Parametric-Control Barrier Function-based Adaptive Safe Merging Control for Heterogeneous Vehicles

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Abstract

With the increasing emphasis on the safe autonomy for robots, model-based safe control approaches such as Control Barrier Functions have been extensively studied to ensure guaranteed safety during inter-robot interactions. In this paper, we introduce the Parametric Control Barrier Function (Parametric-CBF), a novel variant of the traditional Control Barrier Function to extend its expressivity in describing different safe behaviors among heterogeneous robots. Instead of assuming cooperative and homogeneous robots using the same safe controllers, the ego robot is able to model the neighboring robots' underlying safe controllers through different parametric-CBFs with observed data. Given learned parametric-CBF and proved forward invariance, it provides greater flexibility for the ego robot to better coordinate with other heterogeneous robots with improved efficiency while enjoying formally provable safety guarantees. We demonstrate the usage of Parametric-CBF in behavior prediction and adaptive safe control in the ramp merging scenario from the applications of autonomous driving. Compared to traditional CBF, Parametric-CBF has the advantage of better capturing drivers' characteristics, which also allows for richer description of robot behavior in the context of safe control. Numerical simulations are given to validate the effectiveness of the proposed method.

1 Introduction

Although there have been strong strides in the area of heterogeneous robot control, how to obtain theoretically guaranteed safety in heterogeneous robots interaction is still an open challenge. Among the model-based safe control approaches, Control Barrier Function (CBF) is a popular choice due to its forward invariant property. In recent progress on safe design for autonomous robots using CBF Ames et al. [2014, 2019], Wang et al. [2017], Zeng et al. [2020], Lyu et al. [2021], surrounding robots are often assumed to be fully cooperative or passively moving with constant velocity in the collision avoidance scenario. However, in a more realistic setting when robots operate in an unknown environment and interact with other autonomous systems, it is desired for the robots to take observations and infer the underlying safe behavior strategy for the other robots, so that the ego robot could behave more safely and efficiently during such interactions.

When adopting safety-critical control into the domain of autonomous driving, new challenges arise. Considering the fact that autonomous vehicles will have to share roads with human drivers for a very long time, how to improve the expressivity of autonomous vehicles' safe-critical controllers, so that the generated behavior can be better understood by human drivers, remains an open problem. As a milestone towards the final goal of mixed human-robot autonomy, it is desired to explore methods to

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help heterogeneous autonomous vehicles to achieve richer behavior description and characterization while safely interacting with each other.

Motivated by these considerations, we focus on the learning and safe design for interaction of heterogeneous autonomous vehicles in ramp merging scenario. This paper extends the previous work on CBF-based safety-assured merging control in Lyu et al. [2021] and presents the following **contributions**: 1) We propose the novel idea of Parametric-CBF, a variant of traditional CBF that gives a richer behavior description and preserves the property of rendering a forward invariant safe set for the robots; 2) We present a novel safe adaptive merging algorithm that integrates the safe behavior prediction of heterogeneous robots and safe control for the ego robot using the learned parameters of the Parametric CBF, yielding improved task efficiency for the ego robot with safety guarantee. 3) We demonstrate the effectiveness of the Parametric-CBF based behavior predication and safe control through experimental results in the ramp merging scenario in the autonomous driving domain; Our mechanism enables the robot to model the behavior of other entities first and take appropriate actions accordingly, which makes it generally applicable to other robotics applications.

2 Related Work

Safe control in terms of collision avoidance is critical for designing interactive robot behaviors. To minimize the deviation from robots' primary task execution due to safety consideration, reactive collision avoidance methods such as reciprocal velocity obstacles (RVO) Van den Berg et al. [2008], Gopalakrishnan et al. [2017], Alonso-Mora et al. [2012], safety barrier certificates Wang et al. [2017], Wang et al. [2017], Luo et al. [2020] and buffered Voronoi cells Wang et al. [2019], Angeris et al. [2019] have been presented to minimally revise the robot's task-related controller subject to collision avoidance constraints. The collision-free motion often relies on the key assumption of reciprocal or passive behaviors, where each robot is assumed to either employ the homogeneous safe controller (fully cooperative) or move with constant velocity (non-cooperative). When a robot is interacting with unknown agents that employing different behavior design, it is necessary to leverage the observations on the other agents to build effective models for their behaviors and achieve reliable safe interactions.

