

Estimation, Modeling, and Control of Mixed Material Flows on Variable-Speed Conveyor Belt Systems with Applications in Recycling

James Robert Maier
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The Robotics Institute
School of Computer Science
Carnegie Mellon University
Pittsburgh, PA

Thesis Committee:

Dr. Matthew Travers, *chair*
Dr. Howie Choset
Prasanna Sriganesh

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To my wife Sidney and my dog Maisie :)

Abstract

In modern solid waste recycling, Material Recovery Facilities ("MRFs") separate post-consumer waste streams into single material products for use in manufacturing. MRFs face a number of challenges including difficulties with staffing, contamination, and highly variable process infeeds, to the point where the average American MRF only achieves roughly 60% of its design throughput. To address these issues, in this work we describe a novel framework for state representation of transient material flows in MRFs and we provide examples of how our framework can be used to design systems for real-time measurement, state estimation, and control of these systems. We provide results both in simulation and in physical experiment which suggest process throughput increases on the order of 40%. We then discuss the implications of this work for the recycling industry as well as suggest direction for future work.

Acknowledgments

This thesis contains work previously published in "Longitudinal Control Volumes: a Novel Centralized Estimation and Control Framework for Distributed Multi-agent Sorting Systems", of which James Maier was the primary author and contributor.

Contents

1	Introduction	1
1.1	Recycling and MRFs	1
1.2	Optimization of MRF operations via conveyor belt speed control . . .	2
2	Related Work	5
3	Longitudinal Control Volumes: Mathematical Formulation	11
3.1	Longitudinal Control Volumes (LCV)	11
3.2	State Transition Model	12
4	System Design	15
4.1	State Estimation	15
4.2	Model Predictive Control	18
5	Experiments and Results	21
5.1	Simulation-based evaluation of MPC performance	21
5.2	Full-system Experiments on Physical Sorting Equipment	22
5.3	Results	22
6	Conclusions	29
6.1	Discussion	29
6.2	Future Work	30
	Bibliography	31

When this dissertation is viewed as a PDF, the page header is a link to this Table of Contents.

List of Figures

1.1	Materials on an experimental conveyor belt at Carnegie Mellon being sorted by a human	3
1.2	A series of sorting agents with dynamics coupled by a single conveyor belt	4
2.1	Block diagram of binary Bayesian sorting unit (figure from [18])	8
2.2	Example for Q-Matrix for simple linear system (figure from [26]) . . .	8
2.3	Composition of binary Bayesian units into a model of a MRF process along with corresponding Q matrix for a single material (figure from [26])	8
3.1	Control volumes overlaid on conveyor belt serving a team of human and robot sorters	12
4.1	A block diagram of the proposed system to optimize belt speeds for sorting	16
4.2	Control volume boundaries overlaid on image with bounding box detections from object detection system. The red and blue bars at the bottom help illustrate binning for different materials and the measurement vector generated from this input image.	17
5.1	Physical conveyor belt experimental setup showing the RealSense D435 overhead camera, human sorter and the sorted materials	23
5.2	Result of a simulated 3-material sorting run. (a) Plot showing the instantaneous amount of material passing through a sort agent station (b) Plot showing the output belt speeds for the MPC and the baseline. (c) Plot showing the accumulated value as a result of successfully sorting.	25
5.3	Result of a 2-material sorting run. (a) Plot showing the instantaneous amount of material passing through a sort agent (b) Plot showing the output belt speeds for the MPC and the baseline. (c) Plot showing the accumulated value as a result of successfully sorting.	26

5.4	Result plot showing profit rate for optimized speed vs constant speed control	27
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Chapter 1

Introduction

1.1 Recycling and MRFs

Single stream recycling in the United States struggles with economic sustainability. Single Stream (where all recyclable end-of-life products and packaging are commingled at the collection point and are then sorted and sold at Material Recovery Facilities (MRFs)) originated as a concept in the late 1980's and early 1990's as an alternative to consumer-sort (where the consumer separates recyclables prior to collection, common in areas where social responsibility is high such as Northern Europe) and curb-sort (workers sort items manually at the collection point, common in areas where the value of recycled material is high when compared to labor cost. We see this in the United States primarily in the bulk scrap metal market). The core idea behind Single-Stream is that by making the recycling process as easy as possible for consumers, consumer participation rates would be as high as possible. The effort to sort the commingled recyclables would then be put on large, centrally located MRFs which would be able to take advantage of economies of scale and high-tech industrial automation to drive costs down and recycling rates up.

This system has struggled with viability largely due to the difficulty of extracting pure materials free of contaminants from the dirty, highly mixed waste streams that arrive at MRFs. In 2019, the Northeast Recycling Council estimated the average

1. Introduction

value of post-consumer recycled waste in the northeastern US at \$45.83/ton and the average processing cost for a MRF at \$82/ton [15]. These costs are passed on to the municipalities and institutions who collect the recyclable materials, and in the face of rising costs stemming from a tight American labor market as well as international political factors which decrease the viability of shipping unsorted recyclables overseas, many municipalities have ended recycling programs while others have increased costs to residents or restricted what materials would be accepted [4]. Faced with these stark economics, municipalities and the American recycling industry are left with three options: 1) Dramatically decrease the cost to sort recyclables at existing MRFs, 2) Convince the American public to correctly sort recyclables in their homes and to bring sorted material to drop off locations or 3) Allow rising costs to slowly bring an end to American public recycling programs entirely.

Since the price of sorted recyclable material is set by the market, the only way for MRFs to increase profitability is to decrease the cost to sort. According to current research ([23], [21]) along with our own conversations with MRF operators, one of the core issues that MRFs face is that they are generally designed around the average material flow through the facility over the course of a year, but that in reality material inflow condition and composition is highly variable. When inflow composition and condition strays from the norm, issues such as pile-ups on conveyors, machine clogs, idle machines and process sections, and general system inefficiencies abound. [24][9].

