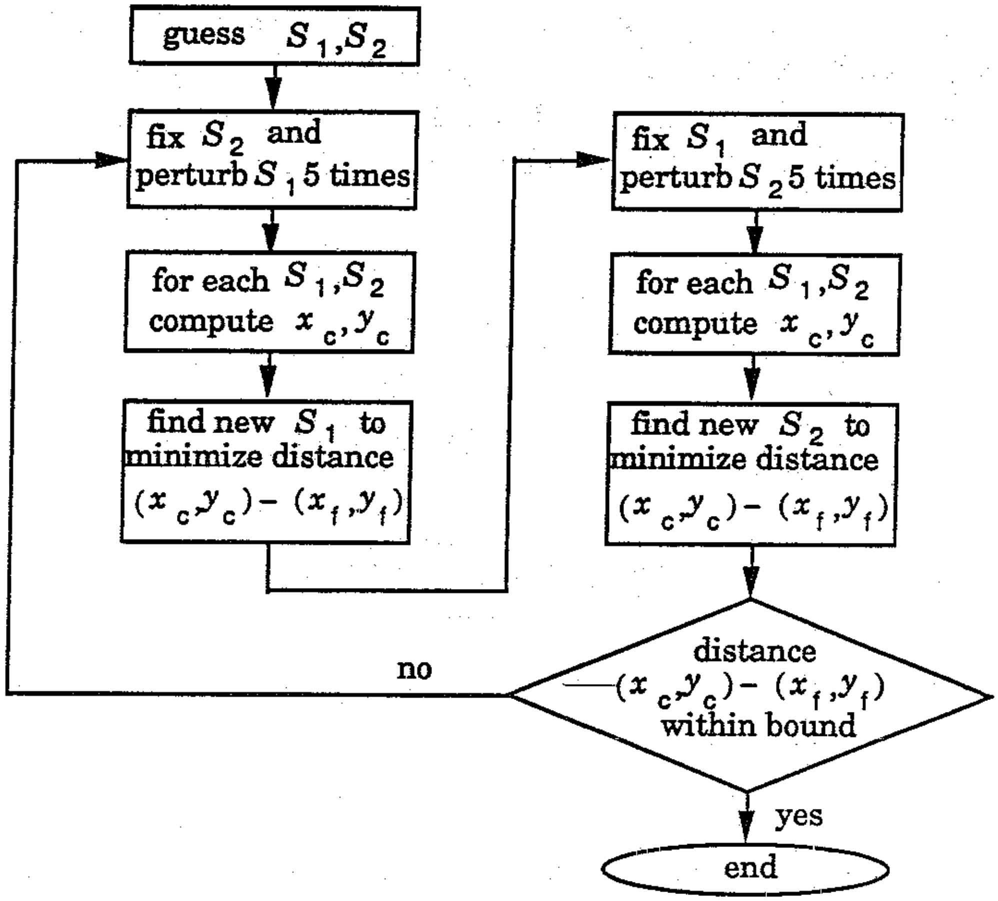


Figure 13: A Numerical Method to Compute the Fresnel Integrals

Initial values for s_1 and s_2 are chosen to be $\frac{1}{3}$ of the average of the lengths of two of the arcs that connect P_i and P_f. Equations (9) and (10) are used to compute k and s. Then, x_c, y_c can be computed for the quadruple (k, s_1, s_2, s_3) using Simpson's approximation. Ideally $x_c, y_c = x_f, y_f$ and in fact values of s_1 and s_2 are adjusted until the difference is within a threshold.

4 Results

Connecting a pair of neighboring associated postures was accomplished successfully, as in Figure 12, using three clothoid curves. Figure 14 shows the graphic result of a posture-continuous path generated by clothoid curves through the given seven points. As the first step of the proposed method, a sequence of seven postures were generated. (Seven arrows in Figure 14). Then, three clothoid segments are used to interpolate between neighboring postures.