Predicting behavior patterns of interactive heterogeneous multi-robot systems is often modeled as parameter identification problem. Grover et al. [2020] presents a method to estimate the system dynamics parameter of a robot using an optimization-based controller, assuming that the CBF-based safety constraint of the optimization-based controller is known. This would require foreknowledge of what kind of safe controller the robot is using, and therefore, the same identification framework cannot be applied directly to other robots with different safe controller parameters. Wang et al. [2016] presents a decentralized control framework for heterogeneous multi-robot interaction by assuming the same reciprocal behaviors but with different mobility capability to identify, and the causality of the difference in generated safe behavior is simplified as the difference in robots' acceleration limits. To capture different safe behaviors produced by heterogeneous safe constraints, it is desired to directly characterize and infer the behavior pattern modelled by learned safe constraints, e.g. CBF-based control constraints, so that the safe behavior of unknown agents could be better predicted.

There have been some recent efforts to construct variants of CBFs with parameter identification. Jagtap et al. [2020b,a] introduced techniques to search for parametric CBFs to synthesize controllers for optimal control, but the focus was on the parameter computation of an assumed particular form of CBF, rather than providing a formal definition of parametric CBF that can be applied generally. Djaballah et al. [2017] introduced a systematic way to construct parametric barrier functions using interval analysis. However, the heavy computation associated with solving the complicated formulation of the parameterized CBFs, which varies case-by-case, prohibits the applicability to large-scale multi-robot systems in real time. In this paper, we propose a general formulation of the Parametric-CBF to leverage a linear combination of candidate CBFs characterizing various risk tolerance levels when the autonomous system is approaching the safety set boundary, therefore enabling distinct and richer behavior expressions for efficient learning and safe control.

3 Method

In this section, we start by introducing the formal definition of Parametric-CBF followed with Parametric-CBF based safe controller design, and Parametric-CBF based behavior style prediction is introduced. Integrating the two tasks together, the safe adaptive merging control algorithm is presented.

3.1 Formal Definition of Parametric-CBF

The main difference between the proposed Parametric-CBF and traditional CBF is the different choice of the mapping function $\kappa(h(x))$. See Appendix A.1 for background of traditional CBF.

Definition 1. *(Parametric-Control Barrier Function) Given a dynamical system (2) and the set $\mathcal{H}$ defined in (3) with a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$, then h is a Parametric-Control Barrier Function (Parametric-CBF) for all $x \in \mathcal{X}$ such that*

$$\sup_{u \in \mathcal{U}} \{\dot{h}(x,u)\} \geq -\alpha H(x) \tag{1}$$

where parameter vector $\alpha = [\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \dots \quad \alpha_n] \in \mathbb{R}^n$ with $\forall \alpha_p \in \mathbb{R}^{\geq 0}$ for $p \in [n]$. $H(x) = \begin{bmatrix} h(x) & h^3(x) & h^5(x) & \dots & h^{2n-1}(x)\end{bmatrix}^T$, $n \in \mathcal{N}$.

In Parametric-CBF, $\kappa(h(x))$ is constructed as a polynomial function $\kappa(h(x)) := \alpha_1 h(x) + \alpha_2 h^3(x) + \alpha_3 h^5(x) + \cdots + \alpha_n h^{2n-1}(x)$ with $H(x)$ as a set of basis functions containing independent odd-powered power functions $h^{2p-1}(x)$. These odd-powered power functions themselves are candidate class $\mathcal{K}$ functions Wang et al. [2017]. The intuition behind the polynomial design is that the relative weighting of its basis, composed of different-order component functions, regulates how fast the states of the system can approach the boundary of the safe set, and therefore how well the polynomial function can capture the system's behavior near the boundary. Note that the proposed Parametric-CBF is a more general formulation that includes all the $\kappa(h(x))$ choices used in Ames et al. [2019], Wang et al. [2016], Wang et al. [2017], Zeng et al. [2020], He et al. [2021].