1.2 Optimization of MRF operations via conveyor belt speed control

Recycling sorting facilities generally are comprised of a series of material sorting agent stations coupled by conveyor belt network. Fig. 1.1 shows a simple example of a material sorting operation. Sorting system operators improve the performance of individual sorting agents by using computer vision systems to detect, classify, and track individual waste items. However, due to the large number of waste items in a sorting system (measured in millions of individual items), these approaches become



Figure 1.1: Materials on an experimental conveyor belt at Carnegie Mellon being sorted by a human

computationally intractable when they are applied to optimize the system's operation as a whole.

The control of the flow rate of material through the system is a key aspect of centralized multi-agent sorting system control. Since the rate at which material is presented to a sorting agent directly affects the sorting agent's ability to remove target material from the stream, a centralized authority can affect the efficacy of sorting agents throughout the system by varying flow rates. Dynamically varying the flow rate throughout the system presents a problem for distributed control systems because the system throughput will be limited to that of the slowest agent. In contrast, a central authority can avoid bottlenecks by dynamically varying material flow rates in response to varying infeed conditions.

In this work, we present a novel modeling framework that divides the entire conveyor into smaller Longitudinal Control Volumes (LCV) which enables a low-dimensional representation and prediction of material flows throughout the sorting facility. We present a method for converting detections of individual waste items from images into sparse measurements in our state space. We use these measurement

1. Introduction

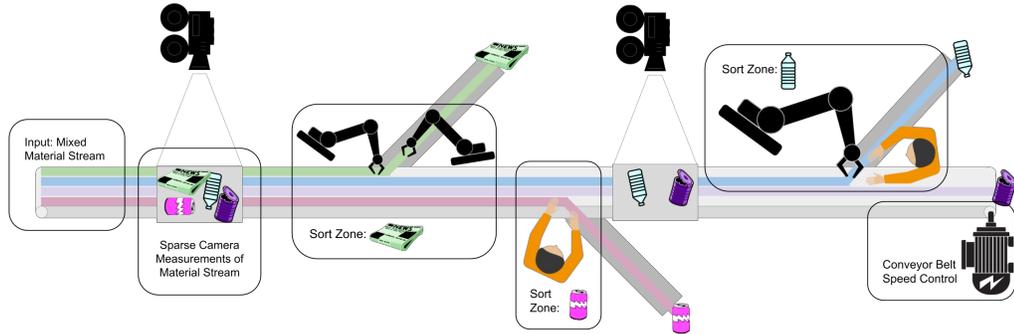


Figure 1.2: A series of sorting agents with dynamics coupled by a single conveyor belt and prediction framework with a Kalman Filter to compute a global estimate of the system state.

This modeling framework is used in conjunction with the global estimation result in a model predictive control (MPC) module that optimizes the material flow to maximize the overall value generated for successful sorting. The controller reacts to the incoming material over a receding time horizon and changes the speed of the conveyor to allow for efficient sorting of materials.

We demonstrate the efficacy of our novel estimation and control approach in simulation and on a physical laboratory sorting system. We show improvements on the order of 40-100% with respect to constant belt speed approaches. We later discuss applications of our framework to diverse multi-agent systems including autonomous mobile robots and over-actuated control systems.

Chapter 2

Related Work

Profitability requirements drive decision-making in most recyclers. Even if a recycler is being run as a non-profit, operations should still maintain profitability to the extent that it is possible [26]. The ability of a MRF to sort material effectively is crucial to its ability to maintain profitability. As the MRF's ability to sort material drops, the MRF is forced to landfill more of the material processed. In order to estimate performance of the core sorting system in a MRF, a tool to convert a proposed system design along with historical information on material inflows and proposed operating parameters into an estimation of process efficiency is needed[22][26][24].

In order to estimate performance of the core sorting system in a material recovery facility (MRF), the authors in [22][26][24] develop tools to estimate sorting process efficiency based on historical data on material flow compositions. These tools leverage long-term historical data and are used to calculate steady-state optimal material flow rates for a given system. In this system-level offline modeling approach, a network flow diagram for the MRF is created by approximating the dynamics of mixed material flows through sorting machines and connecting them via a directed graph. A system of linear equations is then constructed and solved to calculate material stream compositions throughout the system at steady state [8][26].

The work presented in [24][9][11] expands on the foundational work by [8] and [26] by estimating the percentage of material in the main stream that a single sort agent

2. Related Work

can remove. They then set up a centralized optimization problem to find nominal steady-state optimal material flow rates. These offline modeling and optimization techniques represent a method for achieving global system coordination. However, they do not capture dynamics associated with material flow characteristics changing in time. They also rely on assumption of static sorting parameters which are often not representative of real sorting operations [3][25].

A second line of research involves distributed control techniques which can run in real-time. In these distributed control approaches, individual sorting machines leverage local system information to improve their performance. A literature review of these approaches for distributed optimization of sorting systems was conducted by [12].

Of particular relevance to the content of this work, [17][13][16] leverage techniques from computer vision and machine learning along with Kalman Filtering and multi-target tracking. They track each object entering a sorting machine and optimize that sorting machine's operation around real-time flow characteristics. These approaches show promising improvements to the operations of individual sorting machines, but they have not been applied to solving the global coordination problem. We hypothesize that the computational complexity of simultaneously tracking millions of individual pieces of recycled material presents an obstacle to use in centralized control methods.

Furthermore, as noted in [18] and [22], real MRF operations are much less efficient than would be expected. The authors of [23] investigate the discrepancy between theoretically expected sorting operations and real MRF sorting data and conclude that the high degree of variability in the material stream means that solutions calculated with offline centralized optimization systems do not translate well to real operation. The authors in [5] study the variability in the input material stream and provide statistics on flow variability from real MRF operations, and they establish that online control techniques are necessary to cope for the variations in the input material flows.