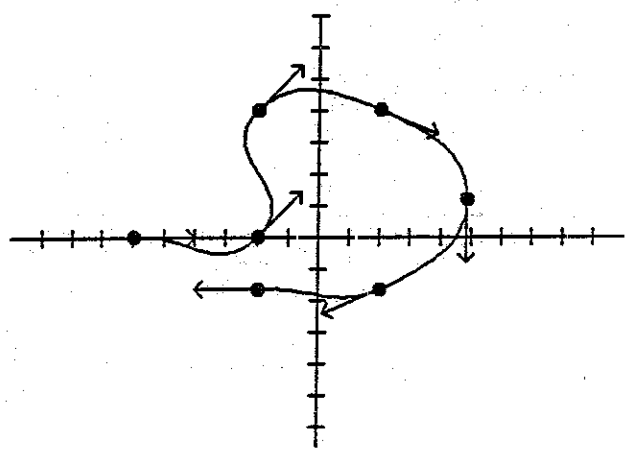
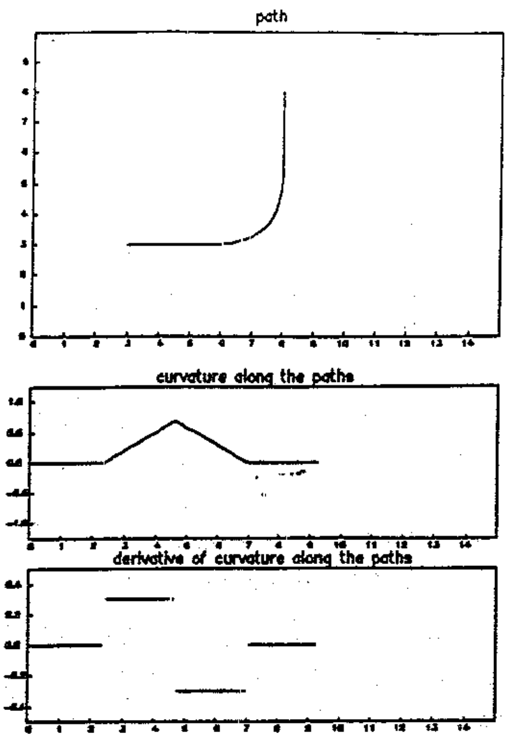


Figure 14: Posture-continuous Path From a Sequence of Seven Points

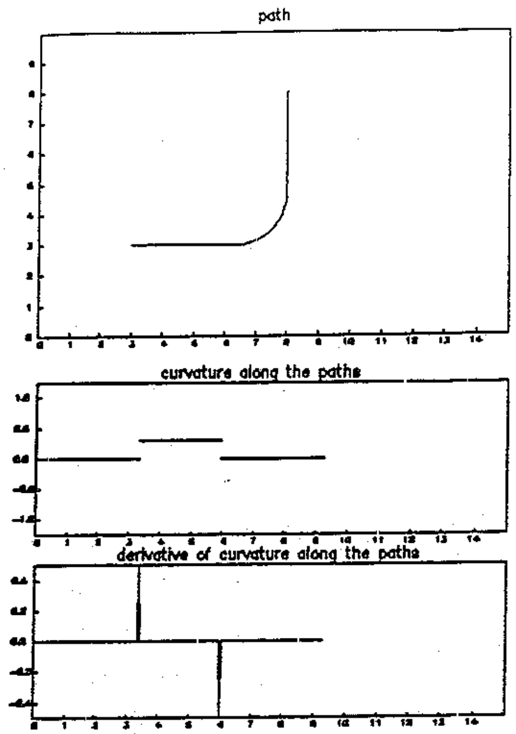
Figure 15 shows a comparison of the performance of the proposed method using arcs, splines, and Kanayama's method [Kanayama85]. Parameters of curvature and sharpness were constrained equally for all methods. The maximum sharpness of Kanayama's method and the maximum curvature used in the arc method are set at the same levels as in the proposed method. Paths, curvatures and sharpness along the paths are compared. Paths resulting from the proposed method have the following advantages over other methods:

- The method proceeds from an arbitrary sequence of points. Generation of postures is essential to exploratory planning where goals are commonly posed as an evolving string of points. Paths generated by the proposed method pass through all the objective points, as in Figure 15; whereas paths from Kanayama's method and the arc method are only proximate to many of the points because these methods start from a sequence of postures that must be modified to ensure curvature at the via points is zero.