3.2 Parametric-CBF based Safe Controller

In this work, we use the same system dynamics to describe vehicles as in Lyu et al. [2021]. The safe controller is formulated as a quadratic programming for heterogeneous multi-vehicles with the control input u_i. The objective function $\min_{u_i \in \mathcal{U}_i} ||u_i - \bar{u}_i||^2$ represents minimally deviation control from the nominal control input $\bar{u}_i$, where i, j are the indices of the pairwise vehicles. The actuation constraint $U_i^{min} \leq u_i \leq U_i^{max}$ represents the maximum and minimum allowed acceleration. The Parametric-CBF based safety constraint is formulated as $\dot{h}_{ij}(x,u) + \alpha_i H_{ij}(x) \geq 0$, where $\alpha_i = [\alpha_{i,1} \dots \quad \alpha_{i,n}]$, $H_{ij}(x) = \begin{bmatrix} h_{ij}(x), \dots, h_{ij}^{2n-1}(x)\end{bmatrix}^T$, $n \in \mathcal{N}$. We consider the particular choice of pairwise vehicle safety function $h_{ij}(x)$ and safety set $\mathcal{H}_i$ as: $\mathcal{H}(x) = \{x \in \mathcal{X} : h_{ij}(x) = ||x_i - x_j||^2 - R_{safe}^2 \geq 0, \forall i \neq j\}$, where x_i, x_j are the positions of each pairwise set of vehicles and $R_{safe} \in \mathbb{R}^+$ is the minimum allowed safety distance.

3.3 Heterogeneous Robots Behavior Prediction through Parameter Learning

We assume each heterogeneous robot carries the aforementioned Parametric-CBF based safe controller with different parameters α_i reflecting their various safe control behaviors, e.g. how aggressive they are in engaging collision avoidance scenario. Here we consider the behavior prediction task for ego vehicle i over prediction object j, who is interacting with a set of other vehicles. To that end, the ego vehicle is able to observe the behavior of the prediction object j and obtain the interactive dataset $\mathbb{D} = \{\dot{h}_{jk}^t, h_{jk}^t\}_{t=1}^m$ calculated from the observations during m time steps (the position and velocity of the prediction object and its surrounding vehicles). Given that, the ego robot i could perform ridge linear regression to find estimated $\bar{\alpha}_j$ for the prediction object j as: $\bar{\alpha}_j = \arg\min_{\alpha_j} \sum_{t=1}^m \left\| \dot{h}_{jk}^t - \alpha_j H_{jk}^t \right\|_2^2 + r\|\alpha_j\|_F^2$, where r is a regularizer parameter and $\|\alpha_j\|_F$ is the Frobenius norm of the estimate parameter α_j.

3.4 Safe Adaptive Merging control Algorithm using Parametric-CBF

The framework of the proposed safe adaptive merging algorithm using Parametric-CBF is presented in Algorithm 1. The high level idea is to leverage the prediction knowledge of the prediction object's driving style based on observations to improve the ego vehicle's reactive behavior generation, in terms of behavior adaptiveness and task efficiency. See Appendix A.3 for more details. Evaluating the proposed framework, Parametric-CBF has advantages in terms of richer descriptive behavior information and broader applications. Evaluation details can be found in Appendix A.4.

4 Application Simulation

As proved in Wang et al. [2016], heterogeneous robots regulated by CBF with different parameters α are guaranteed to be safe. The new challenge here is, how to improve the task efficiency when heterogeneity is present. To demonstrate the claimed advantages of Parametric-CBF (Appendix A.4), we choose the tasks of control and task of prediction in autonomous driving. Three different examples are demonstrated, and more simulation details can be found in Appendix A.5.

Behavior Pattern Prediction The interactive driving scenario is shown in Fig. 2. The task here is to predict the driving style, i.e., the Parametric-CBF parameter vector α, of the prediction object, based on observations over its interaction with surrounding vehicles. The prediction result is shown in Fig. (1(a)). It is observed that the prediction results converge over time as more data points are collected. The learned parameter vector α reaches convergence in only 10 time steps, which is 0.1s, indicating it is computationally efficient enough to be applied in real-time applications.

Richer Safe Behavior Characterization To demonstrate the advantage of Parametric-CBF over traditional CBF in terms of behavior description richness in the safe control task, comparison of traditional CBF (Fig. 1(c)) and Parametric-CBF (Fig. 1(d)) is conducted. The two vehicles interaction scenario has the ego with various underlying safe controllers on the main road and the merging vehicle with the same driving style α on the ramp. It is observed that, Parametric-CBF outperforms the traditional CBF in terms of richer description of the generated behavior with more distinct variations caused by changes in relative weights of different order component functions.