In this work, we draw particular inspiration from [23] and [5] who establish that MRF sorting performance suffers in the face of variable input material flows. We develop a state modeling approach which follows ideologically from the systems described in [26] and [24], but which provides a measure of system state with enough

fidelity for real-time dynamic control. We use this state estimation result as a basis for real-time centralized control of material flows throughout the multi-agent sorting system.

In order to estimate the efficiency of a given MRF configuration, there are two possible approaches. First, individual pieces of material can be tracked/simulated and the physics of each sorting station can be used to solve for a piece of material’s trajectory through the MRF. For example, the trajectory of a plastic water bottle through a MRF can be calculated by taking the geometry and mass of the bottle and simulating its motion through the various sorting machines in the MRF taking into account aerodynamic forces from air pressure sorting stations, contact forces from the bottle bouncing along conveyor belts, gravity, and contacts with other pieces of material. Due to the complexity of the MRF process and the heterogeneity of material flows, this quickly becomes intractable [24]. This will be referred to as “Physical Modeling”. In the second approach, the dynamics of mixed material flows through sorting machines can be approximated by creating a network flow diagram for the MRF as a whole and assigning a control volume to each sorting machine. A system of equations can then be constructed and solved which ensures that mass flow is conserved throughout the system. [24], [26], and [8] establish this field of MRF modeling and describe it as “Probabilistic Modeling”.

The basic building block of the probabilistic MRF model is the Bayesian separation unit. The Bayesian separation unit comprises a unit which divides a single input stream into two (binary Bayesian unit) or more (multi-output Bayesian unit) material flows given some separation parameters [24]. Consider a sorting unit which takes in one input stream and outputs two output streams. We represent each stream as a vector of mass flow rates broken down by material where μ_m represents the mass flow rate of material m in the input stream and f_i^m represents the mass flow rate of material m in output stream i [18]. The effects of the sorting process are parameterized by the probability of the target material correctly sent to the primary stream (r in $[0, 1]$) and the probability of non-target materials correctly sent to the secondary stream (q in $[0, 1]$). In this case, if we take material 1 as the target material and output 1 as the primary stream, we can calculate $r = f_1^1/\mu_1$ and $q = (f_2^2 + f_2^3)/(\mu_2 + \mu_3)$.

2. Related Work

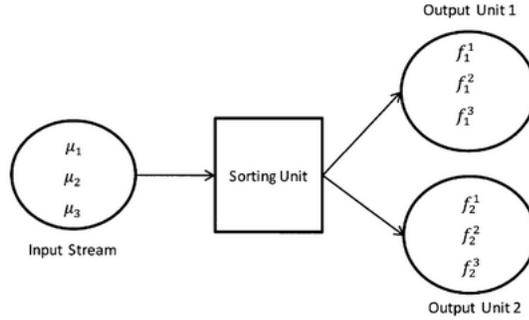


Figure 2.1: Block diagram of binary Bayesian sorting unit (figure from [18])

Under this framework, it is straightforward to compose these building blocks into system level models. Take for example, the case where we have a system that only carries a single material and is made up of of three units, one input unit, one sorting unit, and one output unit (figure 2.2)

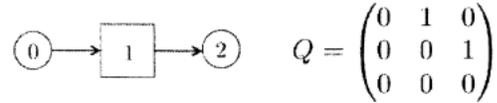


Figure 2.2: Example for Q-Matrix for simple linear system (figure from [26])

Here our scalar separation factor r from the binary Bayesian sorting unit becomes a matrix Q that is built up of per-unit scalar separation factors. Reference [24] shows how more complicated graphical representations of MRF processes can be represented as Q matrices (figure 2.3).

Figure 2.3: Composition of binary Bayesian units into a model of a MRF process along with corresponding Q matrix for a single material (figure from [26])

Testa also establishes that the steady state flows at any edge in the graph can be solved for when the input material vector and the Q matrix are specified:

This tooling documented originally by [8] [26] and expanded on by [24],[9], [11] and others allows us to calculate material flow compositions throughout a MRF when provided with separation parameters for sorting units. These techniques are

$$\bar{f}^0 = (I - (Q^0)^T)^{-1} \bar{\mu}^0.$$

often used in conjunction with configuration optimization techniques to aid in MRF design and layout as well as to estimate overall system efficiency and profitability, however they are not without their downsides. These methods rely on a priori knowledge of the separation parameters needed to generate Q matrices, and these separation parameters are commonly calculated based on test data from different types of machines in different operational environments [3][25].

In order to further investigate the dynamics of these separation parameters, [21] performs a meta-study of 33 MSW separation datasets in the literature. The authors find a number of limitations of using these types of datasets to simulate flows in a real MRF. First, more than 80% of the datasets available in the literature do not mention the operating conditions under which the data was collected (e.g. total flow rate, material composition, machine settings, flow composition), and these conditions can have significant effects on separation parameters [21][14][11][9]. This is especially important since when separation parameters from the literature are used in plant configuration studies such as described in [26][24][9], the assumption is made that these separation parameters are constant. In this case, the different plant configurations that are studied are associated with different flow compositions at individual sorting units and with the assumption of constant separation parameters, any differences in separation parameters associated with these changes in flow composition go unaccounted for [21][23]. Further, [11] and [9] show that the final sorting efficiency of the MRF is highly sensitive to separation parameters, with some output characteristics varying nearly linearly with changes in specific separation parameters.

Since invariance assumptions for separation parameters likely lead to mismatches between predicted and real sorting operations, [23] introduces a mixed modeling framework for modeling separation parameters as a combination of static separation parameters and simple functions of environmental parameters such as flow composition or belt speed. These functions of environmental parameters can be derived from test data or explicit physical modeling [23]. Even with these more adaptive separation parameters, solving for MRF efficiency and profitability using system level modeling

2. Related Work

techniques still fails to reflect day-to-day plant operations since MRF inflows vary significantly in material composition and condition, both between material source transfer stations and in time for a given transfer station [18] [22].