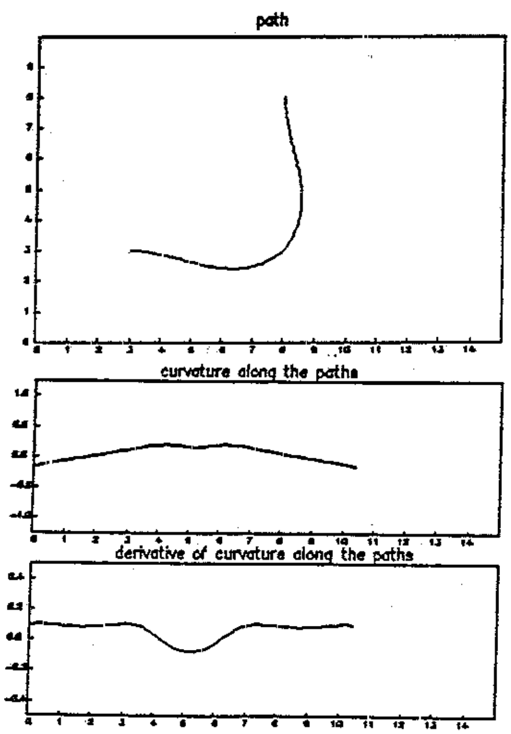
- The method guarantees continuity of position, heading, and curvature along the path. Further, sharpness is piecewise constant.
- Paths generated by the method always sweep outside the acute angles formed by straight line connection of the way points, as in Figure 15. The resulting paths are especially useful for interpolating around obstacles that are commonly on the inside of angles. In contrast, Kanayama's paths are always inside path angles.



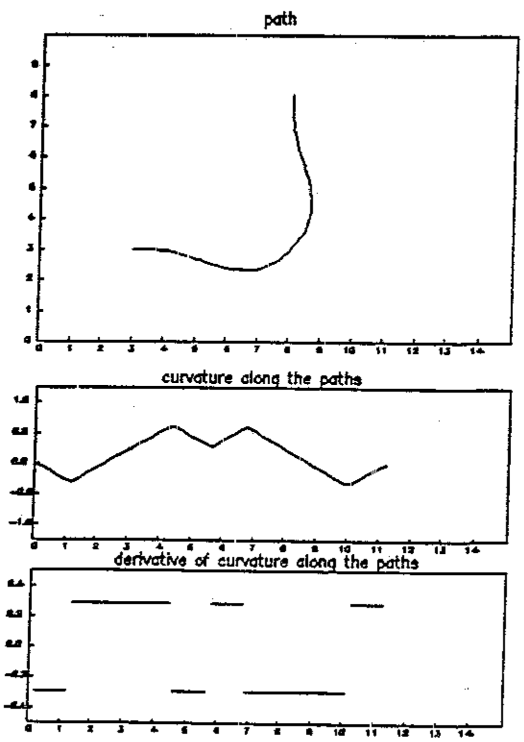
(1) Clothoid with zero curvature transition



(2) Circular arcs and straight lines



(3) Cubic splines



(4) Proposed method

Figure 15: Comparison of the Proposed Method with Three Others.

5 Conclusion

Posture-continuity is presented as the condition for a path that is intrinsically easy to track. In addition, the linearity of curvature of the path is correlated to linear steering motion, facilitating path tracking. Clothoid curves are good path candidates to satisfy the condition. Simulation shows better performance of a vehicle tracking paths of clothoids versus paths of arcs.

A method for generating a continuous path was developed. This method uses clothoid segments and consists of two steps: First, a sequence of the postures is obtained using the objective points. Then, each pair of neighboring postures is connected with three clothoid curve segments.

The method provides additional advantages in that preprocessing of the objective points is not necessary, as with arc and zero curvature clothoids. Further, the geometry of the paths generated always sweeps outside the acute angles formed by a straight-line connection of the way points. These are especially useful for interpolating around obstacles that are commonly on the inside of angles.