Improved Task Efficiency To show how the Parametric-CBF-based prediction can contribute towards more efficient safe control, safe behaviors generated with prediction knowledge and without prediction are compared in Fig. (1(b)). The difference between two trials is whether the ego vehicle estimates the merging vehicle's driving strategy through observations during interactions and choose adaptive strategy in terms of driving aggressiveness accordingly. It is observed that the use of prediction significantly improve the ego vehicle's task efficiency by 39.6% and the overall coordination task efficiency by 16.1%, greatly reducing traffic congestion.

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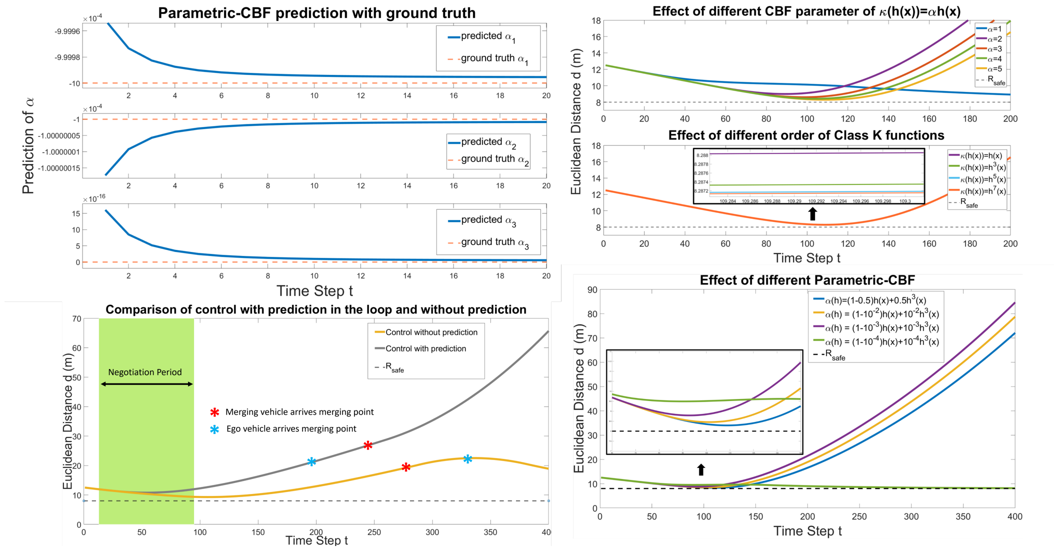

Figure 1: **a.(top left)** Learnt driving style α of the prediction object. **b.(bottom left)** Comparison of control with prediction in the loop and without prediction. **c.(top right)** Performance comparison of traditional CBF with different choices of $\kappa(h(x))$. **d.(bottom right)** Comparison of different constructions of Parametric-CBF. **Please see the supplementary video for enlarged figures.**

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A Appendix

A.1 Background of Control Barrier Function

Control Barrier Functions (CBF) Ames et al. [2019] are used to define an admissible control space for safety assurance of dynamical systems. One of its important properties is its forward-invariance guarantee of a desired safety set. Consider the following nonlinear system in control affine form:

$$\dot{x} = f(x) + g(x)u \tag{2}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ and $u \in \mathcal{U} \subset \mathbb{R}^m$ are the system state and control input with f and g assumed to be locally Lipschitz continuous. A desired safety set $x \in \mathcal{H}$ can be denoted by the following safety function:

$$\mathcal{H} = \{x \in \mathbb{R}^n : h(x) \geq 0\} \tag{3}$$

Thus the control barrier function for the system to remain in the safety set can be defined as follows Ames et al. [2019]:

Definition 2. *(Control Barrier Function) Given a dynamical system (2) and the set $\mathcal{H}$ defined in (3) with a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$, then h is a control barrier function (CBF) if there exists a class $\mathcal{K}$ function for all $x \in \mathcal{X}$ such that*

$$\sup_{u \in \mathcal{U}} \{L_f h(x) + L_g h(x)u\} \geq -\kappa\big(h(x)\big) \tag{4}$$

where $\dot{h}(x, u) = L_f h(x) + L_g h(x)u$ with $L_f h, L_g h$ as the Lie derivatives of h along the vector fields f and g.