Chapter 3

Longitudinal Control Volumes: Mathematical Formulation

In this chapter, we present a method to represent the current state of a sorting system using a framework called Longitudinal Control Volumes. We then describe a model that can track the material composition flow as it moves across the conveyor.

3.1 Longitudinal Control Volumes (LCV)

The state representation of a sorting system needs to provide information about the composition of the material flow as it evolves along the conveyor. To compute this, we first longitudinally split the sorting system into small control volumes arranged sequentially along the direction of travel of the belt (Fig. 3.1). The size of these control volumes is tuned based on the spatial resolution of the perception system and the computational load of having a high number of control volumes. The amount of material in the control volume forms the basis for constructing the state vector.

We construct the state vector by first concatenating smaller vectors \mathbf{x}_k^i , for every material i . Each vector element in \mathbf{x}_k^i corresponds to a control volume and indicates the quantity of a material occupying that control volume at timestep k . Lastly, the current speed of the conveyor belt, r_k is added to the end of the concatenated vector

3. Longitudinal Control Volumes: Mathematical Formulation



Figure 3.1: Control volumes overlaid on conveyor belt serving a team of human and robot sorters

which forms the full system state. The total amount of a given material in the entire system can be computed by summing the elements of the state vector corresponding to that material. Given a total of m control volumes and n different material types to be sorted, the state vector is of the size $nm + 1$ and can be written as follows

$$\mathbf{X}_k = [\mathbf{x}_k^1, \mathbf{x}_k^2, \dots, \mathbf{x}_k^n, r_k]^T \quad (3.1)$$

$$\text{Total amount of material } i \text{ in the system} = \sum \mathbf{x}_k^i$$

3.2 State Transition Model

The motion of the conveyor belt and the sorting process change the composition of material in the control volumes. To predict this change in the system state, we decompose the state transition matrix to model both the dynamics associated with the conveyor motion ($\mathcal{L}(r_k)$) and the dynamics associated with the sorting process ($\mathcal{F}(\mathbf{X}_k)$) to model a linear approximation of these processes. The control input to the system is the change in conveyor speed Δr . This makes the matrix B a $mn + 1$ long zero vector with the last element equal to 1 as the control input affects only the

3. Longitudinal Control Volumes: Mathematical Formulation

last element of the state. This state transition model is written as follows,

$$\begin{aligned}\mathbf{X}_{\mathbf{k}+1} &= A(\mathbf{X}_{\mathbf{k}})\mathbf{X}_{\mathbf{k}} + Bu_k \\ &= \mathcal{F}(\mathbf{X}_{\mathbf{k}})\mathcal{L}(r_k)\mathbf{X}_{\mathbf{k}} + B\Delta r\end{aligned}\quad (3.2)$$

The motion matrix $\mathcal{L}(r_k)$ models the motion of material across different control volumes in the system carried by conveyor belts. This is dependent on the current speed of the conveyor belt. To construct this matrix for any arbitrary belt speed, we compute these motion matrices for integral belt speeds and perform linear interpolation between these integral speed motion matrices. Given an integer belt speed \hat{r}_k , we can construct a motion matrix $\mathcal{L}(\hat{r}_k)$ as follows,

$$\mathcal{L}(\hat{r}_k) = \begin{bmatrix} L(\hat{r}_k) & 0_{m \times m} & \dots & 0_{m \times m} & 0 \\ 0_{m \times m} & L(\hat{r}_k) & \dots & 0_{m \times m} & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0_{m \times m} & 0_{m \times m} & \dots & L(\hat{r}_k) & 0 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

where $L(\hat{r}_k)$ is a $m \times m$ modified shift matrix given by

$$L(\hat{r}_k)_{ij} = \delta_{i,j+\hat{r}_k}$$

where $\delta_{i,j}$ is the Kronecker delta

This motion matrix can only be defined for an integral belt speed due to the Kronecker delta. Hence we do a linear interpolation of the lower and upper integral motion matrices to compute the motion matrix for an arbitrary belt speed. Given a belt speed r_k , we can compute the motion matrix $\mathcal{L}(r_k)$ as follows,

$$\mathcal{L}(r_k) = \frac{\lceil r_k \rceil - r_k}{\lceil r_k \rceil - \lfloor r_k \rfloor} \mathcal{L}(\lfloor r_k \rfloor) + \frac{r_k - \lfloor r_k \rfloor}{\lceil r_k \rceil - \lfloor r_k \rfloor} \mathcal{L}(\lceil r_k \rceil)$$

where $\lceil x \rceil = \text{ceil}(x)$ and $\lfloor x \rfloor = \text{floor}(x)$

We model the dynamics associated with removing material at sorting agents using

3. Longitudinal Control Volumes: Mathematical Formulation

the sort matrix $\mathcal{F}(\mathbf{X}_k)$. As each sorting agent only spans a few control volumes in the entire system, they can sort a limited amount of material in each time step. To capture this behavior, we define a separation parameter p_j^i for each sort agent [21][8]. This separation parameter is defined per material for every control volume based on the maximum number of items that can be picked up by the corresponding sort agent.