The method of obtaining postures, as in Figure 9, requires that the circles formed by the radii of curvature of two postures intersect. Relaxing such a constraint would require heuristics to determine intermediate postures. Assuming intermediate postures could be found, such that the associated circles intersect, the method could then be used on the new set of postures. Thus far, the search for a completely general method that would generate a path between two completely arbitrary postures has not been fruitful.

Directions for future research include the following: (1) optimization of the lengths of more than three clothoid segments on the premise that some cost function can be used to find a better solution than the results here; (2) improvement of the numerical method to connect postures through the Fresnel Integral to improve speed and accuracy.

Appendix

Determination of the Sign of Sharpness

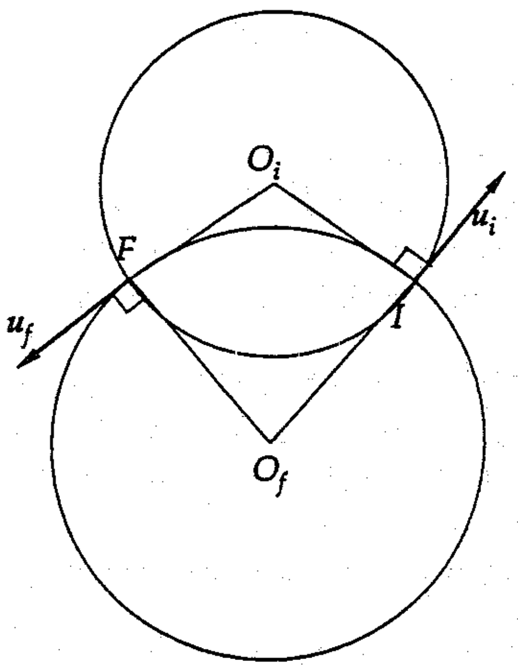


Figure A1: Relative Geometry of Associated Postures

When a pair of neighboring associated postures are connected with clothoid curves as in Section 3.3.2, the sign of sharpness for the first and the last clothoid segments can be determined quantitatively as follows. Let u_i, u_f be unit vectors which have the direction of P_i and P_f respectively, as in Figure A1. A straight forward test can be applied to determine whether the posture is *inward* or *outward* from C_i or C_f:

If $u_f \bullet FO_i < 0$, P_f is *outward* from C_i.

If $u_f \bullet FO_i > 0$, P_f is *inward* into C_i.

If $u_f \bullet FO_i = 0$, P_f is *on* C_i.

If $u_i \bullet IO_f < 0$, P_i is *outward* from C_f. (A3.1)

If $u_i \bullet IO_f > 0$, P_i is *inward* into C_f.

If $u_i \bullet IO_f = 0$, P_i is *on* C_f.

While the *If* condition of the Proposition is described as Equation (A3.1), the *then* result is quantitatively described, as in Table 1. For example, if the first clothoid segment lies inside C_i, its sign is determined by c_i (the curvature of C_i):

If $c_i > 0$, $k_i > 0$

If $c_i < 0$, $k_i < 0$ (A3.2)

If $c_i = 0$, the sign of k_i is determined as the sign of k_f.

Combining (A3.1) and Table 1, the sign of sharpness is determined as follows:

If $u_f \bullet FO_i \bullet c_i < 0$, $k_i > 0$

If $u_f \bullet FO_i \bullet c_i > 0$, $k_i < 0$

If $u_f \bullet FO_i \bullet c_i = 0$, the sign of k_i is determined as the sign of k_f.

If $u_i \bullet FO_f \bullet c_f < 0$, $k_f > 0$

If $u_i \bullet FO_f \bullet c_f > 0$, $k_f < 0$

If $u_i \bullet FO_f \bullet c_f > 0$, the sign of k_f is determined as the sign of k_i.

Note that the proposed method cannot be generalized to the case where both neighboring curvatures are zero. However, this configuration is never generated by the first step shown in Figure 8. If necessary, this case can be solved using clothoids with zero curvature transitions [Kanayama85].

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