A commonly selected class $\mathcal{K}$ function is $\kappa(h(x)) = \gamma h(x)$ Ames et al. [2019], Zeng et al. [2020], He et al. [2021], where $\gamma \in \mathbb{R}^{\geq 0}$ is a CBF design parameter controlling system behaviors near the boundary of $h(x) = 0$. Hence, the admissible control space in (4) can be redefined as

$$\mathcal{B}(x) = \{u \in \mathcal{U} : \dot{h}(x, u) + \gamma h(x) \geq 0\ \} \tag{5}$$

It is proved in Ames et al. [2019] that any controller $u \in \mathcal{B}(x)$ will render the safe state set $\mathcal{H}$ forward-invariant, i.e., if the system (2) starts inside the set $\mathcal{H}$ with $x(t = 0) \in \mathcal{H}$, then it implies $x(t) \in \mathcal{H}$ for all $t > 0$ under controller $u \in \mathcal{B}(x)$. However, this particular form as $\kappa(h(x)) = \gamma h(x)$ is limited in describing complicated system behaviors when approaching to the boundary of $h(x) = 0$, so does the other particular form as $\kappa(h(x)) = \gamma h^3(x)$ used in Wang et al. [2016]. Thus a more general form capturing a richer nonlinear behavior descriptions is needed.

A.2 Properties Proof of Parametric-Control Barrier Function

In this section we prove that function $\kappa(h(x)) = \alpha H(x)$ is a class $\mathcal{K}$ function whose definition is as follows Wang et al. [2016]:

Definition 3. *A continuous function $\beta : [0, a) \to [0, \infty)$ for some a is called a class $\mathcal{K}$ function if (1) it is strictly increasing and (2) $\kappa(0) = 0$.*

To verify the validity of the proposed Parametric-CBF, we need to prove that the function $\kappa(h(x)) = \alpha H(x)$ has the properties of strictly increasing and passing the origin.

Proof. **Strictly increasing**: The easiest way to prove this statement is to calculate the first-order derivative of $\kappa(\cdot)$ w.r.t h:

$$\begin{aligned} \frac{\partial \kappa}{\partial h} &= \frac{\partial}{\partial h}(\alpha_1 h(x) + \alpha_2 h^3(x) + \cdots + \alpha_n h^{2n-1}(x)) \\ &= \alpha_1 + 3\alpha_2 h^2(x) + \cdots + (2n-1)\alpha_n h^{2n-2}(x) \end{aligned} \tag{6}$$

With non-negative parameters $\alpha_1, \ldots, \alpha_n$ and the even-powered power functions $h^2, \ldots, h^{2n-2}$, it is straightforward that $\frac{\partial \kappa}{\partial h} \geq 0$ for $\forall h \geq 0$ and $\frac{\partial \kappa}{\partial h} > 0$ for $\forall h > 0$, indicating that $\kappa(\cdot)$ is a strictly increasing function.

Passes the origin: Substituting $h(x) = 0$ into the polynomial function $\alpha H(x)$, we have

$$H(x) = \begin{bmatrix} h(x) \\ h^3(x) \\ h^5(x) \\ \vdots \\ h^{2n-1}(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \implies \kappa(0) = 0 \cdot \alpha = 0 \tag{7}$$

Therefore, the constructed polynomial function $\kappa(h(x)) = \alpha H(x)$ is proved to be a class $\mathcal{K}$ function. □

Hence, the admissible control space (1) can be redefined as

$$\mathcal{C}(x) = \{ u \in \mathcal{U} : \dot{h}(x, u) + \alpha H(x) \geq 0 \} \tag{8}$$

and with our $\kappa(\cdot)$ being a class $\mathcal{K}$ function, it is proved in Ames et al. [2019] that any controller $u \in \mathcal{C}(x)$ will render the safe state set $\mathcal{H}$ forward-invariant using the comparison lemma, i.e., if the system (2) starts inside the set $\mathcal{H}$ with $x(t = 0) \in \mathcal{H}$, then it implies $x(t) \in \mathcal{H}$ for all $t > 0$ under controller $u \in \mathcal{C}(x)$. Now we conclude the properties proof of Parametric-CBF.