If α^i represents the subset of control volumes spanned by a sort agent and γ_i represents the maximum amount of material that can be picked up in a single timestep, we can compute the separation parameters based on ratio of maximum pick rate and amount of material present on the control volumes of a sort agent (η). We can also represent imperfections inherent in the sorting process by a discount factor q^i with values ranging from 0 (i.e. no successful sorting) to 1 (perfect sorting).

$$\eta^i = q^i \frac{\gamma^i}{\sum_{\alpha^i} \mathbf{x}^i} \quad p_j^i = \begin{cases} 1, & \text{if } j \notin \alpha^i \\ \max(0, 1 - \eta^i), & \text{otherwise} \end{cases}$$

The sorting $\mathcal{F}(\mathbf{X}_k)$ matrix consists of smaller diagonal matrices $F(\mathbf{x}_k^i)$ that capture the sorting process for every material in each control volume. This allows us to represent the dynamics associated with all sorting machines in the system in a single linear element. The matrix $F(\mathbf{x}_k^i)$ is a $m \times m$ diagonal matrix with the diagonal elements equal to the separation parameter p_j^i . We can write this as follows:

$$\mathcal{F}(\mathbf{X}_k) = \begin{bmatrix} F(\mathbf{x}_k^1) & 0_{m \times m} & \dots & 0_{m \times m} & 0 \\ 0_{m \times m} & F(\mathbf{x}_k^2) & \dots & 0_{m \times m} & 0 \\ \dots & \dots & \dots & \dots & \cdot \\ 0_{m \times m} & 0_{m \times m} & \dots & F(\mathbf{x}_k^n) & 0 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix} \quad F(\mathbf{x}_k^i) = \begin{bmatrix} p_1^i & 0 & \dots & 0 \\ 0 & p_2^i & \dots & 0 \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & p_m^i \end{bmatrix}$$

Chapter 4

System Design

We create a system to make an optimal tradeoff between sorting task completion and system throughput in the context of a recycling sorting operation. As the material flows through the conveyor, there is uncertainty in the state induced by the sorting process and material moving relative to the conveyor belt. To incorporate this uncertainty, we present a framework to use a computer vision system to detect materials and estimate the system's state using an Kalman Filter based on the LCV state transition model. We then use this estimated state as input for a centralized model predictive control module, which maximizes the difference between the reward associated with successful sorting and the cost associated with failing to sort potentially valuable material. The controller outputs an optimized conveyor belt speed that maximizes the system's throughput while maintaining high rewards for sorting. A block diagram of this system is shown in Fig. 4.1.

4.1 State Estimation

In order to incorporate real-time data into our state estimate, we devise a computer vision system to detect individual objects on a subset of control volumes. An Intel Realsense D435 RGB camera provides top-down images to a YOLO-based object detection system that classifies different materials and returns bounding boxes that

4. System Design

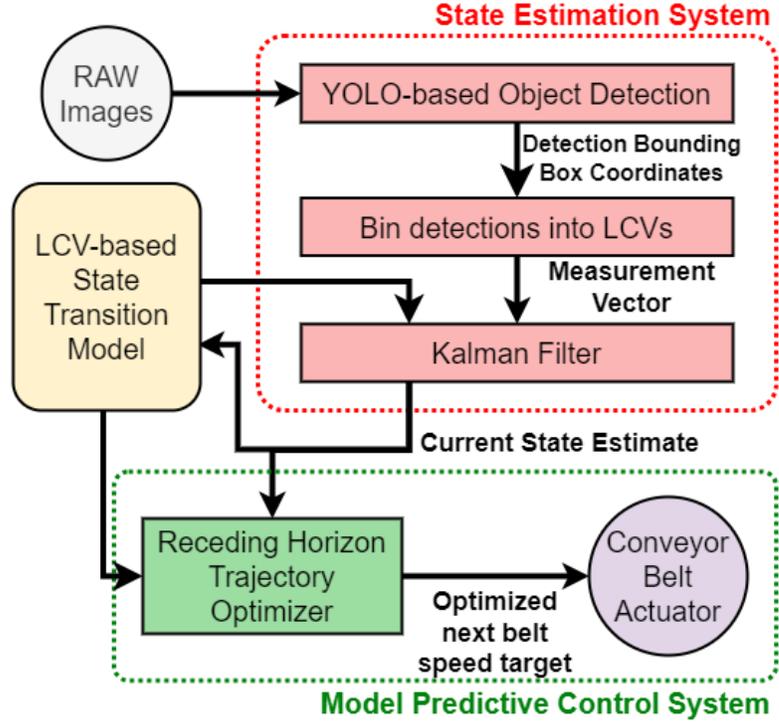


Figure 4.1: A block diagram of the proposed system to optimize belt speeds for sorting

describe the location and type of individual items [19]. The detection from every frame is incorporated as a measurement vector.

To compute the measurement vector, we project the visible control volumes into the camera image and add the counts of detected objects into the control volumes associated with their locations. If a detected object spans multiple control volumes, we add portions of the detected object to each control volume spanned by the object. If λ is the number of control volumes in the camera's view and W^i is the set of detections of material i , we can compute the measurement vector \mathbf{z}_k as follows,

$$\mathbf{z}_k = \left[\bar{\zeta}^0 \quad \bar{\zeta}^1 \quad \dots \quad \bar{\zeta}^n \right] \quad \bar{\zeta}^i = \left[\beta_0^i \quad \beta_1^i \quad \dots \quad \beta_\lambda^i \right]$$

4.2 Model Predictive Control

We construct a Model Predictive Control (MPC) problem which uses the state-transition model described in ?? and the estimated state to optimize sorting operations and control the conveyor belt speed in real-time. We choose to maximize an objective function that can maximize the value of sorted material which is obtained using a Value matrix V and an Opportunity Cost matrix O . The Value matrix maps the price of a material ρ_i onto the sort matrix($F^i(\mathbf{x}_k)$). The Opportunity Cost matrix encodes the material that we fail to sort and are forced to sell for a lower price as mixed material. These matrices are computed as shown below.

$$\begin{aligned} V &= \begin{bmatrix} \bar{v}^0 & \bar{v}^1 & \dots & \bar{v}_n & 0 \end{bmatrix} & \bar{v}^i &= \rho^i F(\mathbf{x}_k^i) \mathbf{1}_m \\ O &= \begin{bmatrix} \bar{o}^0 & \bar{o}^1 & \dots & \bar{o}_n & 0 \end{bmatrix} & \bar{o}^i &= \rho^i (I - F(\mathbf{x}_k^i)) \mathbf{1}_m \end{aligned}$$

where $\mathbf{1}_m$ is a 1-vector of size $m \times 1$

For the optimization, we set the horizon to the time it would take material to fully transit the system with all belts at their slowest speed. Eq. 4.1 describes our objective function that seeks to maximize the difference between the total sale price of material sorted over the horizon and the value lost by selling a material with contamination.

$$\max_{X_{1:T}} \sum_{l=1}^T [V - O] \mathbf{X}_l \tag{4.1}$$

s.t.

$$X_{l+1} = \mathcal{F}(\mathbf{X}_l) \mathcal{L}(r_l) X_l + B u_l \quad \text{for } l = 1, \dots, T - 1$$

$$u_{min} \leq u_l \leq u_{max}$$

We set up a quasi-Newton solver with a back-tracking line search to maximize our objective function. We compute the gradients by calculating finite-difference derivatives for each control variable, and use the gradient for to pick the update direction. The update step is then scaled using a BFGS approximation of the

inverse Hessian matrix in the traditional Newton update equation [2][6][7][20]. The optimization is terminated based on fulfillment of the Armijo Condition[1]. The resulting optimized control input (conveyor belt speed) corresponding to the next timestep is then applied to the system. This process is repeated at every timestep.

4. *System Design*

Chapter 5

Experiments and Results

We conduct experiments in simulation and on a physical conveyor belt sorting system and evaluate our system’s performance by comparing it to that of a baseline approach. For the baseline, we run the conveyor at a constant speed, similar to approaches currently used in industrial MRFs. This optimal constant speed is estimated based on the historical average material flow through the system. We compare the total value generated by this baseline to the value generated by running the system at the optimized variable belt speed generated by the MPC module.

5.1 Simulation-based evaluation of MPC performance

We create a simulated environment where infeed material flow is procedurally generated in a manner representative of actual material flows in an MRF according to material variability statistics described in [5]. The LCV-based state transition model described previously is used to forward-simulate the sorting operation. We generate 30 runs that each represent an hour of continuous operation. We take the average optimized belt speed produced by the MPC module for each run and repeat the experiment with a constant belt speed equal to that average optimized speed.

5. Experiments and Results

We compare the final accumulated value at the end of each optimized run with the accumulated value from its corresponding constant speed run. We repeat this process for sorting systems with 2, 3, and 4 distinct materials to be sorted.

5.2 Full-system Experiments on Physical Sorting Equipment

We also conducted experiments on a physical conveyor belt sorting system to evaluate the performance of our end-to-end system. Different material streams can be fed to the conveyor belt using a hopper on one end. A computer with a ROS framework captured overhead images from the Intel RealSense D435 camera, executed the state estimation and MPC modules in real-time, and controlled the speed of the conveyor belt using a servo motor. At the end of this conveyor, a human sorter was tasked with removing one material from the flow. This setup is shown in the Fig. 5.1.

In these physical experiments, we use two materials: empty green bottles and empty red cans. In order to investigate the ability of our framework to react to changing infeed conditions during these short runs, we loaded the hopper with varying distributions of the material flows as the MPC module optimized the conveyor belt speed in real-time. For each of these runs, we computed the value generated for sorting and compared this to the value generated using a constant speed baseline experiment. The constant speed was equal to the average of the optimized speeds generated by the MPC module.

5.3 Results

The results of our proposed framework for a three-material sorting simulation is shown in the Fig. 5.2. We compare the total profit generated by our method to that of the constant speed baseline model. Fig. 5.2(a) shows the instantaneous distribution of the materials in their respective sort stations, along with their price. Fig. 5.2(b) shows shows the target conveyor belt speed by the MPC module (cyan line) and the



Figure 5.1: Physical conveyor belt experimental setup showing the RealSense D435 overhead camera, human sorter and the sorted materials

target belt speed by the constant speed baseline (magenta line). Fig. 5.2(c) shows the total profits accumulated for successfully sorted materials for both versions.

From the plots shown in the figures 5.2(a) and 5.2(b), it is evident that the MPC module is reacting to the amount of material coming into the system. As more Material 0 (which has a higher price) enters the system, the controller slows down the conveyor to allow the sorters to pick up more material, increasing the profits accumulated. For the same case, since the constant speed model does not change the belt speeds, the plant incurs a loss as the sorters fail to sort this high-value material, and the impure mixture is worth much less. We see the same behavior even in the 2-material runs as shown in the Fig. 5.3.

We notice similar trends across all the different runs for varying amounts of materials in the system. When different materials are distributed evenly, the constant speed baseline and our proposed framework perform similarly. In contrast, the profit rate of our method goes up quickly when the distribution varies quickly. This was true across all the 90 runs we conducted in simulation. Fig. 5.4(a) shows the average profit

5. *Experiments and Results*

rate along with the variance in profit rate for the proposed method and the constant speed baseline for varying number of simulated material runs. In the 2-material case, our optimized speeds improved the margin by around 40 percent while we see around two times the profit in the 3-material and 4-material cases. In the physical experiments with 2-materials, the optimized speed run improves the profit margins by two times as shown in Fig. [5.4\(b\)](#).