A.3 Parametric-CBF-based Adaptive Safe Merging Control Algorithm

Algorithm 1 Safe Adaptive Merging Algorithm using Parametric-CBF

```
Require: Δx_ij, Δx_jk, Δv_ij, Δv_jk, Δt, R_safe
Ensure: ᾱ_j, u_i
  for t = 1 : m do
    if ᾱ_j not converged then
      h^t_jk(x) = ||x^t_j − x^t_j||^2 − R^2_safe ≥ 0
      inverse calculation of ḣ^t_jk
      add {ḣ^t_jk, h^t_jk} to dataset 𝔻
      learn Parametric-CBF parameter α_j from 𝔻 (3.3) and obtain the estimate ᾱ_j
    end if
  end for
  choose the appropriate α_i based on ᾱ_j
  for t = m : N do
    compute safety constraint parameter A^t_ij, b^t_ij
    u^t_i = arg min_{u_i} ||u_i − ū_i||^2 with constraints in (3.2)
  end for
```

i, j, k are the indexes of the ego vehicle, the prediction object, and the interactive surrounding vehicle of the prediction object. The two for loops correspond to the time period of the prediction task and the control task respectively. It is assumed that the observation period is long enough to achieve a converged prediction result. once the prediction of α_j is achieved, the appropriate α_i of the ego vehicle will be chosen. The general rule is to resolve potential conflicts in both parities' driving strategy to avoid deadlock situation. If the prediction object is using a very aggressive driving style, the ego vehicle is set to be relative conservative, and vice versa. In the second loop, the safety constraint parameter A^t_{ij} and b^t_{ij} are state-depedent variables, referring to the Parametric-CBF constraint in (Sec. 3.2) which can be reorganized and written in the form of $A_{ij} u_i \leq b_{ij}$. By integrating Parametric-CBF based prediction and control together, we are able to achieve an adaptive merging control while accounting for safety.

A.4 Evaluation of Parametric-CBF

Richer Descriptive Information The biggest advantage of Parametric-CBF is that, compared to traditional CBF, Parametric-CBF can describe more diverse autonomous system behavior patterns. With the polynomial function $\kappa(h(x)) = \alpha H(x)$, Parametric-CBF can capture the system behavior characteristics to differing degrees. It especially benefits from the design of the linear combination

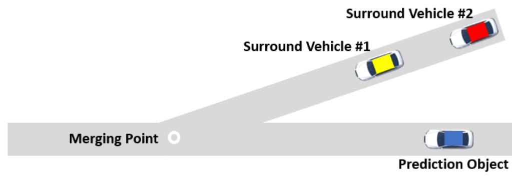

Figure 2: Prediction Scenario.

of parameter vector α and high-order safety measurement vector $H(x)$, which re-formulate the nonlinear mapping relationship to be linear by changing the basis $h(x)$ in traditional CBF to $H(x)$.

Broader Applications As introduced in the earlier section, Parametric-CBF can be simplified as $\dot{h}(x) = \alpha H(x)$, which makes it more generally applicable to various kinds of application scenarios, instead of limited to control tasks. One example is using machine learning approaches, e.g., ridge linear regression, to learn and predict the behavior of an autonomous system using a Parametric-CBF-based controller. More detailed examples will be provided in the following simulation section. Benefiting from this feature, the application of Parametric-CBF can be extended to different domains and has a broader impact.

A.5 Simulation Details

Prediction Scenario Illustration The blue car on the main road is the prediction object whose driving style needs to be estimated. The yellow car and the red car are the surrounding vehicles on the ramp. All three vehicles are using Parametric-CBF-based controllers and are interacting with each other.

A.5.1 Prediction of Driving Style

Different from the symmetric setting where all robots share the same CBF parameters as in Borrmann et al. [2015], in the context of heterogeneous vehicles, vehicles do not have the same responsive behavior to potential collisions. Therefore, it is important to figure out how much responsibility the other vehicle would take for avoiding collision during interaction, which calls for the need of driving style prediction.

In this example, we choose the basis $H(x) = \begin{bmatrix} h(x) & h^3(x) & h^5(x) \end{bmatrix}^T$. The ground truth parameter α used by the prediction object is $\begin{bmatrix} 10^{-3} & 10^{-4} & 0 \end{bmatrix}$. It is observed that the prediction results converge over time as more data points are collected. The learned parameter vector α reaches convergence in only 10 time steps, which is 0.1s, indicating it is computationally efficient enough to be applied in real-time applications.