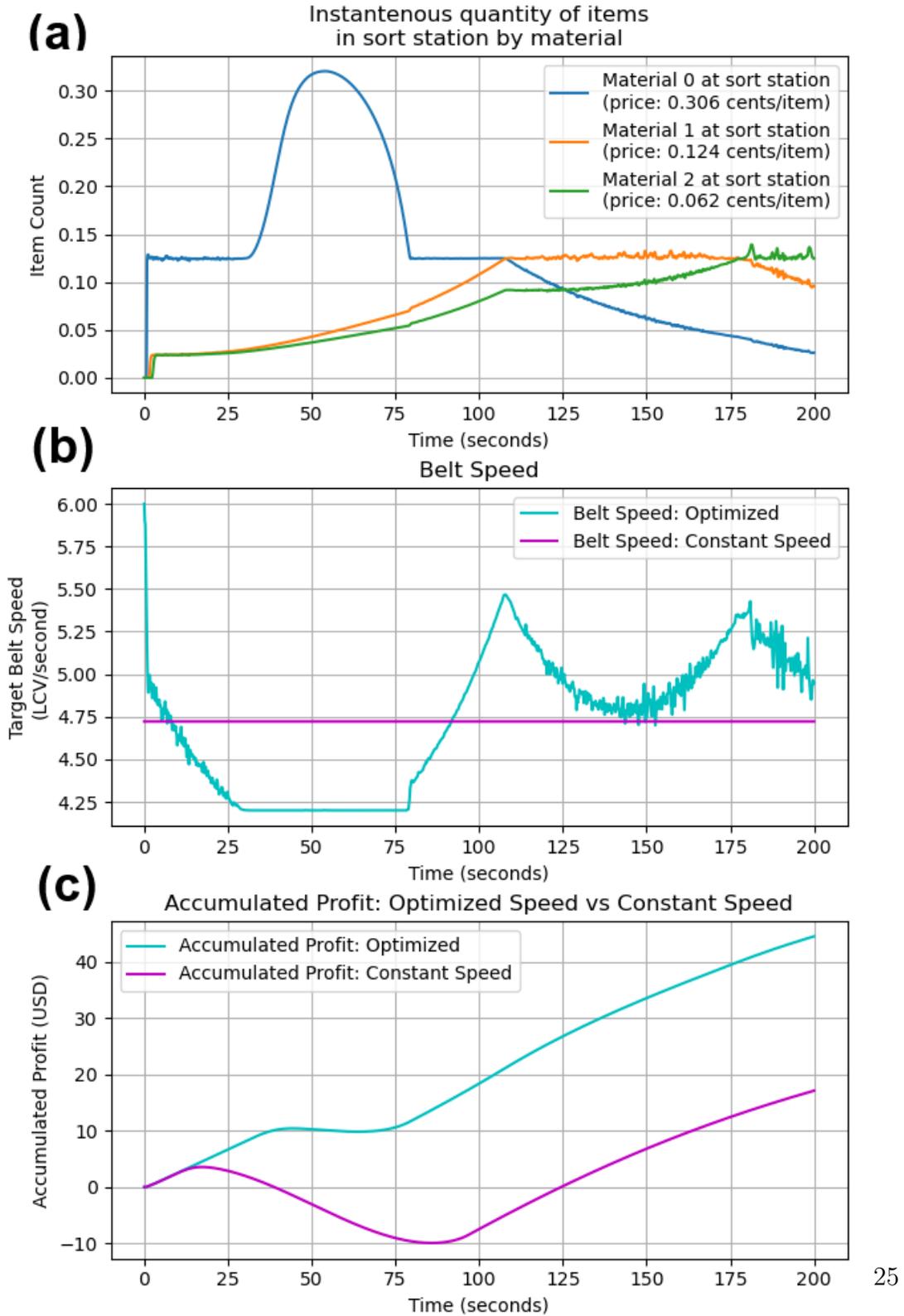


Figure 5.2: Result of a simulated 3-material sorting run. (a) Plot showing the instantaneous amount of material passing through a sort agent station (b) Plot showing the output belt speeds for the MPC and the baseline. (c) Plot showing the accumulated value as a result of successfully sorting.

5. Experiments and Results

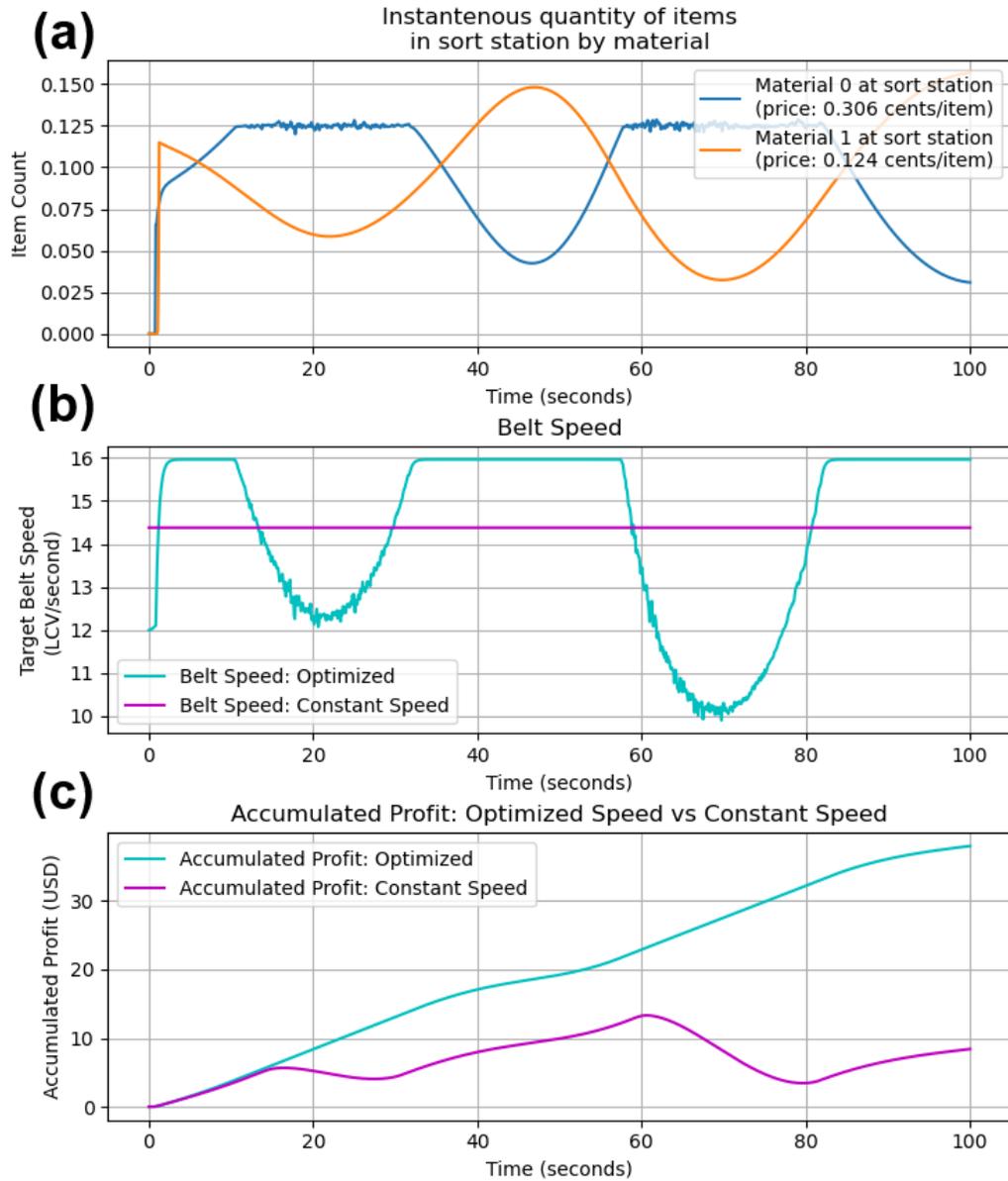


Figure 5.3: Result of a 2-material sorting run. (a) Plot showing the instantaneous amount of material passing through a sort agent (b) Plot showing the output belt speeds for the MPC and the baseline. (c) Plot showing the accumulated value as a result of successfully sorting.

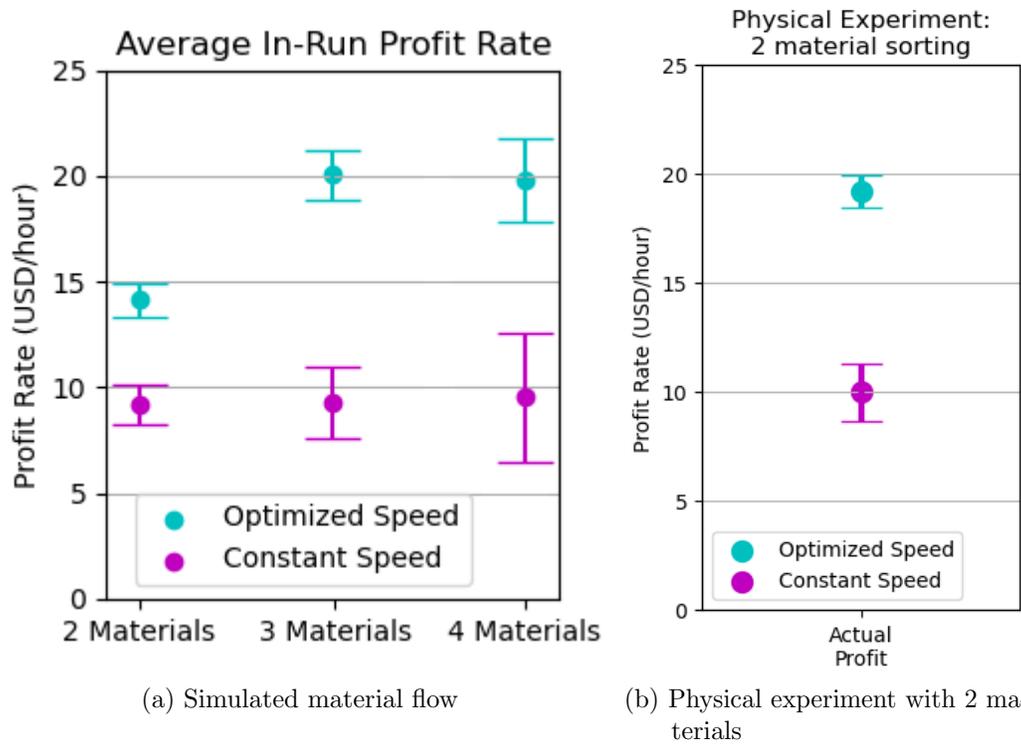


Figure 5.4: Result plot showing profit rate for optimized speed vs constant speed control

5. *Experiments and Results*

Chapter 6

Conclusions

6.1 Discussion

In this work, we present a novel framework that modeled the movement of material along a conveyor belt in a sorting system. We presented an end-to-end system that successfully detected and estimated these material flows while optimizing the conveyor belt speed for maximizing value in the sorting process. We conducted experiments both in simulation and on a physical conveyor belt with varying numbers of materials and compositions. We achieved around 40 to 100% increase in the profit rates using the proposed model predictive controller that accounted for the entire state of the system.

Overall, we see this work as a compelling demonstration of the potential benefits of real-time centralized control of conveyor belt sorting systems. In that light, we see it of paramount importance to determine the degree to which these results will scale up to a full size MRF. In particular, in this work we have made the computer vision problem simpler by using a controlled sample of different materials. In an operational MRF, the material streams present will be much more highly varied and so the difficulty of deploying a functional computer visions system will increase.

6.2 **Future Work**

Although the presented work shows dramatic improvements to process profitability, we identify several potential avenues for further research. As the MPC module does not leverage the uncertainty information from the state estimation module, we intend to explore incorporating the uncertainty in the optimization process along with trying to model an objective function derived from operations research. We would like to investigate learning-based approaches to directly infer measurement vectors from camera images. Further, we are in the process of scaling our experiments at an operational MRF sorting facility. Future work also includes integrating this system with a fully mechanised sorting line in order to eliminate any inconsistency associated with using a human sorting worker.

Beyond the development of the core tech, a substantial body of possible future work lies in the application of this technology to real-world conveyor-based sorting systems. The clearest application is in the municipal MRF, as that was what this work was designed to address. In addition to municipal MRFs, possible future work could include applications as varied as rock-crushing mills and package sortation systems.

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