As mentioned, one of the advantages of Parametric-CBF is its general formulation, which includes the aforementioned particular choices of $\kappa(h(x))$ used in other papers. To verify the generality, 30 tests trials with randomly generated Parametric-CBF in different forms are tested on the same prediction task, where three categories of different $\kappa(h(x))$ are used as the ground truth choices of the prediction object. 30 trials include 5 trials with $\kappa(h(x))$ in the form of $\lambda h(x)$ Ames et al. [2019], 10 trials with $\kappa(h(x))$ in the form of $h^3(x)$ Wang et al. [2016], and 15 trials with $\kappa(h(x))$ in the form of $\alpha H(x) = \alpha_1 h(x) + \alpha_2 h^3(x)$. All the trials return converged prediction to ground truth value with an average root mean square error of 6.32×10^6. It is observed that for the special cases Ames et al. [2019], Wang et al. [2016] of the proposed Parametric-CBF with a single term of the component function, the Parametric-CBF-based prediction framework does not lose its generality by achieving a consistent prediction performance as it does on the trials of the third category (real Parametric-CBFs).

A.5.2 Adaptive Behavior in Control

In this example, the purpose is to demonstrate the advantage of Parametric-CBF over traditional CBF in terms of behavior description richness in the safe control task. A two-vehicle interactive driving scenario is considered here, where both the ego vehicle on the main road and the other vehicle on the ramp use Parametric-CBF based safe controllers. The other vehicle's parameter α remains the same, meaning the driving style is kept the same in all the trials, and the construction of the ego vehicle's Parametric-CBF is varied.

First, the performance of traditional CBF with different choices of $\kappa(h(x))$ is compared in Fig. (1(c)). In the top plot, different α-values are chosen, and it is observed that the larger the parameter α, the more aggressive the driving strategy. Note that the two different kinds of curvatures correspond to two merging results: merging in front of the other vehicle or merging behind it. In the bottom plot, different orders of Class $\mathcal{K}$ functions are chosen, and it is observed that the main difference lies in their tolerance to the change of the safety function $h_{i.j}(x, u)$. The higher the order of class $\mathcal{K}$, the more it can tolerate movement towards the boundary of the safe set, and therefore the more aggressive its driving strategy.

Compared to traditional CBF, Parametric-CBF demonstrates its advantage of richer behavior description, as shown in Fig. (1(d)). The purpose of this experiment is to reveal the effect of changing the relative weighting of the corresponding components on the generated behavior. Here we consider $H(x) = \begin{bmatrix} h(x) & h^3(x) \end{bmatrix}^T$. The green line has the largest relative weight (close to 1) on $h(x)$ and the smallest relative weight (close to 0) on $h^3(x)$. The achieved behavior is similar to $\alpha = 1$ (the blue line on the top) in Fig. (1(c)). As we increase the relative weight of the higher-order components, and decrease the relative weight of the first-order component, it becomes easy for the ego vehicle to merge in front of the other car, as it does in Fig. (1(c)) (top plot) when large α is applied. At the same time, it outperforms the choice of using a single Class K function component (bottom plot) by achieving clearly diverse behavior, instead of the minor difference shown in the enlarged view in Fig. (1(c)).

A.5.3 Adaptive Behavior Improvement using Prediction

The goal of this example is to show how the Parametric-CBF-based prediction can contribute towards more efficient safe control. The scenario we consider here is: the ego vehicle is on the main road and the merging vehicle is on the ramp. Before these two vehicles perform interactive driving, the ego vehicle gets the chance to observe the merging vehicle's behavior while it is interacting with another car. The comparison result is shown in Fig. (1(b)). The green zone indicates the time interval when the ego vehicle and the merging vehicle are negotiating, meaning their Parametric-CBF-based safety constraints are active, and the outputs of the nominal controllers are modified.

It is observed that when the ego vehicle initially started with no prediction of the merging vehicle, for safety's sake, the ego vehicle sets its Parametric-CBF α to be small, trying to be conservative. The different curvatures indicate that the ego vehicle merges behind the merging vehicle when it does not have any prediction knowledge.

Things change when the ego vehicle chooses to observe the interaction between the merging vehicle and another vehicle, and makes its prediction on the merging vehicle's driving style α. This time the ego vehicle merges in front of the merging vehicle when it leverages its prediction result and adjusts its own parameter vector α.

Comparing the time steps when the two vehicles complete the merging task, it is observed that with the involvement of prediction, the ego vehicle spent 39.6% less time to pass the merging point, and the merging vehicle also benefits from the interaction with 10.1% reduced time to pass the merging point. If we mark the time step when the last vehicle passes the merging point as the time completing the merging task, it is observed that by leveraging prediction results, the overall task completion time is further reduced 16.1%. Therefore, it is obvious that Parametric-CBF-based prediction greatly contributes to improving merging task efficiency and reduce the road congestion.